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BY

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CHAPTER I

INTRODUCTION

Nearly a half-century ago, Einstein advanced an ingenious thought experiment in order to expose what he called the "incompleteness" of quantum mechanics.[1] Recent years have seen modified versions of that very thought experiment adapted for actual laboratory settings and performed. Ironically, the results of those experiments may have quietly shattered a world-view that contains the essential features of Einstein's own philosophy of nature.[2]

The controversy surrounding the incompleteness of quantum mechanics has had to do with how the "non-classical" features of the theory are to be understood. Questions of the following sort have marked the debate:

> --Is the statistical character of quantum-theoretical predictions a consequence of an irreducible indeterminism in nature or is it instead merely an indication of the imperfect state of our knowledge of a deterministic world?

> --Do physical entities possess definite attributes for whose precise description quantum mechanics is simply inadequate or does

[1]The thought experiment was introduced in a 1935 paper ([16]) bearing the title "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?" and authored jointly by Albert Einstein, Boris Podolsky, and Nathan Rosen. The central ideas of the now famous "EPR" paper are unmistakably Einstein's. Though a sizable literature could be recommended for informative commentary on the EPR argument, Clifford Hooker offers some especially pertinent insights in [23].

[2]While Einstein is well known to have held steadfastly to a belief in the ultimate determinism of all physical phenomena, the world-view in question (to be referred to as "local realism") is entirely compatible with indeterminism, and includes determinism as a special case.

the very notion of "definite attributes," at least under certain conditions, fail to apply in any strict sense to the entities which populate our world?

--Even if there are intrinsic limits to physical observability (as Heisenberg's Principle is generally taken to assert), can we not at least meaningfully refer to that which defies arbitrarily precise measurement? For example, can we not ascribe to particles simultaneously precise values (albeit unknown and perhaps unknowable values) of position and momentum?; i.e., can we not assert that these quantities each have at least some definite value, regardless of our facility at (or the very possibility of) discovering them?; or, rather, is the assertion that a given particle simultaneously has some particular definite position and some particular definite momentum not, at worst, merely false, but an out-and-out abuse of language?

--Cannot all phenomena be represented in terms of interacting but independently existing (and so, in principle, independently describable) entities--whether they be "particles" or "fields" or whatever --even if the laws of physics themselves do insuperably limit our ability to acquire complete knowledge of them?

According to Niels Bohr's doctrine of complementarity, the nature of quantum-mechanical reality imposes on us, as an epistemological requirement (i.e., a logical prerequisite for the possibility of knowledge), that the description of phenomena be relativized to the experimental conditions of observation. Such conditions are taken to govern the very applicability of the concepts in terms of which phenomena may be described and therefore enter ineliminably into the preconditions for knowledge. On this view, the physical incompatibility of experimental arrangements necessary for the measurement of certain pairs of quantities (those represented in the theory by non-commuting operators) precludes the meaningfulness of simultaneously precise values of those quantities. In Bohr's words:

> Indeed we have in each experimental arrangement suited for the study of proper quantum phenomena not merely to do with an ignorance of the value of certain physical quantities, but with the impossibility of defining these quantities in an unambiguous way ([9]: 699).[1]

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[1] This quotation is from Bohr's identically titled response to the EPR paper.

> . . . , an independent reality in the ordinary physical sense can neither be ascribed to the phenomena nor to the agencies of observation [sic] ([10]: 580).[1]

Einstein, on the other hand, considered the goal of physics to be a full account of "objective reality"; that is to say, he sought a theoretical framework which permits the description of phenomena without making essential reference to the conditions of measurement. According to Einstein, "Physics is an attempt conceptually to grasp reality as it is thought independently of its being observed. In this sense one speaks of 'physical reality'" ([14]: 81).[2] Perhaps the clearest expression of the relevant aspects of Einstein's personal world-view appears in part II of his essay, "Quantum Mechanics and Reality,"[3] which part is reproduced here in its entirety:

> If one asks what, irrespective of quantum mechanics, is characteristic of the world of ideas of physics, one is first of all struck by the following: the concepts of physics relate to a real outside world; that is, ideas are established relating to things such as bodies, fields, etc., which claim a 'real existence' that is independent of the perceiving subject - ideas which, on the other hand, have been brought into as secure a relationship as possible with the sense-data. It is further characteristic of these physical objects that they are thought of as arranged in a space-time continuum. An essential aspect of this arrangement of things in physics is that they lay claim, at a certain time, to an existence independent of one another, provided these objects 'are situated in different parts of space.' Unless one makes this kind of assumption about the independence of the existence (the 'being-thus') of objects which are far apart from one another in space - which stems in the first place from everyday thinking - physical thinking in the familiar sense

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[1]This quotation is from the published version of Bohr's 1927 Como address.

[2]Einstein wrote this in the essay he referred to as being "something like my own obituary." The translation is by P. A. Schilpp.

[3]This essay originally appeared (in German) in an issue of Dialectica devoted to the concept of complementarity. Einstein sent a copy of the essay to Max Born and Born included it in their published correspondence, from where this excerpt is taken.

> would not be possible. It is also hard to see any way of formulating and testing the laws of physics unless one makes a clear distinction of this kind. This principle has been carried to extremes in the field theory by localising the elementary objects on which it is based and which exist independently of each other, as well as the elementary laws which have been postulated for it, in the infinitely small (four dimensional) elements of space.
>
> The following idea characterises the relative independence of objects far apart in space (A and B): external influence on A has no direct influence on B; this is known as the 'principle of contiguity,' which is used consistently only in the field theory. If this axiom were to be completely abolished, the idea of the existence of (quasi-) enclosed systems, and thereby the postulation of laws which can be checked empirically in the accepted sense, would become impossible ([15]: 170-71).

Exceedingly complex issues underlie disputes about the incompleteness of quantum mechanics. Fundamental questions concerning principles of determinism, realism, and locality ("continguity") all arise in the analysis of the quantum-mechanical description of the world. Efforts to bring these issues into focus will be postponed until a relevant experiment has been introduced. The world-view to be called "local realism" constrains theories which purport to account for the results of that experiment, and the analysis of those constraints will afford some clarification of these issues.

While the local realist position will be characterized more precisely in the context of that experiment, an informal outline of that position might profitably be considered now.[1] Put roughly, the local realist picture of the world is distinguished by the following main features:

(i) All physical phenomena are produced by the interactions of various entities whose existence and whose attributes are independent of our knowledge of them.

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[1] No attempt will be made to characterize scientific theories with precision sufficient for a proper general formulation of the constraints of local realism. A more formal approach is taken by Geoffrey Hellman in [21] and [22].

(ii) The physical state of each entity or system of entities may be exhaustively characterized by a list of exactly specified physical quantities. No particular classical physical quantities need be included in the list or definable in terms of those which do occur in the list. In order that there be contact between such a state description and our experience, there must, of course, be some connection between the observable properties of macroscopic bodies and the physical quantities posited in state descriptions. However, it will not be assumed a priori that any particular physical quantities (e.g., the position and momentum of a particle with respect to a given frame of reference) are well-defined in domains which extend arbitrarily far beyond our always limited experience. Therefore, it is assumed only that there is some (not necessarily unique) set of "real" properties of physical entities, where different sets of properties may be associated with entities of different types, and this whether or not it is known what those properties are or what the values of the quantities which measure them are. The complete state description of a system of entities consists in the precise specification of these "real" properties. Such a state description includes, at least implicitly, everything that physically "is the case" about the system so long as it remains in that state.

(iii) The laws governing the interactions of physical entities, whether those laws are deterministic or not, respect considerations of locality. In particular, no causal disturbance can propagate between space-like related events.

Regarded as a program in natural philosophy, local realism is the position according to which all physical phenomena are to be embedded into a picture of the sort sketched above. Of course, this characterization of local realism requires considerable sharpening if any particular phenomenon is to be assessed in detail with respect to it. Nevertheless, a certain "naturalness" (or, perhaps, lenience) of the strictures of local realism is suggested by the difficulties encountered in the effort to imagine a phenomenon which could not be reconciled with some such picture of the world. One might even be tempted to regard compatibility with local realism (in some suitably refined formulation) as a minimal requirement for a theory to have explanatory value.

In the years since the appearance of Einstein's thought experiment, persistent efforts to show that quantum mechanics cannot be "completed" in any suitable way produced a succession of increasingly powerful arguments.[1] Those efforts have culminated in an impressive case against local realism. Quite general arguments support the claim that certain experiments cannot be fit into any picture of the sort sketched in (i)-(iii) above. Various proofs, now collectively known as "Bell's Theorem,"[2] show that any theory compatible with local realism must make certain testable predictions (the so-called "Bell-type inequalities") concerning the results of experiments of a certain type. Since the predictions quantum mechanics makes for such experiments do not satisfy the Bell-type inequalities, Bell's Theorem establishes the incompatibility of local realism and quantum mechanics. However, while this result has tremendous significance for those issues belonging to what has traditionally been called "the interpretation problem" of quantum mechanics, that no local realistic interpretation can be provided for quantum mechanics is not the most serious implication of Bell's Theorem for local realism. Bell-type inequalities have been put to the test in actual experiments and there is now strong evidence that they are violated, thereby incriminating local realism quite independently of its incompatibility with quantum mechanics (whose predictions are in accord with the experimental results).

The focus of this dissertation is on certain aspects of the case against local realism. Different versions of Bell's Theorem correspond to different sets of premises, which are regarded as precise statements

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[1]Belinfante [3] gives a detailed account of these arguments up to the early 1970's. Additional useful information may be found in Jammer [24], particularly in Chapter 7. An excellent review of more recent work is included in Clauser and Shimony [12].

[2]The name is for John S. Bell, who presented the first such proof in 1964 ([5]).

(appropriate for the aforementioned special class of experiments) of constraints demanded by local realism and from which Bell inequalities may be derived. Each version of the theorem employs as such a premise some locality condition (i.e., some formulation or other of a constraint to be interpreted as a prohibition against "action-at-a-distance"). Some versions of Bell's Theorem, namely those restricting consideration to deterministic local realistic theories, employ as a locality condition the requirement warranted by relativity theory, that superluminal signals not be permitted.[1] More general versions of Bell's Theorem, those which apply to stochastic local realistic theories, make use of a stronger locality condition. The physical content of this latter condition and the precise relationship between the relativistic locality constraint and this stronger locality condition are currently controversial topics about which there seems to be considerable confusion.

