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THE ARROW OF TIME

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ABSTRACT

Four distinct classes of physical processes define a preferred direction in time: entropy-generating processes, which define the thermodynamic arrow; information-generating processes, which define the historical arrow; the cosmic expansion, which defines the cosmological arrow; and the decay of neutral kaons, which defines the microscopic arrow. The theory presented here shows that the thermodynamic, historical, and cosmological arrows may be derived from a pair of closely related cosmological postulates. The first postulate, a strong version of the familiar cosmological principle, states that no statistical property of the Universe serves to define a preferred position or direction in space. This postulate is shown to imply that microscopic information about the state of the Universe is objectively absent in the sense that two descriptions that agree in all statistical properties are operationally indistinguishable. The second cosmological postulate states that local thermodynamic equilibrium prevails near the cosmological singularity. This postulate is justified by the behavior of microscopic thermalization rates as $t \to 0$.

Expansion from the singularity is shown to generate macroscopic information as well as entropy; the widely held view that the Universe is running down rests upon a mistaken assumption concerning the relationship between information and entropy. When self-gravitating systems separate out, their initial states are characterized by the presence of macroscopic information (defined as information generated by cosmogonic processes) and the absence of microscopic information. These properties also characterize the initial states of subsystems of self-gravitating systems. The precise definition of macroscopic information for any given physical system depends on its history (i.e., on how the system was "prepared"). These conclusions link up in a natural way with modern statistical theories of irreversibility like those of van Hove, Bogoliubov, Prigogine and Balescu, and others, which have established that, for certain well-defined classes of systems, the coarse-grained entropy is nondecreasing with time when specific kinds of microscopic information are absent in the initial state.

The objective absence of microscopic information represents a new kind of indeterminacy, distinct from quantal indeterminacy. In any finite system this indeterminacy may be reduced through the expenditure of free energy, but it cannot be reduced in the Universe as a whole. The growth of macroscopic information in the Universe therefore implies that the future is, in principle, not wholly predictable.

Subject heading: cosmology

I. THE THERMODYNAMIC ARROW

a) *The Paradox of Irreversibility*

The second law of thermodynamics defines a preferred direction in time; yet it applies to physical systems whose detailed microscopic behavior is governed by equations that are invariant under time reversal. Moreover, a well-known theorem due to Poincaré states that in a closed many-particle system occupying a finite region of phase space, any given initial state is bound to recur, to any specified degree of accuracy, infinitely often. Thus a gas initially confined to one corner of a box and then released must eventually make its way back into the corner. Poincaré's theorem is important because it shows that solutions of the governing dynamical equations (as well as the equations themselves) are time-reversible, in the sense that for every set of initial conditions there exists an arbitrarily closely matching set of final conditions. How can this kind of microscopic reversibility be reconciled with the macroscopic irreversibility demanded by the second law?

The classical answer, given by Boltzmann and Gibbs, contains two distinct elements, illustrated by the following example. Consider a time-reversed film of the diffusion experiment mentioned in the last paragraph. In the reversed film, molecules initially occupying a large volume V spontaneously reassemble in a much smaller volume v. Since the governing equations are invariant under time-reversal, the reversed film makes dynamical sense; it is only the initial state of the gas that does not make sense, because its *a priori* probability is exceedingly low—about $(v/V)^N$, where N is the number of molecules. Under typical macroscopic conditions this number is practically indistinguishable from zero, so it is safe to assume that the putative initial state is actually a final state and that the film has been reversed. Yet the true initial state (in which

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the molecules are confined to a small volume v) has precisely the same low probability as the initial state in the reversed film. (This fact, a consequence of Liouville's theorem, was used to calculate the probability of the initial state in the reversed film.) Hence the *quantity* of information needed to specify the initial state is the same as that needed to specify the final state. But the *quality* of the information is different in the two cases: the information needed to specify the initial state is macroscopic (each molecule lies in a macroscopic region of volume v); that needed to specify the final state is microscopic (the velocity of each molecule must lie in a certain narrow range, depending on its position). This distinction between the microscopic and macroscopic levels of description is one of the two essential elements in the Boltzmann-Gibbs account of irreversibility. It corresponds, in Gibbs's terminology, to the distinction fine-grained/coarse-grained.

The second essential element is a statistical assumption about the initial states of macroscopic systems. In our example this assumption says that all possible microstates consistent with a given set of macroscopic properties (for example, prescribed values of the temperature and pressure) are equally probable. It was this assumption that led us to question the provenance of the initial state in the reversed film. More generally (but less precisely), the assumption may be formulated in the following terms: "Naturally occurring initial states of macroscopic systems are deficient in microinformation." Examples of microinformation are many-particle correlations in classical gases and the relative phases of state vectors in quantal systems.

b) *Formal Derivations of Irreversible Statistical Theories*

The transition from a reversible microscopic description of a many-particle system to an irreversible description involves the following three key steps.

i) *Introduction of Microuncertainty*

Each microscopic state k is assigned a probability p_k. This step is obviously necessary (a closed system initially in a definite microstate remains in that state indefinitely), but it is not sufficient, for Liouville's theorem tells us that the probabilities p_k do not change with time. The microentropy,

$$H = -\sum_k p_k \log p_k\,, \tag{1}$$

also does not change with time.

ii) *Introduction of Macrouncertainty*

This is the step that Gibbs called coarse-graining. Each coarse-grained (or macro-) state α is a collection of fine-grained (or micro-) states i, and every microstate i belongs to one and only one macrostate α. The probabilities p_α are then given by

$$p_\alpha = \sum_{i\in\alpha} p_i\,, \tag{2}$$

and the coarse-grained (or macro-) entropy is given by

$$\bar{H} = -\sum_\alpha p_\alpha \log p_\alpha\,. \tag{3}$$

It follows from equations (1)–(3) that

$$H = \bar{H} + H'\,, \tag{4}$$

where

$$H' = \sum_\alpha p_\alpha\Big(-\sum_i p_{i|\alpha} \log p_{i|\alpha}\Big) \equiv \sum_\alpha p_\alpha H_\alpha\,, \tag{5}$$

the conditional probability $p_{i|\alpha}$ being given by

$$p_i = p_{i|\alpha} p_\alpha\,. \tag{6}$$

Equation (4) expresses the important property of decomposability: the total entropy is the sum of the entropies associated respectively with the macroscopic and microscopic levels of the statistical description. As mentioned above, the total entropy H stays constant, so that if $\bar{H}$ changes with time, H' must change in a compensating manner: entropy must flow from macroscopic to microscopic degrees of freedom or vice versa.

iii) *Introduction of Irreversibility*

The original deterministic description has now been replaced by a two-level statistical description, but the description remains invariant under time-reversal. An additional assumption is needed to make it irreversible. Many such assumptions have been suggested, nearly all of which fall into two broad categories that I shall label "strong" and "weak."

