6. On an improvement of Wien's equation for the spectrum[1]

by M. Planck

(read at the meeting of 19 October 1900)
(cf. above p. 181)[1a]

The interesting results of long wave length spectral energy measurements which were communicated by Mr. Kurlbaum at today's meeting,[2] and which were obtained by him and Mr. Rubens, confirm the statement by Mr. Lummer and Mr. Pringsheim, which was based on their observations that Wien's energy distribution law is not as generally valid as many have supposed up to now,[3] but that this law at most has the character of a limiting case,[4] the exceedingly simple form of which was due only to a restriction to short wave lengths and low temperatures.* Since I myself even in this Society have expressed the opinion that Wien's law must be necessarily true,[5] I may perhaps be permitted to explain briefly the relationship between the electromagnetic radiation theory developed by me and the experimental data.

The energy distribution law is according to this theory determined as soon as the entropy S of a linear[6] resonator which interacts with the radiation is known as a function of its vibrational energy[7] U. I have, however, already in my last paper on this subject† stated that the law of increase of entropy is by itself not yet sufficient to determine this function completely;[8] my view that Wien's law would be of general validity, was brought about rather by special considerations, namely by the evaluation of an infinitesimal increase of the entropy of a system of n identical resonators in a stationary radiation field by two different methods which led to the equation‡

$$dU_n \,.\, \Delta U_n \,.\, f(U_n) = n\, dU \,.\, \Delta U \,.\, f(U),$$

where $\quad U_n = nU \quad$ and $\quad f(U) = -\dfrac{3}{5}\dfrac{d^2S}{dU^2}.$

* Mr. Paschen has written to me that he also has recently found appreciable deviations from Wien's law.

† M. Planck, *Ann. Phys.* **1** [=**306**], 730 (1900).

‡ l.c.p. 732.

From this equation Wien's law follows in the form[9]

$$\frac{d^2S}{dU^2}=\frac{\text{const}}{U}.$$

The expression on the right-hand side of this functional equation is certainly the above-mentioned change in entropy since n identical processes occur independently, the entropy changes of which must simply add up. However, I could consider the possibility, even if it would not be easily understandable and in any case would still be difficult to prove, that the expression on the left-hand side would not have the general meaning which I attributed to it earlier, in other words: that the values of U_n, dU_n and ΔU_n are not by themselves sufficient to determine the change of entropy under consideration, but that U itself must also be known for this.[10] Following this suggestion I have finally started to construct completely arbitrary expressions for the entropy which although they are more complicated than Wien's expression still seem to satisfy just as completely all requirements of the thermodynamic and electromagnetic theory.

I was especially attracted by one of the expressions thus constructed which is nearly as simple as Wien's expression[11] and which would deserve to be investigated since Wien's expression is not sufficient to cover all observations. We get this expression by putting*

$$\frac{d^2S}{dU^2}=\frac{\alpha}{U(\beta+U)}.$$ [12]

It is by far the simplest of all expressions which lead to S as a logarithmic function of U—which is suggested from probability considerations[14]—and which moreover for small values of U reduces to Wien's expression mentioned above. Using the relation

$$\frac{dS}{dU}=\frac{1}{T}$$

and Wien's "displacement" law† one gets a radiation formula with two constants:[15, 16]

* I use the second derivative of S with respect to U since this quantity has a simple physical meaning[13] (l.c.p. 731).

† The expression of Wien's displacement law is simply[19] $S=f(U/\nu)$, where ν is the frequency of the resonator, as I shall show elsewhere.

$$E=\frac{C\lambda^{-5}}{e^{c/\lambda T}-1},$$

which, as far as I can see at the moment, fits the observational data, published up to now, as satisfactorily as the best equations put forward for the spectrum, namely those of Thiesen,*[17] Lummer–Jahnke,† and Lummer–Pringsheim.‡ (This was demonstrated by some numerical examples.[18]) I should therefore be permitted to draw your attention to this new formula which I consider to be the simplest possible, apart from Wien's expression, from the point of view of the electromagnetic theory of radiation.

* M. Thiesen, *Verh. Deutsch. Phys. Ges.* **2**, 67 (1900).

One can see there that Mr. Thiesen had put forward his formula before Mr. Lummer and Mr. Pringsheim had extended their measurements to longer wave lengths. I emphasise this point as I have made a somewhat different statement before this paper was published. (M. Planck, *Ann. Phys.* **1** [=**306**], 719 (1900).)

† O. Lummer and E. Jahnke, *Ann. Phys.* **3** [=**308**], 288 (1900).

‡ O. Lummer and E. Pringsheim, *Verh. Deutsch. Phys. Ges.* **2**, 174 (1900).