The major objective of this dissertation is to provide some clarification of these matters by calling attention to a certain "decomposition" of the stronger locality condition into two weaker conditions, each of which has a rather straightforward physical interpretation. More specifically, it will be shown that the stronger locality condition is logically equivalent to the conjunction of the relativistic locality constraint and another condition which has to do with the "completeness" of the state descriptions allowed by the theory. This logical equivalence will be exploited to isolate those features of local realism which cannot be realized by any theory whose predictions accord with those of quantum mechanics.

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[1]In fact, the locality condition usually employed in these versions of the argument is not quite that of relativity theory. (This point will be clarified a bit in Chapter III.) Nevertheless, expressions such as "relativistic locality constraint" will be used loosely to refer to it.

It will be argued that there is a class of local realistic theories for which the standard formulation of the stronger locality condition is not an appropriate constraint. The most general theories of this class permit indeterministic states that evolve stochastically. These are states which, while determining at each instant only probabilities for outcomes of possible measurements, are themselves subject to an indeterministic temporal evolution. However, though the extension of the Bell-type arguments to cover such theories is not altogether unproblematic, the prospects for local realism will be seen to remain extremely poor.

If local realism is no longer tenable and a more adequate world-view is to be constructed, then it ought to be of value to understand clearly how the assumptions of local realism lead to conflicts with experimental results. It is in this spirit that this analysis of the case against local realism is offered.

CHAPTER II

THE EXPERIMENTAL SETUP/NOTATION[1]

In what follows, we will be concerned with the measurement of components of the spin of spin-½ particles.[2] Since the quantum-mechanical spin has no analogue in classical physics, one might suspect there is some possibility for circularity in the argument, brought about through the use of tacit assumptions having to do with this purely quantum-mechanical notion. However, as will be apparent, the correctness of the quantum-mechanical characterization of spin is nowhere presupposed in the argument against local realism.

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[1]At least a partial justification can be given for the regrettably cumbersome-looking notation introduced in this chapter. It has become customary to represent the relevant probability functions in a way which builds in a certain locality condition (namely, the one to be discussed in Chapter III). For many purposes, this practice is convenient and entirely appropriate. However, for present purposes, a notation is needed which permits the expression of a certain condition ("completeness"--to be discussed in Chapter VI) which is independent of the locality condition and this requires a departure from the customary, more wieldy formalism. This desideratum still could be met without explicitly including the full microstates of the measuring devices (as will be done here). Nevertheless, since a properly general treatment of Bell's Theorem must give due consideration to the full microstates of the measuring devices, notation of this sort does permit some added flexibility.

[2]David Bohm, in his classic textbook ([7]), converted the EPR thought experiment into the kind of spin correlation experiment to be discussed here. As a matter of fact, most of the actual experiments which have tested the predictions of local realistic theories have been of a different sort, involving correlated polarizations of pairs of photons. The two sorts of experiments are identical in all relevant respects. Also, see Mermin [26] for a derivation, in the context of a more general EPR-Bohm-Bell correlated spin experiment, of a Bell-type inequality which applies to particles of arbitrary spin.

Consider an arrangement of magnets as shown schematically in Figure 1. It is an experimental fact that under appropriate conditions, certain kinds of particles (namely, spin-½ particles), when passed through the region between the magnets, are invariably deflected upwards or downwards with respect to the direction d defined by the alignment axis of the magnets. This is inferred from the fact that of an array of particle detectors placed beyond the magnets, only those in two well-defined spatial regions, one above the line defined by the direction of collimation (and intersecting the plane defined by that direction and d) and the other symmetrically below it, are ever triggered.

This setup, a so-called "Stern-Gerlach apparatus," is said to measure the d-component of spin. A spin-½ particle is said to have "spin up" ("spin down") in direction d just in case the particle is deflected upward (downward), relative to d, by the magnets. Any component of spin may be measured by rotating the apparatus so that the alignment axis of the magnets is parallel to the chosen direction.

The property spin-½ is regarded here as nothing more than the disposition to respond to certain configurations of magnets in the way described. Surely nothing of quantum mechanics is smuggled into the argument by this use of the term 'spin.' Because spin is to be regarded in this highly phenomenalistic way, no unwarranted constraints will be placed upon the concepts local realistic theories may adopt to frame reality. In particular, since no notion of angular momentum will be invoked to explicate spin phenomena, it is not presupposed that "intrinsic angular momentum" is definable in terms of whatever physical quantities local realism acknowledges.[1]

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[1]This is relevant because, in quantum mechanics, the spin of a

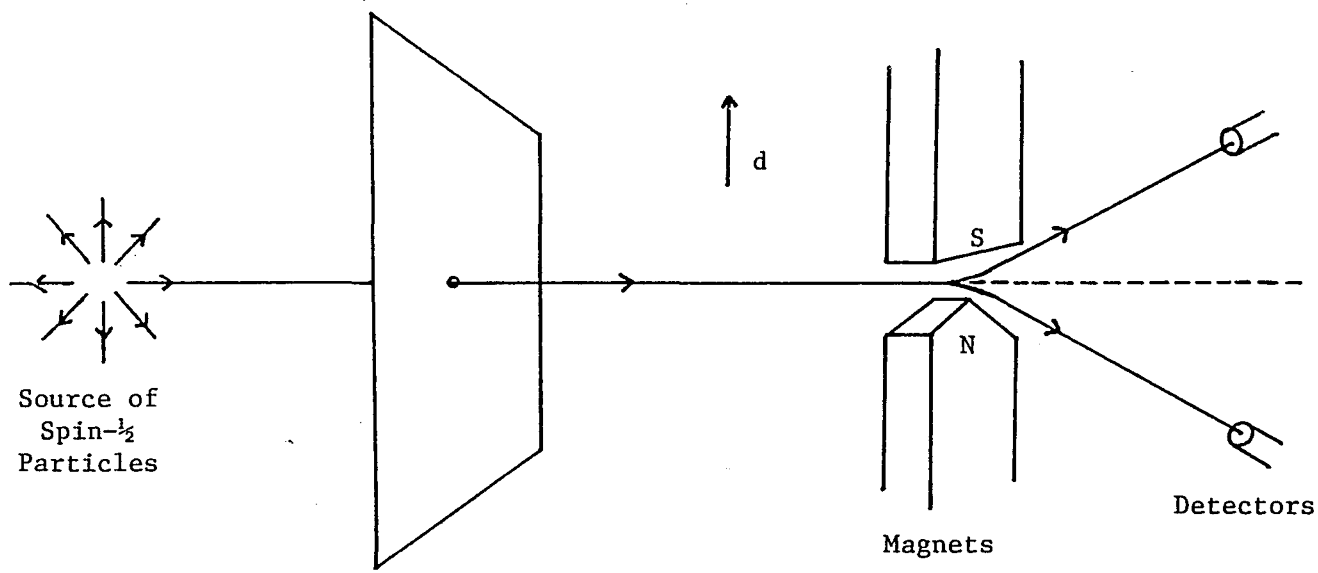

Fig. 1.--A Stern-Gerlach Apparatus

Somewhat similar remarks apply to the use of the term 'particle.' Liberal use has been made of this term in the above description. It is important to understand that this does not compromise the generality of the argument to follow (i.e., that this does not constitute an ontological presupposition on the part of local realistic theories). As just noted in the discussion of the use of the term 'spin,' 'particle' may similarly be regarded in a highly phenomenalistic way. Ultimately, only claims about the numbers of counts registered in various detectors are essential and such claims, in principle, might equally well be analyzed in terms of "fields" or in terms of entities of some other type.[1] The 'particle' locutions employed here should be construed in this ontologically innocuous way.

Consider now a system of two spin-½ particles. Suppose that the two particles interact briefly and then are allowed to separate to an arbitrarily great relative distance in a manner which does not (otherwise) disturb the particles.[2] We will be concerned with spin measurements on the two single-particle subsystems. Let 'L' and 'R' (for 'left' and

---

particle is taken to be an "intrinsic angular momentum," and spin is represented mathematically in a peculiar form (from the point of view of classical physics) which is characteristic of quantum-mechanical angular momenta. The less sophisticated use being made here of this term is intended to be neutral with respect to this quantum-theoretical account of spin phenomena.

[1]None of this is intended to suggest that such words as 'particle' and 'field' (without further qualifications or description of assumed properties, modes of interaction, etc.) either unambiguously denote anything or need be names for distinct entities.

[2]It is assumed here that the particles interact appreciably only with each other and (later) with their respective measuring devices. Of course, there is no way to exclude the possibility that other (unknown) interactions do occur, but since quantum mechanics makes accurate predictions of the results of such experiments without violating this assumption, it is appropriate to seek local realistic candidates which likewise do not violate it.

'right') denote Stern-Gerlach measuring devices situated on diametrically opposed sides of the interaction region as indicated schematically in Figure 2. (In what follows, the labels 'L' and 'R' will be used promiscuously to refer to the left-hand member and right-hand member, respectively, of the pair of measuring devices, of a pair of particles, of a pair of measuring device/single-particle subsystems, etc. In each case, the intended use will be made clear by the context.)

Let Λ be the set of states of the two-particle system and let $M_L(M_R)$ be the set of states of measuring device L(R). (For the sake of generality, the exact form of these state descriptions is deliberately left unspecified.) It will be useful to distinguish certain subsets of M_L and M_R as follows: Let d be an arbitrary unit vector in space; then M_L^d is the set containing just those states $\bar{\mu}_L \varepsilon M_L$ which could be occupied by an L measuring device prepared to measure the d-component of spin. M_R^d is defined analogously. Since the macroscopic parameter of primary interest in these state descriptions is the direction along which the Stern-Gerlach magnets are aligned (i.e., the component of spin to be measured), let us represent measuring device states $\bar{\mu}_i \varepsilon M_i^{d_i}$ as ordered pairs $\bar{\mu}_i = (\mu_i, d_i)$, where the subscript i may assume the values 'L' and 'R'. This notation merely collects together into μ_i whatever information in excess of the direction d_i is required to fully specify the state of measuring device i.