Strong assumptions are exemplified by the following rule relating the macroprobabilities p_α evaluated at time $t + \Delta t$ to their values at time t (see, for example, Wu 1966):

$$p_\alpha(t + \Delta t) = \sum_\beta A_{\alpha\beta} p_\beta(t)\,, \tag{7}$$

where the coefficients $A_{\alpha\beta}$ satisfy the conditions

$$A_{\alpha\beta} \geq 0\,, \qquad \sum_\alpha A_{\alpha\beta} = 1, \qquad \sum_\beta A_{\alpha\beta} = 1\,. \tag{8}$$

From the fact that $x \log x$ is a continuous, convex function and from equations (3), (7), and (8), it follows that

$$\bar{H}(t + \Delta t) \geq \bar{H}(t)\,. \tag{9}$$

Thus if conditions (7) and (8) are valid for a given initial value of t and *all* positive values of Δt, the macroentropy $\bar{H}$ is a nondecreasing function of Δt. With the help of (8), we can rewrite equation (7) in the form

$$\frac{dp_\alpha}{dt} = \sum_\beta (a_{\alpha\beta} p_\beta - a_{\beta\alpha} p_\alpha)\,, \tag{10}$$

where $a_{\alpha\beta}$ represents a (time-dependent) transition probability.

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If the matrix A is nonsingular, we may invert equation (7):

$$p_\alpha(t) = \sum_\beta (A^{-1})_{\alpha\beta} p_\beta(t + \Delta t), \qquad (11)$$

but the matrix elements of A^{-1} do not satisfy the conditions (8). Thus equation (7) with the conditions (8) explicitly introduces a preferred direction in time.

A general quantal derivation of equation (10) was given by Pauli (1928), based on the assumption that the off-diagonal elements of the density matrix vanish permanently. Van Hove (1955) pointed out that this condition (the random phase approximation) is too strong: it holds only for systems in thermodynamic equilibrium. This conclusion follows directly from our earlier observation that the total entropy $H = \bar{H} + H'$ of a closed system stays constant in time. If $\bar{H}$ increases, H' must decrease; that is, the microscopic information of the system must increase. But off-diagonal elements of the density matrix express microscopic information, hence they cannot vanish permanently. An exactly analogous criticism applies to Boltzmann's kinetic equation, which has the form of equation (10). The physical assumption underlying this equation (the *Stosszahlansatz*) implies that two-particle correlations are permanently absent, which is impossible for a closed system that is not in thermodynamic equilibrium.

The question now arises whether irreversibility can be established for isolated systems that satisfy substantially weaker irreversibility postulates than those of Boltzmann and Pauli. An explicit answer was given by van Hove (1955), who succeeded in demonstrating that the coarse-grained entropy of quantal systems satisfying the random-phase approximation (and certain other conditions) at some "initial" instant cannot "subsequently" decrease.[1] Bogoliubov (1946) had earlier demonstrated irreversibility for a classical dilute gas under the assumption that many-particle correlations vanish asymptotically in the infinite past ($t \rightarrow -\infty$). Subsequent studies (e.g., Kac 1956; Prigogine and Balescu 1959, 1960; Kohn and Luttinger 1957, 1958) have established irreversibility theorems for other kinds of systems and other kinds of initial conditions.

c) *Unresolved Questions*

Two questions now arise. (1) What is the origin of the initial conditions postulated by current theories of irreversible behavior in many-particle systems? (2) What is the objective significance of these initial conditions? Can microscopic information about the state of a macroscopic system be said to be objectively absent if that information can in principle be obtained?

[1] The quotation marks in this sentence are intended to remind the reader that the theorem itself defines the direction of time for any system to which it applies. Statistical theories of isolated systems cannot explain why the thermodynamic arrow has the same direction in different isolated systems.

II. PREVIOUS ATTEMPTS TO EXPLAIN THE ORIGIN OF THE THERMODYNAMIC ARROW

a) *Random-Perturbation Theories*

The concept of a closed macroscopic system is, of course, an idealization; no finite system can be perfectly insulated from interaction with the rest of the Universe. Many authors, of whom Borel (1912) was perhaps the first, have seen in this circumstance the key to an understanding of the thermodynamic arrow; see Bergmann and Lebowitz (1955), Blatt (1959), Morrison (1966). The argument runs as follows. Suppose we had a box with perfectly reflecting, perfectly insulating walls, and suppose it was technically feasible to reverse instantaneously the velocities of all the molecules within it. Even then we would not be able to create an initial state leading to a significant decrease in entropy, for gravitational interactions between the gas-molecules and distant matter would quickly destroy the microscopic correlations needed to sustain "anti-entropic" macroscopic behavior. In general, interactions between the particles composing a nominally closed system and the outside world may be thought of as contributing a small random component to the Hamiltonian of the system. Microscopic information then flows out of the system as fast as it is generated by the decay of macroscopic information. The objection against "strong" irreversibility postulates raised in § I*b* now becomes irrelevant.