It will also be useful to allow the variable d to assume the value 0 (corresponding to no direction at all) in order to denote those sets of states, M_L^0 and M_R^0, which could be occupied by measuring devices unprepared to measure <u>any</u> component of spin. If Δ is the set of unit vectors in space and if M_i^Δ is defined as the set $\bigcup_{d_i \varepsilon \Delta} M_i^{d_i}$, then, for each i, $M_i = M_i^0 \cup M_i^\Delta$ and $M_i^0 \cap M_i^\Delta = \phi$.

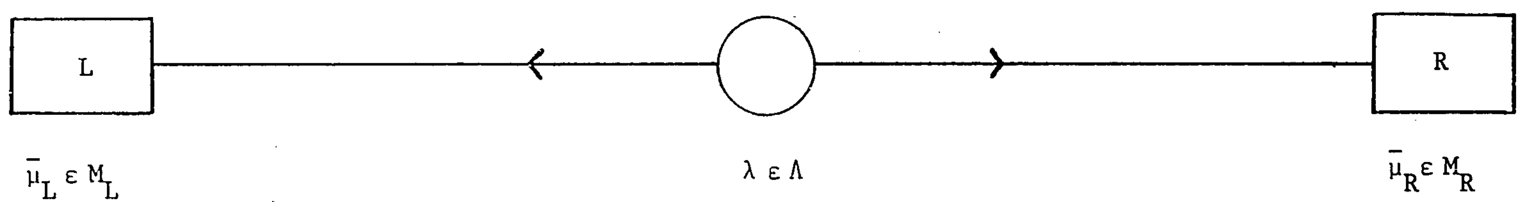

Fig. 2.--A Bell-type Experimental Setup

Suppose that for a given pair of particles, device L is prepared to measure th d_L-component of the spin of particle L and device R is prepared to measure the d_R-component of the spin of particle R. To this point there has been no mention of conditions a theory must satisfy in order that it make minimal empirical contact with such spin measurements. For this purpose, it will be demanded of a theory that it provide (on the basis of the relevant state descriptions) values of probability for each of the possible pairs of particle deflections (upward and downward for each particle) during joint spin measurements. The long-run relative frequencies of counts registered in the appropriate detectors will provide the evidence for these statistical predictions. Any theory which fails to do this much makes no testable claims about these phenomena.

Accordingly, attention will be restricted to theories which employ state descriptions for the two-particle system and measuring devices which determine unique values for the probabilities of the various possible (pairs of) outcomes of joint spin measurements. That is to say, given a state $\lambda \varepsilon \Lambda$ of the two-particle system and states $\bar{\mu}_L \varepsilon M_L^{d_L}$ and $\bar{\mu}_R \varepsilon M_R^{d_R}$ of the measuring devices (where $d_L \varepsilon \Delta$ and $d_R \varepsilon \Delta$), and given a pair x_L and x_R of possible measurement outcomes, the theory must uniquely specify a number $\pi^{\mu_L,\mu_R}(d_L,x_L;d_R,x_R)$ which is to be interpreted as the probability that the outcomes of a measurement of the d_L-component of the spin of particle L and a measurement of the d_R-component of the spin of particle R are x_L and x_R respectively.[1]

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[1]Following Skyrms [29], the assertions of the theory are to be regarded as subjunctive conditionals of the form: If measuring device L, in state $\bar{\mu}_L \varepsilon M_L^{d_L}$, and measuring device R, in state $\bar{\mu}_R \varepsilon M_R^{d_R}$, were to measure, respectively, the d_L-component of the spin of particle L and the d_R-

For the local realist, these π_λ functions are most naturally regarded as single-case propensities.[1] Since these propensities attach to individual measurement trials (as opposed to ensembles), this suggests a sense in which local realism might be regarded as a generalized Einsteinian world-view, one that relaxes the assumption of determinism. Determinism requires that the outcomes of individual measurement trials be deducible from the pre-measurement properties of the systems as encoded in the state descriptions. Given a state $\lambda \varepsilon \Lambda$ of the two-particle system and the states $\bar{\mu}_L \varepsilon M_L^{d_L}$ and $\bar{\mu}_R \varepsilon M_R^{d_R}$ of the measuring devices, a deterministic theory predicts a particular definite outcome for each measurement. Local realism requires that the pre-measurement state of affairs fix the probabilities for the occurrence of the various possible measurement outcomes; deterministic theories are special cases in which all probabilities take on a value of 0 or 1.[2]

For any given direction, the spin up and spin down measurement outcomes will be represented here by the values +1 and -1 respectively. Let us also incorporate into the formalism the trivial measurement "outcome," represented by the value 0, which obtains whenever no measurement is performed (i.e., whenever $\bar{\mu}_L \varepsilon M_L^0$ or $\bar{\mu}_R \varepsilon M_R^0$). Letting $X = \{+1,-1\}$, the

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component of the spin of particle R, for a pair of particles in the state $\lambda \varepsilon \Lambda$, then the (joint) probability for the pair of outcomes, x_L of the L measurement and x_R of the R measurement, would be $\pi_\lambda^{\mu_L,\mu_R}(d_L,x_L;d_R,x_R)$. Note that quantum mechanics is itself a theory of this type.

[1]Note, however, that since the evidence for such attributions of propensity is to be drawn from the long-run relative frequencies of measurement outcomes, it is not necessary to interpret the probabilities along such lines. The neutral term 'probability' will be used here.

[2]This is not to say that probability 0 entails impossibility and that probability 1 entails necessity. It is only a necessary condition for determinism that the probabilities assume no intermediate values.

condition described above can be expressed as follows: For each state $\lambda \varepsilon \Lambda$, there is a function π_λ: $D_\pi \to [0,1]$, where

$$D_\pi = \{(\bar{\mu}_L, x_L, \bar{\mu}_R, x_R) \mid \bar{\mu}_L \varepsilon M_L^\Delta,\ x_L \varepsilon X,\ \bar{\mu}_R \varepsilon M_R^\Delta,\ x_R \varepsilon X\}$$

$$\cup \{(\bar{\mu}_L, x_L, \bar{\mu}_R^\circ, 0) \mid \bar{\mu}_L \varepsilon M_L^\Delta,\ x_L \varepsilon X,\ \bar{\mu}_R^\circ \varepsilon M_R^0\}$$

$$\cup \{(\bar{\mu}_L^\circ, 0, \bar{\mu}_R, x_R) \mid \bar{\mu}_L^\circ \varepsilon M_L^0,\ \bar{\mu}_R \varepsilon M_R^\Delta,\ x_R \varepsilon X\},$$

satisfying the following conditions:[1]

(1) For all $d_L \varepsilon \Delta$, $\bar{\mu}_L \varepsilon M_L^{d_L}$, and $\bar{\mu}_R^\circ \varepsilon M_R^0$,

$$\sum_{x_L \varepsilon X} \pi_\lambda^{\mu_L, \mu_R^\circ} (d_L, x_L; 0, 0) = 1.$$

(2) For all $\bar{\mu}_L^\circ \varepsilon M_L^0$, $d_R \varepsilon \Delta$, and $\bar{\mu}_R \varepsilon M_R^{d_R}$,

$$\sum_{x_R \varepsilon X} \pi_\lambda^{\mu_L^\circ, \mu_R} (0, 0; d_R, x_R) = 1.$$

(3) For all $d_L, d_R \varepsilon \Delta$, $\bar{\mu}_L \varepsilon M_L^{d_L}$, and $\bar{\mu}_R \varepsilon M_R^{d_R}$,

$$\sum_{x_L, x_R \varepsilon X} \pi_\lambda^{\mu_L, \mu_R} (d_L, x_L; d_R, x_R) = 1.$$

The domain of these probability functions includes pairs of measuring device states corresponding to cases in which one (but not both) of

[1]Note that although the elements of D_π have the form $(\bar{\mu}_L, x_L, \bar{\mu}_R, x_R)$, the associated probability will be expressed as $\pi_\lambda^{\mu_L, \mu_R}(d_L, x_L; d_R, x_R)$, where $\bar{\mu}_L = (\mu_L, d_L)$ and $\bar{\mu}_R = (\mu_R, d_R)$. This is done merely to isolate the four parameters of greatest interest: the two components of spin to be measured and the two measurement outcomes.

the measuring devices is in a non-measurement state. Conditions (1) - (3) above merely insure that the relevant probabilities summed over the possible outcomes add up to unity. Conditions (1) and (2) express this requirement for those cases in which only one of the measuring devices is prepared to perform a spin measurement and condition (3) expresses it for cases in which both measuring devices are so prepared.

Some brief comments broadly outlining the structure of the argument to follow may be useful at this point. (See Figure 3.) It has two major stages. The first stage (Chapters III-V) shows that in the context of the experimental situation described above, any deterministic theory which satisfies two conditions, to be called "locality" and "conservation," must assign probabilities (for the possible outcomes of possible pairs of spin measurements on the two particles) which satisfy a certain relation, the Bell-Wigner inequality. The second stage (Chapters VI-IX) shows that any

$$\begin{array}{ccccc} & & \longleftarrow\text{STAGE ONE}\longrightarrow & & \\ \left.\begin{array}{l}\text{STRONG LOCALITY}\\ \text{CONSERVATION}\end{array}\right\} & \Longrightarrow & \left\{\begin{array}{l}\text{DETERMINISM}\\ \text{CONSERVATION}\\ \text{LOCALITY}\end{array}\right\} & \Longrightarrow & \text{BELL-WIGNER INEQUALITY} \\ \longleftarrow\text{STAGE TWO} & \longrightarrow & & & \end{array}$$

Fig. 3.--The Structure of the Argument

theory which satisfies conservation and a condition to be called "strong locality" must be deterministic.[1] Since locality (as will be shown) is a

[1] This claim has to be qualified. It will be shown only that any theory satisfying conservation and strong locality must assign a probability of 0 or 1 to each possible pair of possible spin measurement outcomes. However, since the first stage of the argument will invoke only

consequence of strong locality, it follows that any theory which satisfies conservation and strong locality--and, as will be seen, this encompasses all plausible local realistic theories[1]--must make predictions which accord with the Bell-Wigner inequality. It is basically a more general form of this inequality which has been most severely tested experimentally and found to be violated.[2] It will be shown (in an appendix) that the Bell-Wigner inequality does conflict with quantum mechanics, whose predictions accord well with the results of actual experiments. Considerations regarding the physical significance of the assumptions introduced will form a major part of the presentation.

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this characteristic of deterministic theories, there will be no need to maintain (within this context) a distinction between theories which predict certain measurement outcomes to occur with certainty and those which only assign those outcomes probability 1.

[1]While conservation is not grounded in local realism, there is good empirical evidence in support of it.

[2]Derivations of these "generalized Bell inequalities" proceed from strong locality, without the assumption of conservation, and they do not take the detour through determinism indicated for the derivation to be presented here. Although conservation is in good agreement with the evidence of experiments, these more general arguments do yield a stronger conclusion. See Clauser and Shimony [12], pp. 1897-98, for a derivation of the Clauser-Horne inequality for local deterministic theories and pp. 1894-95 for a derivation of that inequality for strongly local indeterministic theories. (The latter derivation is a summary of the original given in Clauser and Horne [11], which contains a good discussion of the connections between this result and Bell's 1971 argument in [4].) Neither of these derivations invokes the conservation condition. The particular form of the argument to be used here (making use of the conservation condition) has been chosen so as to emphasize the relevance, for those familiar with it, of the original EPR argument.

CHAPTER III

LOCALITY

For purposes of the present discussion, a theory will be said to be local (or to satisfy locality) just in case the probability functions it specifies meet the following constraints: For all $\lambda \varepsilon \Lambda$, $d_L, d_R \varepsilon \Delta$, $\bar{\mu}_L \varepsilon M_L^{d_L}$, $\bar{\mu}_R \varepsilon M_R^{d_R}$, $\bar{\mu}_L^{\circ} \varepsilon M_L^0$, $\bar{\mu}_R^{\circ} \varepsilon M_R^0$, and $x_L, x_R \varepsilon X$,

$$(4) \qquad \pi_{\lambda}^{\mu_L,\mu_R^{\circ}}(d_L,x_L;0,0) = \sum_{x_R' \varepsilon X} \lambda_{\lambda}^{\mu_L,\mu_R}(d_L,x_L;d_R,x_R')$$

and

$$(5) \qquad \pi_{\lambda}^{\mu_L^{\circ},\mu_R}(0,0;d_R,x_R) = \sum_{x_L' \varepsilon X} \pi_{\lambda}^{\mu_L,\mu_R}(d_L,x_L';d_R,x_R).$$

Locality requires that the probability for the outcome $x_i \varepsilon X$ of a spin measurement on particle i be determined "locally"; i.e., that it depend only on the state $\lambda \varepsilon \Lambda$ of the two-particle system and on the state $\bar{\mu}_i \varepsilon M_i^{\Lambda}$ of the i measuring device. In particular, that probability must be independent of which (if any) component of spin the distant measuring device is set to measure.

It should be noted that locality does not exclude the possibility of acquiring information about subsystem R(L)--information bearing on the outcome of a subsequent R(L) measurement--by performing a measurement on particle L(R). Since λ presumably contains information about both particles, a measurement on one particle, from the outcome of which partial knowledge of λ may be inferred, may therefore yield information about the other particle as well. Strictly correlated pairs of measurement outcomes represent the extreme situation of this sort. Such correlations are entirely compatible with locality.

What locality <u>does</u> exclude, however, is the possibility that the preparation of either measuring device in some particular state can exert a causal influence on the other subsystem so as to affect the probabilities for the possible outcomes of measurements performed on that other subsystem. Since these two events, the preparation of one measuring device in a given state and the measurement executed by the other measuring device, can be space-like related, locality is a requirement of relativity theory.

In order to establish the relativistic basis for locality, it suffices to show that if enough control over the state preparations is assumed, violations of the locality condition provide (at least in principle) the means for superluminal signal transmission.[1] For this purpose, it will be convenient to treat deterministic theories and indeterministic theories separately.

Consider first an arbitrary deterministic theory. Suppose this theory violates (4). (One could just as well suppose instead the theory violates (5) and give a similar argument.) Then for some $\lambda \in \Lambda$, $d_L, d_R \in \Delta$, $\bar{\mu}_L \in M_L^{d_L}$, $\bar{\mu}_R \in M_R^{d_R}$, $\bar{\mu}_R^{\circ} \in M_R^{0}$, and $x_L \in X$,

$$\pi_\lambda^{\mu_L, \mu_R^{\circ}}(d_L, x_L; 0, 0) \neq \sum_{x_R \in X} \pi_\lambda^{\mu_L, \mu_R}(d_L, x_L; d_R, x_R).$$

Since, under the assumption of determinism, all probabilities are either 0 or 1, there are only two cases to consider:

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[1]This point was made previously by Eberhard in [13]. See Hellman [22] for another account of the relativistic locality condition.

Case (i) $$\pi_\lambda^{\mu_L,\mu_R^\circ}(d_L,x_L;0,0) = 0$$

and

$$\pi_\lambda^{\mu_L,\mu_R}(d_L,x_L;d_R,+1) = 1 \quad \text{or} \quad \pi_\lambda^{\mu_L,\mu_R}(d_L,x_L;d_R,-1) = 1.$$

Case (ii) $$\pi_\lambda^{\mu_L,\mu_R^\circ}(d_L,x_L;0,0) = 1$$

and

$$\pi_\lambda^{\mu_L,\mu_R}(d_L,x_L;d_R,+1) = \pi_\lambda^{\mu_L,\mu_R}(d_L,x_L;d_R,-1) = 0.$$

Suppose a pair of particles is prepared in the state λ.[1] Suppose also that experimenters O_L and O_R arrange to perform measurements of the d_L and d_R components (respectively) of the spin of the L and R particles (respectively). Let it be agreed beforehand that O_L is to perform his measurement by preparing his measuring device in the state $\bar{\mu}_L = (\mu_L,d_L)$, but that O_R has the choice of performing his measurement by preparing his measuring device in the state $\bar{\mu}_R = (\mu_R,d_R)$ or he can perform no measurement at all by preparing his measuring device in the state $\bar{\mu}_R^\circ = (\mu_R^\circ,0)$.[2] Moreover, O_R is not to make his decision until such time as the making of that decision and the measurement by O_L are space-like related events. (See Figure 4.)

Now O_L can use the outcome of his measurement to determine whether or not O_R decided to perform his measurement.[3] If case (i) above obtains

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[1]While it here assumed that the experimenters are able to prepare systems in the desired states, the inability to do so in practice does not undermine the argument being made.

[2]See n. 1 above.

[3]Since a probabilistic formalism is here being used to represent the

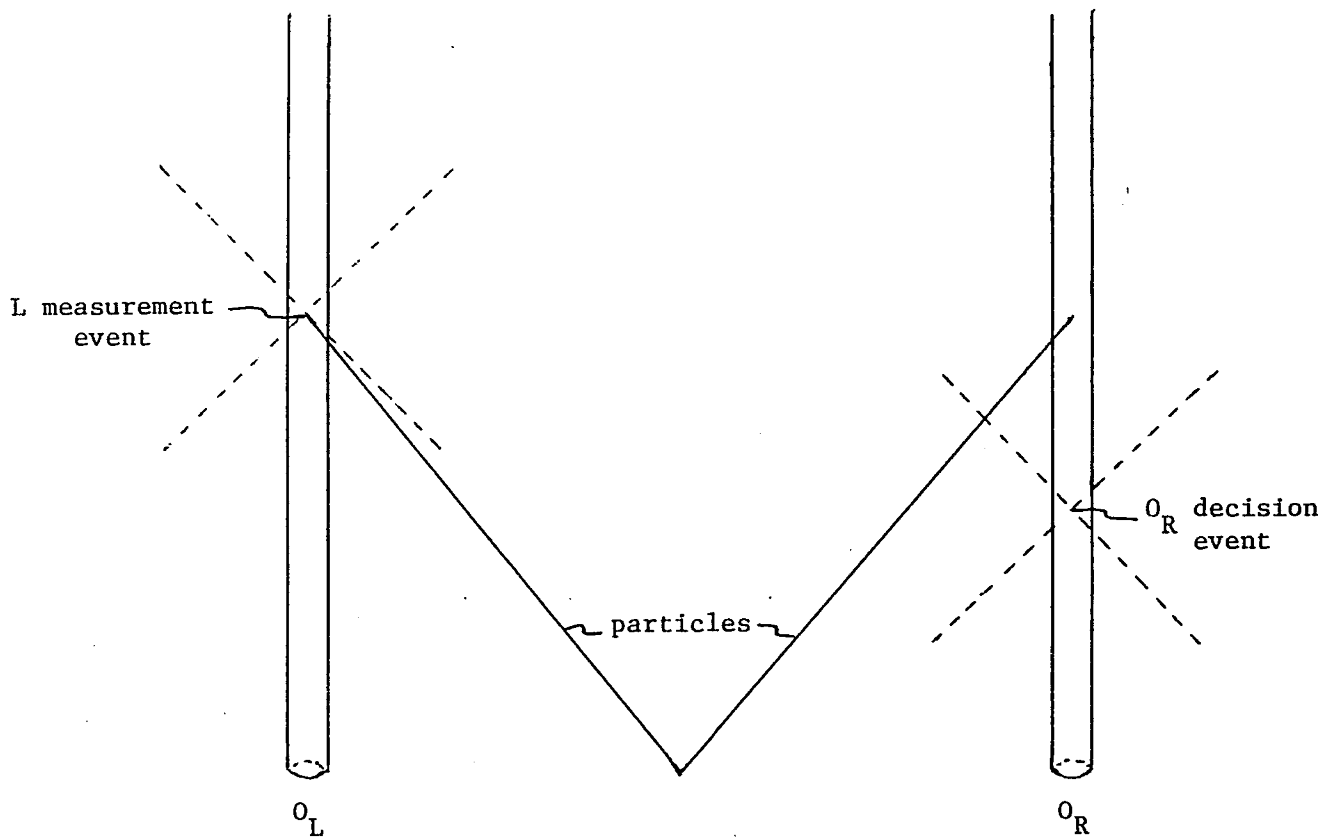

Fig. 4.--Spacetime Diagram of Superluminal Transmission Scheme. (Note that neither labeled event lies inside the future light cone of the other.)

and O_L finds outcome x_L, then he knows that O_R must also have performed a measurement. On the other hand, if O_L finds outcome $-x_L$, then (by equations (1) and (3)) he can be certain that O_R did not perform his measurement. Similarly, if case (ii) above obtains and O_L finds outcome x_L, he can infer that O_R did not perform his measurement. If O_L finds outcome $-x_L$, however, he can infer that O_R did perform his measurement. In this way, O_R could communicate his decision to O_L in a time less than that in which light would traverse the distance between them.