The difficulty with this argument lies in the concept of a random perturbation. What is intended by the argument is clear enough: perturbations are random insofar as they do not conspire to produce anti-entropic behavior. But what grounds are there for assuming that perturbations of finite systems by the rest of the Universe do not in fact conspire to produce anti-entropic behavior? If the Universe as a whole admits a complete microscopic description, there is no obvious reason why the interactions we have chosen to classify as perturbations should not on occasion give rise to anti-entropic episodes in nominally closed systems—or even why such episodes should not be as common as entropic episodes. The only reason for assuming that anti-entropic episodes hardly ever occur seems to be the Second Law itself.

b) *Irreversibility and the Cosmic Expansion*

It has been suggested by Gold (1958, 1962) that the thermodynamic arrow is a consequence of the cosmic expansion. Gold's argument runs as follows. (1) If the Universe were in thermodynamic equilibrium, there would be no preferred time-direction; hence the fact that the Universe is not in thermodynamic equilibrium is crucial to the explanation of time asymmetry. (2) The Universe as now constituted is an almost perfect sink for radiation. (3) The last fact makes it possible for nonequilibrium states of finite systems to come into being and persist. (4) The Universe is an almost perfect sink for radiation because it is expanding. (5) Hence the time-direction in which entropy increases must coincide with that in which the Universe expands.

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Before examining individual steps in this argument, let us recall the questions formulated at the end of § I: What is the origin of the initial conditions postulated by current theories of irreversible behavior in many-particle systems? And what is the objective significance of these initial conditions? We have seen that current theories postulate that microscopic information is absent and macroscopic information is present at an "initial" instant.[2] When these postulates are suitably formulated for well-defined classes of physical systems, they have been shown to imply that the coarse-grained entropy is a nondecreasing function of the elapsed time. The argument outlined above does not address the questions of what distinguishes microscopic from macroscopic information and why the latter but not the former is "initially" present in natural systems. What it does seek to elucidate are (*a*) the presence of macroscopic information in natural systems (i.e., the observation that natural systems are not in thermodynamic equilibrium), and (*b*) the observation that all natural systems exhibit the *same* thermodynamic arrow.

Suppose, for the sake of the argument, we accept Gold's contention that "without the radiation sink that the universe provides, sub-units would have reached thermodynamic equilibrium" (Gold 1974). This could explain why natural systems have persisted in nonequilibrium states, but it does not explain how they got there in the first place. The most obvious (and perhaps most fundamental) feature of the processes through which natural systems have come into being is that they have acted to concentrate mass. There is no compelling theoretical reason for believing that the transparency of the Universe is responsible for mass-concentration (Layzer 1971*b*, 1976).

Finally, it is not correct that the Universe is a sink for radiation because it is expanding. As Harrison (1965*a*, *b*; see also Layzer 1966) has pointed out, the night-sky is dark not because the Universe is expanding but because the cosmic mass- (or energy-) density is small. In the absence of redshift observations, there would be no valid observational or physical reason for supposing that the Universe is expanding rather than contracting. Hence there is no reason to assume that the cosmic expansion is directly linked to the growth of entropy in isolated systems. And if the Universe is finite, there is no reason to question the naïve expectation that the present correlation between entropy generation and cosmic expansion will be reversed during the contraction phase.

c) *The Electromagnetic Arrow and Its Origin*

Consider a finite system of charged particles. According to classical electromagnetic theory, the field F acting on the ith particle is given by the equivalent formulae

$$F = \sum_{j \neq i} F_{\text{ret}}^{(j)} + \hat{F}^{(i)} + F_{\text{in}}\,, \qquad (2.1\text{a})$$

$$= \sum_{j \neq i} F_{\text{adv}}^{(j)} - \hat{F}^{(i)} + F_{\text{out}}\,, \qquad (2.1\text{b})$$

where $F_{\text{ret}}^{(j)}$ $F_{\text{adv}}^{(j)}$ are the retarded and advanced fields associated with the jth particle, F_{in} denotes the incident field, F_{out} the outgoing field, and $\hat{F}$ the radiation reaction, given by Dirac's (1938) formula

$$\hat{F}^{(i)} = \tfrac{1}{2}(F_{\text{ret}}^{(i)} - F_{\text{adv}}^{(i)})\,. \qquad (2.2)$$

Under time-reversal, equations (2.1a, b) are interchanged, so the description is time-symmetric. To bring out the symmetry we may write F in the form

$$F = \frac{1}{2} \sum_{j \neq i} (F_{\text{ret}}^{(j)} + F_{\text{adv}}^{(j)}) + \frac{1}{2}(F_{\text{in}} + F_{\text{out}}) \quad (2.1\text{c})$$

which follows from equations (2.1a, b).

At the microscopic level, the retarded and advanced descriptions are entirely equivalent, but at the macroscopic level a preferred time-direction emerges. Consider, for example, an antenna radiating into a cavity bounded by opaque walls. The conventional retarded description requires one to specify certain macroscopic properties of the antenna (its geometry and the distribution of electric currents) at some instant of time. Classical electromagnetic theory then enables one to complete the description, given appropriate macroscopic information about the cavity and the walls. A reversed film of this macroscopic absorption/emission process would show wavelets emitted by the wall-atoms combining to form waves converging on the antenna. A description of this process would necessarily require the specification of detailed microscopic information about the individual emitters and their relative phases. Thus the distinction initial/final hinges on the quality of the information needed to characterize the terminal states: macroscopic information is present in the initial state and absent in the final state; microscopic information is absent in the initial state and present in the final state. This explanation of the temporal asymmetry of macroscopic radiation processes is not merely analogous to but formally identical with the explanation of temporal asymmetry in thermodynamics. The qualitative distinction between the retarded and advanced descriptions of macroscopic radiation processes thus hinges on the thermodynamic properties of the matter with which the radiation interacts. The preceding argument was adumbrated by Einstein (1909).

Let us now turn our attention to the fields F_{in}, F_{out}, which are solutions of the homogeneous wave equation. Consider a finite system of charged particles in a region V bounded by a surface S. Call this region "the observable universe." For the sake of definiteness, let us use the retarded description. Then it is clear that we could in principle choose the incident radiation field F_{in} so as to make the electromagnetic arrow point in the opposite direction from the thermodynamic arrow. For example, we could devise incident fields that would excite oscillating currents in antennas. Thus the initial conditions responsible for the thermodynamic arrow are necessary but not sufficient to generate a consistent electromagnetic arrow. If we wish to recover conventional electromagnetic theory

[2] See note 1.