Thus, deterministic theories which violate locality are quite difficult to reconcile with relativity.[1] An analogous argument (actually a generalization of the one just given for deterministic theories) can be given for indeterministic theories, but because of complications owing to the statistical character of the predictions, the situation here is a bit more delicate. Consider an arbitrary indeterministic theory which violates (4). Then, as before, for some $\lambda \varepsilon \Lambda$, $d_L, d_R \varepsilon \Delta$, $\bar{\mu}_L \varepsilon M_L^{d_L}$, $\bar{\mu}_R \varepsilon M_R^{d_R}$, $\bar{\mu}_R^{\circ} \varepsilon M_R^0$, and $x_L \varepsilon X$,

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claims of a genuinely deterministic theory, it is legitimate to associate probability 1 with necessity and probability 0 with impossibility.

[1]Not all such theories are incompatible with relativity. Since the backward light cones of the two events labeled in Figure 4 eventually do overlap, the possibility exists that by some unknown--but purely causal--mechanism, O_R's decision and O_L's measurement outcome are correlated precisely as they would have been if there actually had been a superluminal physical disturbance. Such "conspiratorial" possibilities cannot be excluded, but there are (as yet) no positive grounds--none whatsoever--for taking them seriously. On this point, Shimony, Horne, and Clauser have remarked, ". . . , skepticism of this sort will essentially dismiss all results of scientific experimentation. Unless we proceed under the assumption that hidden conspiracies of this sort do not occur, we have abandoned in advance the whole enterprise of discovering the laws of nature by experimentation" ([28]: 6). Still, even if these "conspiratorial" possibilities are excluded, it must be acknowledged that, because no superluminal transport of matter-energy is actually entailed by the violation of locality, such a violation need not correspond to a violation of relativity theory. However, in this case, it is not clear how the violation of locality could be understood at all.

$$\pi_{\lambda}^{\mu_L,\mu_R^\circ}(d_L,x_L;0,0) \neq \sum_{x_R \varepsilon X} \pi_{\lambda}^{\mu_L,\mu_R}(d_L,x_L;d_R,x_R).$$

Suppose that each pair of a large ensemble of pairs of particles is prepared in the state λ.[1] Suppose also that experimenters O_L and O_R arrange to perform measurements of the d_L and d_R components (respectively) of the L and R particles (respectively) of each pair in the ensemble by means of corresponding ensembles of L and R measuring devices. Let it be agreed beforehand that O_L is to perform his measurements by preparing each L measuring device in the state $\bar{\mu}_L = (\mu_L, d_L)$, but that O_R has the choice of performing his measurements by preparing each measuring device in the state $\bar{\mu}_R = (\mu_R, d_R)$ or he can perform no measurements at all by preparing each R measuring device in the state $\bar{\mu}_R^\circ = (\mu_R^\circ, 0)$.[2] As before, O_R is not to make his decision until such time as the making of that decision is space-like related to each of the L measurements.

If the ensembles are sufficiently large, we can expect the relative frequency of the outcome x_L in the L measurements to be insensitive to the actual number of systems in the ensembles (at least approximately). If O_R decides not to perform his measurements, then this relative frequency "ought" to equal $\pi_{\lambda}^{\mu_L,\mu_R^\circ}(d_L,x_L;0,0)$ approximately. If O_R does decide to perform his measurements, then it "ought" to equal $\sum_{x_R \varepsilon X} \pi_{\lambda}^{\mu_L,\mu_R}(d_L,x_L;d_R,x_R)$ approximately.

Of course, there is no guarantee of this; finite sequences (as well as infinite ones, for that matter) of measurement outcomes need not contain specific outcomes distributed precisely so that the relative frequencies equal or even approximate the corresponding single-case probabilities. For

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[1]See n. 1, p. 22. [2]See n. 1, p. 22.

this reason, O_L cannot infer with certainty O_R's decision. However, this fact is without significance for the point at issue. Either O_R performs the measurements or he does not, and the probabilities for the possible outcomes of O_L's measurements depend on which of these is the case. That one might never have conclusive evidence for the value of a probability[1] does not subvert the relativistic basis for the locality condition.

Thus, locality is (if not strictly, at least nearly enough) a consequence of the relativistic prohibition against superluminal signals. It can also be shown that locality entails the impossibility of sending a superluminal signal with a Bell-type experimental setup. Therefore, in the context of such Bell-type experimental setups, locality is (to all intents and purposes) fully logically equivalent to the impossibility of superluminal signals.[2] In order to complete the demonstration of this equivalence, it will now be shown that if a Bell-type experimental setup could be used to send a superluminal signal, then any theory which adequately described the phenomena would violate locality.

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[1]This is a quite general matter. To whatever extent the confirmation of probabilistic assertions is taken to present serious problems, quantum mechanics is subject to those problems as well.

[2]A more precise evaluation of the status of locality with respect to relativity theory requires a further qualification in addition to that already mentioned in n. 1, p. 24. There it was indicated that locality is, in at least one respect, a bit stronger than relativity demands. However, it is also, in a different respect, a bit weaker. Since no assumptions have been made about the relationship between the exact structure of the elements of Λ and the probability functions they determine, locality permits the probabilities for outcomes of measurement events to exhibit a dependence on the physical properties of spatially distant parts of the system; such a probability would depend on information associated with events outside the light cone of the measurement event in question. However, in the present context, the exclusion of any dependence on the state of the distant measuring device is the condition which bears on the possibility of sending superluminal signals by performing spin measurements. In any case, since the truly relativistic locality condition is (in this sense) stronger than locality, conclusions deduced with locality as a premise (e.g., Bell-type inequalities) follow a fortiori from the truly relativistic constraint (except as qualified in n. 1, p. 24).

Once again, it will be convenient to treat deterministic theories and indeterministic theories separately. Consider the deterministic case first. Suppose experimenter O_R could use a Bell-type setup to send a superluminal signal to O_L, telling him whether or not measuring device R was set to measure some (not necessarily specified) component of the spin of particle R; i.e., suppose that by performing a measurement, e.g., of the d_L-component of the spin of particle L, O_L could infer whether or not O_R performs his measurement on particle R.[1] For this to be possible, a one-to-one correspondence must hold between the outcome of O_L's measurement and O_R's message (i.e., whether or not an R measurement is performed). For example:

TABLE 1

CORRESPONDENCE BETWEEN OUTCOME OF O_L's MEASUREMENT AND O_R's MESSAGE (DETERMINISTIC CASE)

Outcome of Measurement of d_L-Component of Spin of Particle L	O_R's Message
+1	"No Measurement on Particle R"
-1	"Measurement on Particle R"

In this example, O_L finds outcome -1(+1) of his measurement just in case O_R does (does not) perform his measurement.[2] It follows that for some chosen $\lambda \varepsilon \Lambda$, $\bar{\mu}_L \varepsilon M_L^{d_L}$, and $\bar{\mu}_R^{\circ} \varepsilon M_R^0$,

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[1]Note that any superluminal signal must consist of information coded in some such way. The account given here can readily be modified to treat cases in which O_R may measure either one of two different specified components of the spin of particle R.

[2]Analogous treatment can be given to any other such scheme.

$$\pi_\lambda^{\mu_L,\mu_R^\circ}(d_L,+1;0,0) = 1. \tag{6}$$

Otherwise,[1] a +1 outcome of the measurement on particle L would indicate that a measurement had occurred at R. Also, for λ and $\bar{\mu}_L$ the same as above and for some chosen $d_R \varepsilon \Delta$ and $\bar{\mu}_R \varepsilon M_R^{d_R}$,

$$\pi_\lambda^{\mu_L,\mu_R}(d_L,+1;d_R,+1) = \pi_\lambda^{\mu_L,\mu_R}(d_L,+1;d_R,-1) = 0. \tag{7}$$

Otherwise, a +1 outcome of the measurement on particle L would be compatible with a measurement of some component of the spin of particle R. But locality (equation (4)) requires that for all $\lambda \varepsilon \Lambda$, $d_L, d_R \varepsilon \Delta$, $\bar{\mu}_L \varepsilon M_L^{d_L}$, $\bar{\mu}_R \varepsilon M_R^{d_R}$, and $\bar{\mu}_R^\circ \varepsilon M_R^0$,

$$\pi_\lambda^{\mu_L,\mu_R^\circ}(d_L,+1;0,0) = \pi_\lambda^{\mu_L,\mu_R}(d_L,+1;d_R,+1) + \pi_\lambda^{\mu_L,\mu_R}(d_L,+1;d_R,-1).$$

Obviously this cannot be true if both (6) and (7) are true. This completes the argument for the deterministic case.

Now consider the indeterministic case. Suppose experimenter O_R could use a large ensemble of Bell-type setups to send a superluminal signal to O_L, telling him whether or not all of the R measuring devices were prepared to measure some (not necessarily specified) component of the spin of each particle R; i.e., suppose that by performing a measurement, e.g., of the d_L-component of the spin of each particle L, O_L could infer whether or not O_R performs his measurement on each particle R.[2] For this to be possible, a suitable correspondence must hold between the probabilities for the possible outcomes of O_L's measurements (as approximated

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[1]See n. 3, p. 22. [2]See n. 1, p. 27.

by the observed relative frequencies) and O_R's message (i.e., whether or not the R measurements are performed). For example:

TABLE 2

CORRESPONDENCE BETWEEN OUTCOMES OF O_L'S MEASUREMENTS AND O_R's MESSAGE (INDETERMINISTIC CASE)

$\|p - x\|$	O_R's Message
≈ 0	"No Measurement on Particle R"
$>>0$	"Measurement on Particle R"

Here p is the fraction of the systems in which O_L's measurement of the d_L-component of the spin of particle L yields the outcome +1 and x is some previously agreed upon value in the interval from 0 to 1.

In this example, O_L performs his measurements and finds p significantly different from x just in case O_R does perform his measurements.[1] It follows that for some chosen $\lambda \varepsilon \Lambda$, $\bar{\mu}_L \varepsilon M_L^{d_L}$, and $\bar{\mu}_R^{\circ} \varepsilon M_R^0$,

$$\pi_{\lambda}^{\mu_L, \mu_R^{\circ}}(d_L, +1; 0, 0) \approx x. \tag{8}$$

Otherwise, values of p significantly different from x would be compatible with there being no measurements at R.[2] Also, for λ and $\bar{\mu}_L$ the same as above and for some chosen $d_R \varepsilon \Delta$ and $\bar{\mu}_R \varepsilon M_R^{d_R}$,

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[1]See n. 2, p. 27.

[2]In recognition of the aforementioned general difficulties associated with the confirmation of probabilistic assertions, the word 'compatible' should be understood here in an appropriate (very strong) sense. Obviously, mere logical compatibility will not suffice. Indeed, all probabilistic assertions are logically compatible with any observations whatsoever.

(9) $$\left|\pi_{\lambda}^{\mu_L,\mu_R}(d_L,+1;d_R,+1) + \pi_{\lambda}^{\mu_L,\mu_R}(d_L,+1;d_R,-1) - x\right| \gg 0.$$

Otherwise, values of p approximately equal to x would be compatible with measurements of some component of the spin of the R particles.[1] It remains only to point out that (8) and (9), taken in conjunction, are incompatible with locality. (See equation (4).)

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[1]See n. 2, p. 29.

CHAPTER IV

CONSERVATION

Before presenting the conservation condition, some further specification of the experimental situation is needed. This consists in the requirement that the initial interaction between the two particles leave them in a singlet spin state. As in the previous discussion regarding the use of the term 'spin' itself, the use of the expression 'singlet spin state' will be seen not to constitute an intrusion of purely quantum-mechanical concepts into the case against local realism. For purposes of this argument, while no explicit "operational definition" of the expression is assumed, 'singlet spin state' can be adequately explicated in a sufficiently operational way. Certain procedures for preparing a system in such a state can be described without presupposing any peculiarly quantum-mechanical characterizations.[1]

The specific details of these procedures are of no concern here.

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[1]The claim being made here is not that all laboratory procedures can be described in some "theory-neutral observation language," but rather only that quantum theory need not be assumed in order to specify the procedures in question. In the correlated photon polarization experiments, one set of procedures can be described involving the laser-stimulated excitation of a particular species of atom (which then decays in two stages, emitting two photons). Another has to do with arrangements for the mutual annihilation of an electron-positron pair. In the correlated proton spin experiments, a set of procedures can be described for injecting low-energy protons into a hydrogen target. The point to be stressed is that in each case, regardless of the particular bits of language invoked, what one actually has to do in order to bring about the desired state can be expressed without presupposing the propriety of any peculiarly quantum-mechanical concepts.

What is relevant, however, is that what results from these procedures is a pair of spatially separated particles upon which joint measurements of any single component of spin always (within the experimental uncertainties) yield opposite outcomes; one measurement yields spin up and the other yields spin down.[1] Which device records the outcome +1 and which one -1 varies from one pair of particles to the next, but for each pair of outcomes x_L and x_R of such joint spin measurements, it is found that $x_L + x_R = 0$.

It is not to be assumed that there is a unique singlet spin state. In general, it will be allowed that the state preparation procedures mentioned above may leave the two-particle system in any one of a set of states $\Lambda_o \subset \Lambda$, each of which exhibits the characteristic features of "the" singlet state. For this reason, these underlying states are often called "hidden states" or said to include "hidden variables" or "hidden parameters" which then serve to distinguish the experimentally prepared singlet state<u>s</u>. Hereafter, a pair of particles will be said to be in the singlet state provided only that the particles are in some state $\lambda \varepsilon \Lambda_o$.

If a theory is to confront this observed anti-correlation of measurement outcomes in the most straightforward way, it must satisfy a condition called "conservation."[2] A theory satisfies <u>conservation</u> just in case the probability functions it specifies meet the following constraint:

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[1]Since perfectly efficient detectors and perfectly functioning analyzers (e.g., Stern-Gerlach devices and polarization filters) are idealizations not to be found among real laboratory equipment, this claim is not altogether true. However, given the magnitude of the relevant experimental uncertainties, specific instances of violations of the claim can quite plausibly be regarded as spurious.

[2]As is shown in the Appendix, quantum mechanics itself satisfies conservation. Indeed, in the quantum-mechanical account of these phenomena, this condition is taken to express the conservation of angular momentum. (Hence the name 'conservation,' as used in Skyrms [29].)

For all $\lambda \varepsilon \Lambda_o$, $d \varepsilon \Delta$, $\bar{\mu}_L \varepsilon M_L^d$, $\bar{\mu}_R \varepsilon M_R^d$, and $x \varepsilon X$,

$$\pi_\lambda^{\mu_L,\mu_R}(d,x;d,x) = 0.$$

Theories which satisfy conservation predict that for pairs of particles in the singlet state, joint measurements of the same component of spin yield opposite values as outcomes (with unit probability).

The status of this condition must now be qualified in accordance with a previous remark.[1] Because there are uncertainties in the experimental results[2] (as with any experiment), a theory need not strictly satisfy conservation in order to be compatible[3] with the experimental evidence. This gives rise to a class of local realistic theories which exploit the uncertainties in actual experimental data in order to escape the verdict of Bell-type arguments. These theories typically do violate conservation.[4] Nevertheless, if there is a weak link in the case against local realism, it is not conservation. As previously mentioned,[5] conservation is not employed in derivations of the generalized Bell-type inequalities, which have been well tested experimentally. Such inequalities are violated by theories which predict a sufficiently strong (but never necessarily perfect) correlation between measurement outcomes.

These considerations regarding conservation provide an appropriate

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[1]See n. 1, p. 32.

[2]One might consider here the comparable status of experiments which can only establish an exceedingly small, but non-zero, mass of the photon.

[3]See n. 2, p. 29.

[4]See Fine [17], [18], and [20], Shimony [27], and Lo and Shimony [25] for more on these theories.

[5]See n.2, p. 19.

context in which to address an ambiguity attending the presentation thus far, and to clarify further the approach to be followed in subsequent chapters. Consider that, in general, the state descriptions admitted by a theory may be deterministic (i.e., at each moment, the states may determine the outcome of every possible spin measurement at that moment) or they may be indeterministic (i.e., at each moment, the states may determine only the probability for each possible outcome of every possible spin measurement at that moment). However, in each of these cases, the states of the system may exhibit time-dependence; they may undergo temporal evolution. Moreover, that temporal evolution itself may be either deterministic or indeterministic (stochastic); i.e., the law(s) governing the evolution either may or may not be such that the state, at every moment, is sufficient to determine the state at every future moment.

Thus, the probabilistic or statistical character of theoretical predictions may be associated with states of various types: (1) time-independent indeterministic states; (2) stochastically-evolving deterministic states; (3) deterministically-evolving indeterministic states; and (4) stochastically-evolving indeterministic states. Similarly, deterministic theoretical predictions may be associated with time-independent deterministic states as well as deterministically-evolving deterministic states.

It should be apparent that theories which admit relevantly[1] time-

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[1]A state of the two-particle system is relevantly time-dependent if at least some of its associated probability functions exhibit time-dependence not solely resulting from that of the measuring device states. Two-particle states whose time-dependence resides exclusively in parameters which do not contribute to the probabilities of interest, or whose associated probability functions have no time-dependence apart from that resulting from time-dependent measuring device states, are, for present purposes, time-independent.

dependent states are difficult to reconcile with conservation. If the states of the system do vary with time, then measurement outcomes (or probabilities of such) ought to be sensitive to variations in the time interval between the two measurement events; but a theory which satisfies conservation predicts that in those pairs of particles for which the outcome of a d-component spin measurement on particle L is +1, the outcome of a d-component spin measurement on particle R will be -1 (with probability 1) no matter when the R measurement occurs.

For this reason, the discussion of the immediately succeeding chapters will be tacitly restricted to those theories whose (deterministic or indeterministic) states are time-independent. In particular, the account to be given in Chapter VI of the local realistic basis of the completeness condition will not apply to theories which admit time-dependent states. Since the local realistic justification for the strong locality condition derives from that of completeness (and locality), it too will break down for such theories.

It might reasonably be required of any "respectable" theory aspiring to provide a local realistic account of the Bell-type phenomena, that it respect conservation. However, since conservation is not required for empirical adequacy, it may therefore be of some interest to consider the problem of devising a suitable generalization of the strong locality condition which would allow a version of Bell's Theorem that does apply to theories which admit time-dependent states. So as not to interrupt the flow of the main argument by complicating the exposition in ways that might obscure the issues of primary concern, this problem will be postponed until Chapter IX.

CHAPTER V

A BELL-TYPE INEQUALITY FOR DETERMINISTIC THEORIES

At last the stage is set for the derivation of the Bell-Wigner inequality for deterministic theories which satisfy locality and conservation.[1] Recall that for any local theory, the probability for any given measurement outcome is determined by the state of the two-particle system and that of the participating measuring device (independently of the state of the distant measuring device). Any local deterministic theory specifies the outcome of each possible spin measurement as a function of the state of the two-particle system and that of the participating measuring device. (For what follows, it will only be needed that measurement outcomes are determined with unit probability.)

Under these assumptions, for every direction $d \in \Delta$, there exists a well-defined set $D_L^+(\mu_L) \subset \Lambda$ consisting of just those states of the two-particle system for which the theory predicts (with probability 1) the spin up outcome for a measurement of the d-component of the spin of particle L when the L measuring device is in the state $\bar{\mu}_L \in M_L^d$; i.e., $\lambda \in D_L^+(\mu_L)$ just in case $\sum_{x \in X} \pi_\lambda^{\mu_L,\mu_R}(d,+1;d_R,x) = 1$ for arbitrary $d_R \in \Delta$ and $\bar{\mu}_R \in M_R^{d_R}$. $D_L^-(\mu_L)$, $D_R^+(\mu_R)$, and $D_R^-(\mu_R)$ are defined analogously.[2]

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[1]This inequality was first derived by Wigner in [31].

[2]Note that in all of these definitions, the assumption of locality obviates explicit reference to the states in M_L^0 and M_R^0.