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from a description based on equation (2.1a), we must, as is well-known, impose a suitable condition on the incident field F_{in}. That is, we must introduce an assumption about the action of the unobservable universe on the observable universe. As discussed in § II*a*, such an assumption is also needed to derive the thermodynamic arrow: one must assume that perturbations by the unobservable universe do not conspire to produce anti-entropic behavior in observable systems.

The Sommerfeld radiation condition $F_{\text{in}} = 0$ eliminates by fiat all fields that do not arise from sources within the observable universe. The effects of such fields may in fact be negligible for most purposes. From a theoretical standpoint, however, the Sommerfeld condition is too strong. For example, it would rule out the cosmic microwave background, which (*a*) exists and (*b*) is compatible with the electromagnetic arrow.

Once it is recognized that cosmological considerations are relevant to both the thermodynamic and electromagnetic arrows, the conventional division of electromagnetic (and gravitational) fields into components associated with observed and unobserved regions is seen to be inappropriate. The very existence of the thermodynamic and electromagnetic arrows—the fact that thermodynamics and electromagnetic theory adequately describe macroscopic phenomena in the observable part of the Universe—places an important constraint on the "unobservable" part of the Universe. As in classical relativistic cosmology, we can avoid the troublesome and artificial question of boundary conditions by making appropriate symmetry postulates. The simplest postulate of this kind, discussed in greater detail in § III, is that of statistical spatial[3] homogeneity and isotropy. This symmetry postulate requires the fields that figure in equations (2.1a, b) to be statistically homogeneous and isotropic. Now, the retarded description (2.1a) requires that the positions and velocities of all charged particles, as well as the radiation field, be specified at some "initial" instant t_0. Then the fields F_{ret}, F_{adv}, and F_{in} can be evaluated everywhere at all "subsequent" times. If microscopic information is *not* needed to specify the initial state, then it *will* be needed to specify the initial state in a time-reversed description based on equation (2.1b).

The "standard model" of the early Universe includes a primordial radiation field assumed to be in thermodynamic equilibrium.[4] Thus microscopic information is not needed to specify F_{in}. In the (nonstandard) cold universe, $F_{\text{in}} \to 0$ as $t \to 0$.

d) *Arguments Based on the Absorber Theory of Radiation*

Suppose that in a given (bounded or unbounded) region of spacetime every photon has both an emitter and an absorber. Then the source-free fields F_{in} and F_{out} both vanish, and F is given by the time-symmetric formula

$$F = \frac{1}{2} \sum_{j \neq i} (F_{\text{ret}}{}^{(j)} + F_{\text{adv}}{}^{(j)})\,. \tag{2.3}$$

A theory that takes this formula as its starting point was developed by Wheeler and Feynman (1945). In this theory the electromagnetic field is not an independent physical entity but merely a convenient mathematical construct. An attractive feature of the theory is the nonoccurrence of electromagnetic self-energy.[5] For the purposes of the present discussion, we may regard the Wheeler-Feynman theory as a special case of the Maxwell-Dirac theory discussed in § II*c*. Two questions now arise. (1) Given the Wheeler-Feynman postulate, $F_{\text{in}} = F_{\text{out}} = 0$, what is the origin of the electromagnetic arrow? (2) Under what (cosmological) conditions is this description mathematically equivalent to the conventional one? The two questions are logically independent because the Maxwell-Dirac theory, no less than the Wheeler-Feynman theory, is inherently time-symmetric. Wheeler and Feynman emphasized in their discussion that the source of the electromagnetic arrow must be the same in both theories, namely, interaction between radiation and "entropic" matter. In the Wheeler-Feynman theory no additional restriction needs to be imposed on source-free fields because they never occur.

Wheeler and Feynman confined their discussion of the second question to the hypothetical case of a macroscopically uniform, infinite, nonexpanding universe. In such a universe every emitted photon is ultimately absorbed and every absorbed photon has an emitter; the model is opaque to electromagnetic radiation of all frequencies in both time-directions. Hogarth (1962), followed by Hoyle and Narlikar (1962) and Sciama (1963), discussed the Wheeler-Feynman theory in expanding world models. Open relativistic world models present an obvious difficulty: owing to the ever-decreasing matter density, an open Friedmann universe ultimately becomes transparent and hence contains emitted but nonabsorbed photons. It follows that the Wheeler-Feynman theory cannot reduce to conventional electrodynamics in an open Friedmann universe. In the steady-state universe, on the other hand, every emitted photon is ultimately absorbed, and one can formulate a description of radiation processes that satisfies the requirements of the Wheeler-Feynman theory.[6] Hogarth (1962) and

[3] As is well known, this symmetry postulate is itself the basis for a unique decomposition of spacetime into space and time.

[4] More precisely, the primordial radiation field is assumed to be asymptotically thermal in the limit $t \to 0$.

[5] Divergent contributions to the electron's self-energy arise from a term $\frac{1}{2}(F_{\text{ret}}{}^{(i)} + F_{\text{adv}}{}^{(i)})$ that has simply been omitted from equations (2.1a, b). In the Wheeler-Feynman theory such terms never arise.

[6] This is true only if one does not insist on including the cosmic microwave background in the description. The thermal character of this radiation field demands that the *visible* universe be opaque to photons of the appropriate frequencies at some stage in the history of the field. This condition cannot be met in a steady-state universe.

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Hoyle and Narlikar (1962) made the additional argument that the steady-state universe is partially transparent in the direction of the past, owing to the blueshifting of photons propagating backward in time (the forward time-direction being defined by the cosmic expansion). From this they concluded that the steady-state cosmology is compatible with a retarded description of radiation processes but not with an advanced description. They concluded that the electromagnetic arrow is determined not by thermodynamics but by cosmology.

This argument was criticized informally by Feynman, who remarked that the absorption cross section for γ-rays approaches a finite value with increasing energy. Hence the free path of a photon traveling backward in time in a steady-state universe is finite.