Under the additional assumption of conservation, these set functions are constant with respect to their arguments, as will now be shown for D_L^+. (The proofs for the others have the same form.) Consider arbitrary $d \in \Delta$, $\bar{\mu}_R \in M_R^d$, and $\bar{\mu}_L, \bar{\mu}'_L \in M_L^d$, where $\mu_L \neq \mu'_L$. Then,

$$\lambda \in D_L^+(\mu_L) \Longleftrightarrow \sum_{x \in X} \pi_\lambda^{\mu_L,\mu_R}(d,+1;d,x) = 1 \qquad \text{by locality and the definition of } D_L^+$$

$$\Longleftrightarrow \pi_\lambda^{\mu_L,\mu_R}(d,+1;d,-1) = 1 \qquad \text{by conservation}$$

$$\Longleftrightarrow \sum_{x \in X} \pi_\lambda^{\mu_L,\mu_R}(d,x;d,-1) = 1 \qquad \text{by conservation}$$

$$\Longleftrightarrow \sum_{x \in X} \pi_\lambda^{\mu'_L,\mu_R}(d,x;d,-1) = 1 \qquad \text{by locality}$$

$$\Longleftrightarrow \pi_\lambda^{\mu'_L,\mu_R}(d,+1;d,-1) = 1 \qquad \text{by conservation}$$

$$\Longleftrightarrow \sum_{x \in X} \pi_\lambda^{\mu'_L,\mu_R}(d,+1;d,x) = 1 \qquad \text{by conservation}$$

$$\Longleftrightarrow \lambda \in D_L^+(\mu'_L) \qquad \text{by locality and the definition of } D_L^+.$$

Therefore, given the state $\lambda \in \Lambda_o$ of the two-particle system and the direction $d \in \Delta$, the theory must predict the outcome of a d-component spin measurement on either particle independently of any further specification of the measuring device states. The outcome is determined by λ and d alone. This can now be incorporated into the notation by defining, for example, the set D_L^+ as follows: $D_L^+ = \{\lambda | \lambda \in D_L^+(\mu_L)$ for arbitrary $\bar{\mu}_L \in M_L^d\}$; the sets D_L^-, D_R^+, and D_R^- are defined analogously.[1]

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[1]See n. 2, p. 36.

Since the theories under consideration are deterministic and there are only two possible outcomes to spin measurements, for all $\lambda \varepsilon \Lambda_o$ and $d \varepsilon \Delta$, $\lambda \varepsilon D_L^- \Longleftrightarrow \lambda \notin D_L^+$. It follows from conservation that for all $\lambda \varepsilon \Lambda_o$ and $d \varepsilon \Delta$, $\lambda \varepsilon D_L^- \Longleftrightarrow \lambda \varepsilon D_R^+$. Combining these yields that for all $\lambda \varepsilon \Lambda_o$ and $d \varepsilon \Delta$, $D_L^+ \cup D_R^+ = \Lambda_o$ and $D_L^+ \cap D_R^+ = \phi$. Thus each component of spin (i.e., each direction in space) partitions the set Λ_o of states of the two-particle system into two mutually exclusive and jointly exhaustive subsets. The subsets determined by any given component of spin contain just those states for which the theory predicts (with probability 1) the outcome +1 for L and R measurements, respectively, of the given spin component.

Let a, b, and c be arbitrary unit vectors. It follows that $A_L^+ \cap B_R^+ \subseteq (A_L^+ \cap C_R^+) \cup (C_L^+ \cap B_R^+)$. (See Figure 5.)

Proof: Since $C_R^+ \cup C_L^+ = \Lambda_o$,

$$\begin{aligned} A_L^+ \cap B_R^+ &= (A_L^+ \cap B_R^+) \cap (C_R^+ \cup C_L^+) \\ &= [(A_L^+ \cap B_R^+) \cap C_R^+] \cup [(A_L^+ \cap B_R^+) \cap C_L^+] \\ &\subseteq (A_L^+ \cap C_R^+) \cup (C_L^+ \cap B_R^+). \end{aligned}$$

Therefore, for any probability measure m defined on Λ_o,

$$m(A_L^+ \cap B_R^+) \leq m(A_L^+ \cap C_R^+) + m(C_L^+ \cap B_R^+). \tag{10}$$

Although the theories under consideration are deterministic with respect to the "hidden" states $\lambda \varepsilon \Lambda_o$, the observable evidence the predictions of the theory must confront is statistical in character. From the perspective of these theories, this statistical character of the results of actual experiments is not due to any inherent indeterminism in

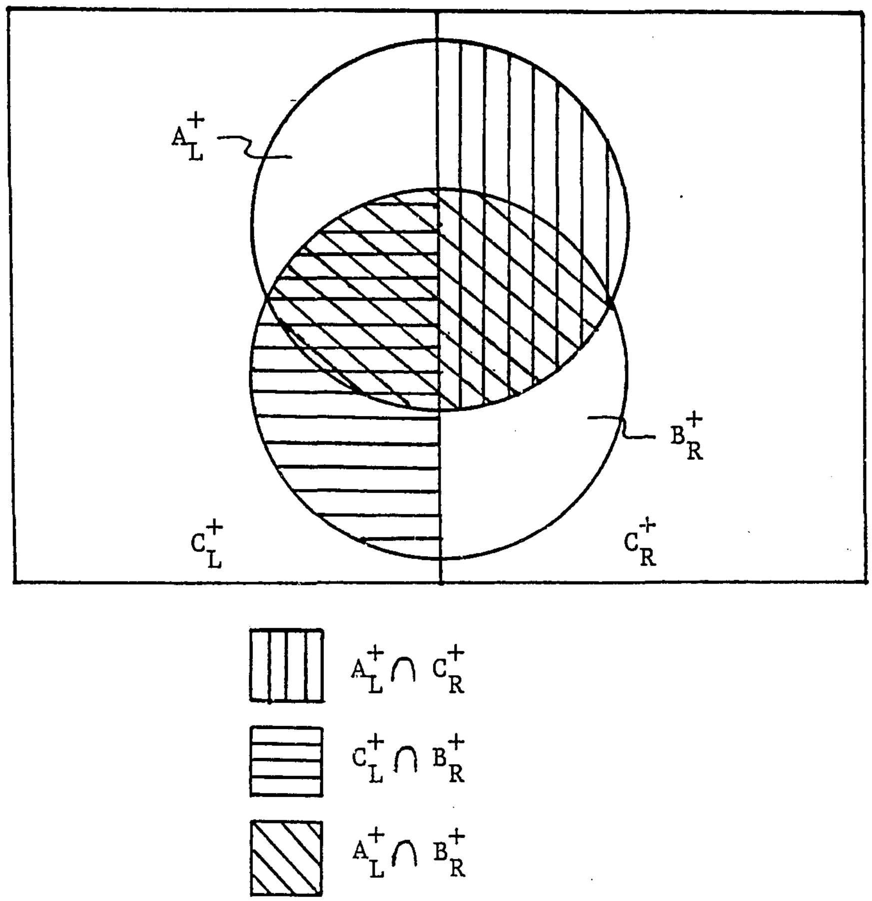

Fig. 5.--Set of States Λ_o Partitioned to Illustrate the Bell-Wigner Inequality.

the world, but rather to our ignorance of the fully specified "hidden" state of each particle pair. For the various combinations of spin measurements performed on any large number of pairs of particles in the singlet state, the observed relative frequencies of measurement outcomes are to be understood in terms of the distribution of these underlying "hidden" states.

For each pair of directions u and v, the probability $P_{LR}(u_L^+ \,\&\, v_R^+)$ a local deterministic theory assigns to the pair of +1 outcomes of joint measurements of the u-component of the spin of particle L and the v-component of the spin of particle R is given by $p(U_L^+ \cap V_R^+)$, where p is the probability measure on Λ_o associated with the distribution of underlying states. (The probabilities for the other pairs of outcomes are determined similarly.) No matter how the underlying states may be distributed (i.e., no matter what probability measure p is assumed), it follows immediately from (10) above that for any three directions a, b, and c,

$$(11) \qquad P_{LR}(a_L^+ \,\&\, b_R^+) \leq P_{LR}(a_L^+ \,\&\, c_R^+) + P_{LR}(c_L^+ \,\&\, b_R^+).$$

This is one form of the Bell-Wigner inequality. The predictions of quantum mechanics violate the Bell-Wigner inequality[1] and, although the experimental evidence against it is not overwhelming, generalized versions of this inequality are violated in strong experimental tests.[2]

This concludes the first stage of the argument. Since locality is warranted by special relativity (which is extraordinarily well-confirmed independently), the experimental evidence against Bell-type inequalities

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[1]See the Appendix.

[2]See Clauser and Shimony [12], pp. 1918-21, for a discussion of the experimental results.

is direct evidence against determinism.[1] If the experimental evidence can be trusted, then any local realistic account of Bell-type phenomena must proceed by way of a genuinely indeterministic theory. Einstein's commitment to determinism has been mentioned. However, most physicists since Einstein's time have reconciled themselves to an indeterministic world-view, largely because of the unprecedented success of quantum mechanics. To them, this result may be of little significance. In the second stage of the argument it will be shown that the retreat to indeterministic theories fails to rescue local realism. To everyone but the most extreme instrumentalists, this result must be seen as a profound enigma.

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[1]As previously noted, derivations of generalized Bell-type inequalities proceed from the assumption of strong locality alone, without invoking conservation. It follows from results to be established in Chapters VI and VII that in the context of deterministic theories, strong locality is logically equivalent to locality. (See n. 1, p. 50.)

It must also be acknowledged here that the claim in the text is, at this point, still something of an overstatement. It is being taken for granted here and throughout that the results of experiments in progress (which include the first in which the state preparations of the two measuring devices take place in mutually spacelike regions) will confirm the predictions of quantum mechanics as expected. However, because the backward light cones of events associated with the state preparations of the two measuring devices must intersect, no such results can ever be altogether decisive.