There is an even more fundamental difficulty with the Hogarth-Hoyle-Narlikar argument. For the sake of the argument, let us grant the assumption that the steady-state universe is partially transparent in the direction of the past. We have seen that in any case there is a description—the retarded description—in which every emitted photon is ultimately absorbed and every photon has an emitter. In the time-reversed (advanced) description, then, every absorbed photon is "ultimately" emitted and every photon has an absorber. This is merely a restatement of the condition that holds in the retarded description. In both descriptions every photon has both an absorber and an emitter; time-reversal merely interchanges the roles of absorber and emitter. (See the discussion by Gold 1967.) But how can we reconcile the conclusion that, in the advanced description, every photon has an absorber with the assumption that the model is partially transparent in the direction of the past? The answer is that, in the advanced description, photons are emitted in such a way that none of them slips through (to $t = -\infty$) although each has a finite probability of doing so. The advanced fields converge on the particles "destined" to absorb them (the particles that emit photons in the retarded description). The essential distinction, then, between the retarded and the advanced descriptions is that a vast quantity of microscopic information must be contained in the advanced description, whereas such information must *not* be present in the retarded description. (If microscopic information were allowed in the initial state of the retarded description, one could program the postulated matter-creation process to generate anti-entropic episodes like the spontaneous excitation of macroscopically organized electric currents in antennas. The nonoccurrence of such episodes is a consequence of the tacit assumption that the spontaneous creation of matter postulated in steady-state cosmology can be completely characterized by macroscopic parameters.) So once again we are led back to the conclusions reached at the end of § I: The time-asymmetry of macroscopic physics is a consequence of the initial conditions that characterize natural systems. The relevant aspects of these initial conditions are (*a*) the absence of microscopic information, (*b*) the presence of macroscopic information. A theory of macroscopic irreversibility must predict these initial conditions and must assign objective meanings to "absence (presence) of microscopic (macroscopic) information," and to the distinction microscopic/macroscopic. The theory outlined below[7] addresses these problems.

III. THE STRONG COSMOLOGICAL PRINCIPLE AND COSMIC INDETERMINACY

In a finite or bounded physical system microscopic information can always be specified in principle, however difficult it may be to obtain in practice. In this section, I shall argue that in an infinite universe with a sufficiently high degree of spatial symmetry, microscopic information is unobtainable *in principle*. This conclusion will be used later to argue that certain kinds of microscopic information are objectively absent in newly formed astronomical systems.

Consider a Poisson distribution of identical point-masses along an infinite straight line. This one-dimensional "universe" is characterized by a single macroscopic parameter, $\bar{n}$, the mean density of mass-points. Suppose we knew that the separation between a given pair of mass-points was precisely 1 cm. This information would distinguish (*a*) the given realization from any other realization of a Poisson distribution with the same statistical parameter, and (*b*) the given pair of mass-points from every other pair of mass-points. For the probability that the interval X between any pair of successive mass-points lies between x and $x + dx$ may be expressed in the form $\Pr\{x \leq X \leq x + dx\} = p(x)dx$, where $p(x)$ is a bounded function. Hence it is almost certain (i.e., it is true with probability 1) that no member of a countably infinite set of intervals assumes some specified real value.

The quantity of information needed to specify the precise separation between two mass-points is $\log_2 2^{\aleph_0} = \aleph_0$, where $\aleph_0$ is the cardinal number of the set of integers. Now, it is a consequence of basic quantal principles that only a finite quantity of information is needed to specify completely the state of a finite system. For example, to specify the state of a gas that occupies a finite volume of phase space, we need to specify the occupation numbers of a finite number of cells of volume h^3. To achieve a more realistic microscopic description of our one-dimensional model universe, we therefore divide the line into equal cells of length h and specify the occupation number of each cell. A microscopic description of a given realization is then specified by a countably infinite set of occupation numbers $\ldots n_{-2}n_{-1}n_0n_1n_2\ldots$. The statistical distribution of occupation numbers is a Poisson distribution with mean occupation $\bar{n}_h = \bar{n}h$. The Strong Law of Large Numbers implies that the average occupation number of a sample containing N cells converges with probability 1 to $\bar{n}_h$ as $N \to \infty$. Thus a single realization contains enough information to specify its defining statistical parameter with

[7] Earlier and less complete discussions are given in Layzer (1967, 1970, 1971*a*, 1974).

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arbitrary precision. (It also contains enough information to verify with arbitrary precision any other statistical property of the Poisson distribution.)

On the other hand, a single realization, specified by a countably infinite set of occupation numbers, contains no *microscopic* information. For such information would distinguish between different realizations having identical statistical properties. Suppose we are given two such realizations. Consider a sequence of occupation numbers of length N in the first realization. The same sequence occurs infinitely often in the second realization (since any finite sequence has a finite probability of occurrence). This is true for all values of N. But if the two realizations were distinguishable, there would exist some value of N for which the matching could not be carried out. Hence any two realizations that have the same statistical properties are microscopically indistinguishable.[8]

These considerations can easily be extended to statistically homogeneous but nonuniform discrete distributions of mass-points on a line. Such distributions may be characterized by a random function $\rho(x)$ representing a variable probability density (Layzer 1956, 1976). The statistical properties of this random function (for example, its mean, its variance, and its autocorrelation function), all of which are independent of x, constitute macroscopic information about the distribution. If the autocorrelation function $f_\rho(x) \to 0$ as $x \to \infty$, the distribution is ergodic, according to a well-known theorem of Birkhoff and Khinchin (see, e.g., Gnedenko 1962). The ergodicity of the distribution ensures that its statistical parameters can be approximated with arbitrary precision by spatial averages. Precisely analogous considerations apply to statistically homogeneous distributions in three-dimensional position space and six-dimensional phase space. We conclude that a complete statistical description of an infinite, unbounded, statistically homogeneous, ergodic distribution determines a unique set of occupation numbers.[9] Conversely, any realization of such a distribution, specified by a given set of occupation numbers, serves to define its macroscopic parameters completely.

The infinite statistical distributions that we have been discussing are microscopically indeterminate in an absolute, objective sense; that is, it is impossible, even in principle, to acquire information that would permit one to distinguish between realizations with the same statistical properties. The probabilities of the individual microstates k are defined by the condition $H = H_{\max}$, where H is given as a function of the probabilities p_k by equation (1) and $H_{\max}$ is to be calculated under the constraints contained in a complete statistical description of the distribution. Thus the condition $H = H_{\max}$ expresses the microscopic indeterminacy of the distribution. The ergodicity of the distribution allows probabilities to be identified with limits of spatial frequencies. For any finite subsystem S there is a definite probability p_k associated with each possible microstate S_k of S. If the system S is effectively closed, we can then argue, as in the classical discussions of Boltzmann and Gibbs, that microscopic states that lead to an apparent decrease of entropy are highly improbable (and hence occur very infrequently) if the number of degrees of freedom of the system is large. The present discussion adds to the classical one (*a*) a prescription for calculating the probabilities[10] p_k and (*b*) an objective interpretation of these probabilities. According to that interpretation, microscopic indeterminacy is both objective and irreducible.

I use "strong cosmological principle" to denote the assumption that no statistical property of the Universe defines a preferred position or direction in space. (The cosmological principle, as it is usually formulated, merely asserts that the Universe is uniform and isotropic apart from local irregularities.) According to the preceding discussion, an infinite universe that satisfies the strong cosmological principle and in which all correlation distances are finite admits a complete statistical description—a description that neither contains nor can be supplemented by microscopic information. (The assumption of statistical isotropy is not needed to establish the completeness of the statistical description, but plays an important part in the subsequent discussion.)

Strictly interpreted, the strong cosmological principle seems at first sight to be inconsistent with a closed universe, for any finite system has distinguishable microscopic states that correspond to the same statistical description. If, however, successive cycles are causally disjoint but have macroscopically identical (or statistically independent) initial states, then microscopic information about a given cycle no longer figures in a complete description. In these circumstances distinct cycles are analogous to the observable universes of observers so far apart that their event horizons do not intersect.

IV. THE ORIGIN OF MACROSCOPIC INFORMATION

It is widely believed that the second law of thermodynamics implies that the Universe was initially more highly ordered than it is now and that the order initially present is gradually being dissipated by irreversible processes. For example: "In the 'big bang' cosmology, the universe must start with a marked degree of thermodynamic disequilibrium and must eventually run down" (Hoyle and Narlikar 1967). I shall argue that, under certain conditions, the cosmic expansion generates information as well as entropy and that the second law does not require the initial state of the Universe to have been highly structured, or indeed to have had any structure at all.

An example will illustrate the general argument.

[8] This inference presupposes that the logic underlying our description is ω-consistent.

[9] The specification of the occupation numbers is understood to include a definite scheme for ordering them, as well as a specification of the cell size.

[10] This prescription was first proposed as a basis for statistical thermodynamics by Jaynes (1957*a*, *b*). Jaynes, however, advocated a subjective interpretation of entropy and of the probabilities that figure in its definition.

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Consider a uniform mixture of nonrelativistic gas and radiation, which at time t_0 is in thermodynamic equilibrium at temperature T_0. The internal-energy density $e = e_g + e_r$ and the pressure $p = p_g + p_r$, where the suffixes g and r refer to the gas and radiation, respectively, satisfy the cosmological energy equation

$$\frac{d[(e_g + e_r)V]}{dt} + (p_g + p_r)\frac{dV}{dt} = 0\,, \qquad (12)$$

where V, the volume of any co-expanding region, is proportional to the cube of the cosmic scale factor $a(t)$. As the mixture expands (or contracts) away from the initial state of thermodynamic equilibrium, it will not in general remain in equilibrium. For example, if the expansion (contraction) rate greatly exceeds the rate of thermalization, each component of the mixture will expand (contract) nearly adiabatically. Since the adiabatic exponents are different for the two components, a temperature difference develops between them. More generally, e_g and e_r satisfy the following pair of equations:

$$\frac{d(e_g V)}{dt} + p_g\frac{dV}{dt} = \frac{\delta Q}{\delta t}\,, \qquad (13a)$$

$$\frac{d(e_r V)}{dt} + p_r\frac{dV}{dt} = -\frac{\delta Q}{\delta t}\,. \qquad (13b)$$

Suppose for the sake of illustration that each component maintains an approximately thermal distribution, so that we may write

$$\frac{\delta Q}{\delta t} = T_g\frac{dS_g}{dt} = -T_r\frac{dS_r}{dt}\,, \qquad (14)$$

where S (the thermodynamic entropy) $\equiv k_B\bar{H}$ (k_B = Boltzmann's constant, $\bar{H}$ = the macroscopic entropy[11]). (This would be a more realistic assumption for a mixture of relativistic electrons and nonrelativistic protons.) Then the rate of change of S is given by

$$\frac{dS}{dt} = \left(\frac{1}{T_g} - \frac{1}{T_r}\right)\frac{\delta Q}{\delta t}\,. \qquad (15)$$

Thus the cosmic expansion (contraction) generates entropy unless the thermalization rate is much smaller or much larger than the expansion (contraction) rate. In the first case $\delta Q/\delta t = 0$; in the second, $T_g = T_r$.

Unless the thermalization rate greatly exceeds the expansion (contraction) rate, information is also generated. The information I is defined by

$$I = H_{\max} - H\,, \qquad (16)$$

where $H_{\max}$ denotes the maximum value of the entropy H subject to given constraints. The information present at any moment is thus equal to the entropy that would be generated if the mixture were to relax instantaneously to a state of thermodynamic equilibrium. Notice that the rates of entropy generation and information generation are not simply related. The rate of information generation assumes both its maximum and minimum values when the rate of entropy generation vanishes.

This example suggests the general rule that information is generated whenever the expansion (contraction) rate exceeds the rate of a local equilibrium-maintaining process.[12] Nucleogenesis during the early stages of the cosmic expansion is the most familiar and the most thoroughly investigated example of this process.

More generally, if the approach to equilibrium is mediated by two-particle interactions, then its rate is proportional to the mean density ρ. The rate at which the macroscopic parameters are changing is comparable in general to the expansion rate $H(t)$, which is proportional to $\rho^{1/2}$ during the early stages of the expansion. Thus the thermalization rate decreases faster with decreasing density than the expansion rate. If the expansion rate is initially smaller than the rate of some equilibrium-maintaining reaction, we may expect the two rates to become comparable at some later moment. The equilibrium concentrations that prevail at this moment will tend to persist through the subsequent expansion.

Analogous considerations apply to the formation of self-gravitating systems by gravitational clustering (Layzer 1971*b*, 1976). The gravitational-energy spectrum associates a binding energy $\epsilon(M)$ with fluctuations of mass M. Under certain conditions that need not be specified here, $\epsilon(M)$ remains constant as the Universe expands. The role of thermalizing interactions is here played by tidal interactions between neighboring fluctuations, whose strength varies as $\rho^{2/3}$. Eventually the self-energy of a condensation exceeds its interaction energy, and fluctuations of that mass begin to separate out. This theory, whose details are given in the references cited above, provides a concrete illustration of the growth of macroscopic structure and information in an expanding universe.[13]

[11] For any definition of macrostates (§ I*b*[ii]), maximizing the corresponding $\bar{H}$ under appropriate constraints yields a system of relations formally identical with classical thermodynamics. Under given experimental conditions, a particular grouping of microstates into macrostates and a particular choice of constraints will be appropriate, and other groupings and constraints will be inappropriate. For a clear discussion of this point and related considerations, see Jaynes (1965).

[12] These considerations are closely related to the following argument of Jaynes (1965). Suppose that a system in thermodynamic equilibrium at time t_0 undergoes an adiabatic change of state, then relaxes into a new state of thermodynamic equilibrium, which it reaches at time t_1. During the entire process, the total (Gibbs) entropy (eq. [1]) remains constant. Hence

$$k_B{}^{-1}S(t_0) \equiv H_{\max}(t_0) = H(t_0) = H(t_1) \leq H_{\max}(t_1) \equiv k_B{}^{-1}S(t_1)\,.$$

Thus the thermodynamic entropy is nondecreasing (under the stated conditions). The argument also shows that an irreversible adiabatic change of state from an initial state of thermodynamic equilibrium generates information: $H_{\max}(t_1) - H(t_1) > 0$.

[13] The gravitational potential energy discussed here is not to be confused with the quantity discussed by Tolman (1934) in a related context. Tolman argued that both the energy and entropy of an oscillating Friedmann universe would increase

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The present discussion of entropy generation in the cosmos supports the orthodox view that the second law of thermodynamics is truly universal; if all local macroscopic processes generate entropy, it follows from the cosmological principle that the total entropy of any region large enough for the inflow of entropy to balance the outflow to a high degree of approximation must continually increase. On the other hand, the present discussion of *information*-generation in expanding or contracting cosmic mass-distributions differs in essential respects from earlier discussions (known to me) of the same problem. Thus Davies (1974, p. 106) writes: "From the cosmological point of view it is most satisfactory that one may expect an accumulation of structure in the universe on thermodynamic grounds." Davies does not explicitly state the "thermodynamic grounds" in question. Indeed his discussion does not make clear why "the accumulation of structure in the universe" is to be "expected" at all. On the contrary, he remarks (Davies 1974, p. 107)—quite correctly—that "at the present stage of the [hot big-bang] theory it appears that appropriate amplitude perturbations must simply be postulated as initial conditions to explain the existence of galaxies."

Davies also appeals to the notion criticized in footnote 11, that the cosmic medium "possesses an infinite reservoir of negative entropy," which he says (Davies 1974, p. 108) "is significant in two separate ways. The first is the possibility of local departures from homogeneity due to gravitational growth of density perturbations.... The second... is that the cosmological fluid itself is partaking of the general cosmological expansion, and so finds itself in a changing gravitational field on the global scale. The increase of entropy through irreversible processes may be thought of as being 'paid for' by the gravitational field of the universe." Let us consider these points in turn. (1) An unbounded, statistically uniform thermal distribution of (idealized) gravitating particles is in no sense unstable against the growth of astronomically significant density fluctuations.[14] Thus there seem to be only two ways of trying to account for the locally nonuniform structure of the Universe: to assume that substantial nonthermal density-fluctuations are present initially; or to postulate that the initial state has zero temperature. It has been argued (Layzer 1976) that the internal energy per unit mass of an initially cold universe ultimately becomes negative, and that the cosmic medium then becomes unstable (in the strict thermodynamic sense) against the growth of density fluctuations. This instability depends on nongravitational forces, as well as on the cosmic expansion; it would not occur in a cosmic medium composed of idealized gravitating particles. It also depends on the postulated initial condition $T \to 0$. (2) Entropy-generation in a universe expanding from an initial state of local (or global) thermodynamic equilibrium takes place not because the Universe has "an infinite reservoir of negative entropy" and hence "unlimited capacity to increase its entropy" but because, as explained in the text, the cosmic expansion generates information. The essential points in this connection are (*a*) that information and negative entropy are not the same thing, and (*b*) that, because the cosmic expansion has a finite rate, it is nonadiabatic and hence generates departures from thermodynamic equilibrium.

from cycle to cycle owing to irreversible entropy-generation. He suggested that the increased thermal energy of the cosmic medium (gas and radiation) would be supplied by the "potential gravitational energy" of the uniform fluid. This argument is elaborated by Davies (1974, pp. 190–191). In fact, the concept of (specific) gravitational potential energy of a uniform cosmic fluid has never been defined in the context of Einstein's theory of gravitation. Einstein's energy-momentum pseudotensor, mentioned in Tolman's discussion, does not provide an acceptable definition of potential energy because it is not a true tensor. There is one well-known exception to this statement: For a finite system asymptotically embedded in Minkowskian spacetime, one can derive, from the conservation equation satisfied by the energy-momentum pseudotensor, a *Lorentz-invariant* conservation law for the total energy and momentum. Tolman's question, Where does the extra thermal energy come from? rests on a false premise: that the principle of energy conservation applies to a statistically homogeneous universe governed by Einstein's theory of gravitation. The gravitational potential energy discussed in the text (Layzer 1963, 1971*b*, 1976) is an extensive quantity defined for any statistically homogeneous mass-distribution having a finite density-autocorrelation scale.

V. THE INITIAL STATE OF THE UNIVERSE

We have seen that macroscopic information may be generated by cosmic expansion *or contraction* from an initial state of thermodynamic equilibrium. The preceding discussion of reaction rates shows that local thermodynamic equilibrium may be expected to prevail asymptotically in the limit $t \to 0$, $\rho \to \infty$, and only in that limit. *It follows that the temporal direction in which entropy and information are generated coincides—at least initially—with the temporal direction in which the Universe expands.* The reason is simply that the temporal direction *from* the state in which microscopic information is postulated to be absent—the singular state—necessarily coincides with the direction of cosmic expansion. From any other cosmic state there are two distinguishable temporal directions; from the singular state there is only one.

The assumption that the Universe is initially in a state of local thermodynamic equilibrium leaves open the question of macroscopic information in the initial state. Is the entropy per baryon finite or zero? Is the density field uniform or nonuniform? These questions cannot be settled by general considerations, but require a detailed cosmogonic theory that would describe the

[14] Lifshitz (1946) showed that the largest mass that could have condensed from thermal fluctuations present at the epoch when the mean cosmic density was comparable to nuclear density is of order 10^6 g. Layzer (1976) has derived an upper limit of $10^{-22}\ M_{\odot}$ for the mass of gravitationally coherent thermal density-fluctuations. One cannot, however, rule out the possibility that an unbounded, statistically uniform, thermal distribution of gravitating masses is *absolutely* stable against the growth of density fluctuations; the upper bound just mentioned comes from a consideration of *necessary* conditions for the growth of thermal fluctuations.

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transformation of initial macroscopic information and the generation of new macroscopic information in the course of the cosmic expansion. Such a theory would describe the genesis of astronomical systems and would supply the information needed to characterize their initial states. It would thus provide a detailed answer to the first of the two questions raised at the end of § I: What is the origin of the initial conditions postulated by current theories of irreversible behavior in many-particle systems? The second question, What is the objective significance of these initial conditions? was answered in § III.

VI. CONCLUSIONS

Four classes of physical processes distinguish between the directions of the past and the future: (1) entropy-generating processes at the macroscopic and cosmological levels; (2) information-generating processes in open macroscopic systems and in the Universe as a whole; (3) the cosmic expansion; and (4) the decay of neutral kaons. Each set of processes may be said to define its own arrow: the thermodynamic arrow, the historical arrow, the cosmological arrow, and the microscopic arrow, respectively. Only the first three of these arrows have figured in the preceding discussion.[15] The most widely accepted interpretation of thermodynamic irreversibility attributes it to certain properties of the initial states of naturally occurring systems.[16] For specific classes of many-particle systems it has been shown (by van Hove, Bogoliubov, and others) that the macroscopic entropy is nondecreasing with time if microscopic information is absent in the initial state. The precise definition of "microscopic information" (which also yields the definition of macroscopic entropy) depends on the system under consideration. The present theory supplements this explanation of thermodynamic irreversibility by supplying objective interpretations for microscopic indeterminacy and macroscopic information. Microscopic indeterminacy has been shown to be an objective and irreducible feature of a universe satisfying the strong cosmological principle. This kind of indeterminacy depends on basic quantal principles but is distinct from quantal indeterminacy. Unlike quantal indeterminacy, which is absolutely irreducible, the indeterminacy resulting from the strong cosmological principle is irreducible only in the Universe as a whole; in finite subsystems it can be reduced through the expenditure of free energy.

The initial conditions for nominally closed finite subsystems of the Universe are connected by a cosmogonic theory to initial conditions for the Universe as a whole. For physical reasons, it is natural to consider the singular state of a Friedmann universe as the initial state and to assume that local thermodynamic equilibrium prevails in its neighborhood. This assumption establishes the initial distinction between microscopic and macroscopic information. Thenceforth, cosmogonic processes generate only macroscopic information (by definition). It has been shown that, with this initial condition, the cosmic expansion generates both entropy and information. Since a Friedmann universe necessarily expands from the singular state, the arrows defined by the generation of entropy and of information coincide initially with the arrow defined by the cosmic expansion. In a closed Friedmann universe, however, entropy continues to increase during the contraction phase, since the thermodynamic and cosmological arrows are linked only by an initial condition.

The full set of initial conditions for the Universe—and hence the cosmogonic theory linking these initial conditions to the initial conditions of astronomical systems—is still a matter of conjecture. The standard model of the early Universe postulates a finite value of the entropy per baryon, as well as a spectrum of primordial density fluctuations (Novikov and Zel'dovich 1973). But it may be possible to account for the properties of self-gravitating systems and of the microwave background by a cosmogonic theory that postulates an initial state in global thermodynamic equilibrium at zero temperature (Layzer and Hively 1973; Layzer 1971*b*, 1976). In either case the present discussion links the thermodynamic arrow in a natural way to the historical and cosmological arrows and shows how all three can be derived from a simple and general postulate concerning the spatial symmetry of the Universe.

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[15] Ne'eman (1970) and Aharony and Ne'eman (1970) have discussed the cosmological implications of the microscopic arrow under the assumption that the temporal direction in which entropy increases always coincides with the direction in which the Universe expands. Under this assumption, if *CP* invariance is violated but *CPT* invariance is preserved, the contracting phase of a closed Friedmann universe is indistinguishable from the expanding phase, except that the definitions of matter and antimatter are interchanged. If *CPT* invariance is violated, however, the physical laws governing the decay of kaons are different in the expanding and contracting phases. According to the theory developed in the present paper, the thermodynamic and cosmological arrows are not linked in the way assumed by Ne'eman. In a closed Friedmann universe, entropy continues to be generated during the contracting phase. The present theory therefore implies (in the light of Ne'eman's discussion) that the microscopic arrow has no direct cosmological implications.

[16] A nominally closed system may be said to come into being when its interaction with the outside world first satisfies some arbitrary criterion. The world lines of any nominally closed system thus originate on a more or less well defined surface t = constant in the finite past (finite because the cosmic expansion imposes an upper limit on the age of any nominally closed system defined in this way).

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