CHAPTER VI

COMPLETENESS

A theory will be said to be <u>complete</u> (or to satisfy <u>completeness</u>) just in case the probability functions it specifies meet the following constraint:

For all $\lambda \varepsilon \Lambda$, $d_L, d_R \varepsilon \Delta$, $\bar{\mu}_L \varepsilon M_L^{d_L}$, $\bar{\mu}_R \varepsilon M_R^{d_R}$, and $x_L, x_R \varepsilon X$,

$$(12) \qquad \pi_\lambda^{\mu_L,\mu_R}(d_L,x_L;d_R,x_R) =$$

$$\sum_{x_R' \varepsilon X} \pi_\lambda^{\mu_L,\mu_R}(d_L,x_L;d_R,x_R') \cdot \sum_{x_L' \varepsilon X} \pi_\lambda^{\mu_L,\mu_R}(d_L,x_L';d_R,x_R).$$

Completeness asserts the stochastic independence of the two outcomes in each pair of spin measurements.[1] This can be interpreted as a natural requirement for theories which represent observable phenomena as the effects of interacting (but otherwise independently existing) entities whose physical state may be exhaustively characterized by the specification of some (not necessarily unique) set of definite, well-defined properties. This recommends completeness as a reasonable constraint on local realistic theories.

For complete theories, given the state of the two-particle system

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[1]To see this, notice that the right-hand side of (12) is a product of two terms. The first term is the probability for the x_L outcome of the L measurement (with unspecified outcome of the R measurement) and the second term is the probability for the x_R outcome of the R measurement (with unspecified outcome of the L measurement).

and the states of both measuring devices, the probability for a given outcome of the given spin measurement at L(R) is independent of the outcome of the other spin measurement at R(L); that is to say, the probability for the given outcome at L(R) is invariant under conditionalization on the outcome of the measurement at R(L). However, unlike local theories, the probability for that outcome of the L(R) measurement may very well exhibit a dependence on the state of the distant measuring device R(L). By including in the state descriptions of the two-particle system and the measuring devices all those properties of the systems (i.e., precise numerical values of whatever physical quantities are appropriate for the types of entities posited by the theory) in virtue of which measurement interactions yield each possible outcome with its assigned probability, such state descriptions automatically "screen off" any correlation of the pairs of measurement outcomes which might arise from the omission from the state descriptions of predictively relevant information.

To be sure, knowledge of the outcome x_R of an R measurement may (depending on the character of the laws governing the interactions) ground inferences about the properties of particle L. In this way, one might learn from the outcome x_R that particle L is characerized by such-and-such values of such-and-such quantities with such-and-such probabilities; but the very most that one could infer from the outcome x_R is that particle L definitely possesses certain properties. If all such information about particle L is antecedently included in the state description, then knowledge of the outcome x_R provides only redundant information about the L subsystem.[1] Hence, the conditional probability of the outcome x_L of the L

[1] On this point, theories which admit relevantly time-dependent states contravene completeness. See Chapter IX.

measurement given the outcome x_R of the R measurement must be equal to the unconditioned probability for x_L.[1] In other words, the two outcomes are stochastically independent.

A simple (and incorrect) model for the Bell-type correlated spin phenomena may serve as a useful illustration.[2] Suppose, purely for the sake of this illustration, that spin is correctly represented as an ordinary classical angular momentum. Suppose further that when a pair of particles is prepared in the singlet state, the spin vectors for the two particles are aligned exactly anti-parallel to each other. Moreover, given an ensemble of such two-particle systems, suppose that each direction in space is equally likely to be the direction of alignment for an arbitrarily selected member of the ensemble. Finally, if unit vector d gives the direction along which the axis of the Stern-Gerlach apparatus is aligned (i.e., the d-component of the spin is to be measured), and if s is the spin vector of the particle (with units chosen so that s has unit magnitude), then the outcome of that measurement is +1 if $d \cdot s > 0$ and -1 if $d \cdot s < 0$.[3]

According to this model, then, the probability for a spin up (or +1) outcome of a measurement of any component of the spin of either particle of

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[1]The issue here is not fundamentally an epistemological one. The point is that if the state descriptions are complete in the relevant sense, then conditionalization on the outcome of a measurement on particle R entails no further restrictions on the physical possibilities that would serve to better define the probabilities for the outcomes of possible measurements on particle L. However, it may be helpful to think of this conditionalization in terms of the acquisition of knowledge.

[2]Bell discussed a model of this sort in his 1964 paper ([5]). It is being used here to illustrate that theories which admit state descriptions which are compatible with more fully specified "hidden" states do not satisfy completeness.

[3]There is no need to consider the set of directions d such that $d \cdot s = 0$, since that set is of measure zero and will not bear on the claims of interest here.

a randomly chosen pair of particles in the singlet state is ½. For cases in which measurements are performed on both particles of a pair in the singlet state (denoted below by 'σ'), this may be expressed as follows:

For all $d_L, d_R \in \Delta$, $\bar{\mu}_L \in M_L^{d_L}$, $\bar{\mu}_R \in M_R^{d_R}$, and $x_L, x_R \in X$,

$$\sum_{x'_R \in X} \pi_\sigma^{\mu_L,\mu_R}(d_L,x_L;d_R,x'_R) = \tfrac{1}{2} = \sum_{x'_L \in X} \pi_\sigma^{\mu_L,\mu_R}(d_L,x'_L;d_R,x_R).$$

However, if a d-component spin measurement is performed on each particle of a pair, the probability of finding the same outcome for both of the measurements must be 0. (This is the conservation condition.) So, for all $d \in \Delta$, $\bar{\mu}_L \in M_L^d$, $\bar{\mu}_R \in M_R^d$, and $x \in X$,

$$\pi_\sigma^{\mu_L,\mu_R}(d,x;d,x) = 0.$$

In this case, then, completeness is clearly violated. (See equation (12).)

The probabilities specified for this model are grounded in a blatantly incomplete description of the two-particle state. In the context of this model, if the theory assigns probabilities only on the basis of the occupancy by the two-particle system of the singlet state, then conditioning on the outcome, let us say, of a d-component spin measurement on particle R(L) may well yield a different probability for the outcome of a spin measurement on particle L(R) than would have been given by the corresponding unconditioned probability (i.e., ½). This is so because, if the outcome of the measurement on particle R is +1, then it may be inferred that $d \cdot s_L < 0$ (with probability 1), where s_L is the spin of particle L. Similarly, if the outcome is -1, then it may be inferred that $d \cdot s_L > 0$ (with probability 1). The outcome of the measurement on particle R provides information about particle L which was not included in the incomplete state description σ.

For a theory which assigns probabilities on the basis of the complete state description (a state description which, for this model, would entail exact values for s_L and $s_R = -s_L$), any information about particle L(R) which may be inferred from the outcome of a measurement on particle R(L) is clearly redundant. In general, the very most that one could infer about an arbitrary spin measurement on particle L(R) from the outcome of a spin measurement on particle R(L) is considerably less than the knowledge of the L(R) measurement outcome; but given a complete state description, that outcome is determined (with probability 1) antecedently by $s_L (s_R)$ and the state of the L(R) measuring device (i.e., the component of spin to be measured).

Indeed, all deterministic theories satisfy the completeness condition. This holds not only for models of the sort just described, but is true quite generally. To see this, consider the table below. It lists in each column one of the four possible sets of numerical assignments (those satisfying equation (3)) to the terms occurring in equation (12) for deterministic theories. Inspection of Table 3 reveals that for every instantiation of numerical values for x_L and x_R in (12), the values in each column do satisfy (12).[1]

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[1]Consider the following constraint:

(Condition C) For all $\lambda \varepsilon \Lambda$, $d_L, d_R \varepsilon \Delta$, $\bar{\mu}_L \varepsilon M_L^{d_L}$, and $\bar{\mu}_R \varepsilon M_R^{d_R}$,

$$\pi_\lambda^{\mu_L,\mu_R}(d_L,+1;d_R,+1) \cdot \pi_L^{\mu_L,\mu_R}(d_L,-1;d_R,-1)$$

$$= \pi_\lambda^{\mu_L,\mu_R}(d_L,+1;d_R,-1) \cdot \pi_\lambda^{\mu_L,\mu_R}(d_L,-1;d_R,+1).$$

That deterministic theories must satisfy completeness may also be seen by

TABLE 3

POSSIBLE VALUES (IN THE DETERMINISTIC CASE) FOR THE TERMS IN EQUATION (12)

$\pi_\lambda^{\mu_L,\mu_R}(d_L,+1;d_R,+1)$	1	0	0	0
$\pi_\lambda^{\mu_L,\mu_R}(d_L,+1;d_R,-1)$	0	1	0	0
$\pi_\lambda^{\mu_L,\mu_R}(d_L,-1;d_R,+1)$	0	0	1	0
$\pi_\lambda^{\mu_L,\mu_R}(d_L,-1;d_R,-1)$	0	0	0	1
$\sum_{x_R'\varepsilon X} \Pi_\Lambda^{\mu_L,\mu_R}(d_L,+1;d_R,x_R')$	1	1	0	0
$\sum_{x_R'\varepsilon X} \pi_\lambda^{\mu_L,\mu_R}(d_L,-1;d_R,x_R')$	0	0	1	1
$\sum_{x_L'\varepsilon X} \pi_\lambda^{\mu_L,\mu_R}(d_L,x_L';d_R,+1)$	1	0	1	0
$\sum_{x_L'\varepsilon X} \pi_\lambda^{\mu_L,\mu_R}(d_L,x_L';d_R,-1)$	0	1	0	1

This accords well with the interpretation of the completeness condition just put forward. If the state descriptions go so far as to <u>determine</u>

first showing that completeness is logically equivalent to condition C and then noting that for deterministic theories, of the four probabilities occurring in the statement of condition C, three must vanish (and the other must be 1).

measurement outcomes (and not just the probabilities for such), then those state descriptions include a fortiori (at least implicitly) a full account of whatever features of the physical situation contribute to the probabilities for the possible measurement outcomes.

The underlying deterministic character exhibited by the model discussed in this chapter is by no means essential for completeness; genuinely indeterministic theories certainly may satisfy (12). However, it is true that models which attribute strictly anti-correlated spins to the particles can be adequately described by a local complete theory only if that theory is deterministic (at least in the sense that it makes probability assignments of 0 and 1 only).[1] The proof that local complete indeterministic theories are incompatible with such models is given in Chapter VIII.[2]

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[1]This would not necessarily be the case if measurements did not minimally have to disclose the anti-correlation. For the claim to be strictly true, it must be assumed that the theories in question satisfy conservation.

[2]It will first be shown in the next chapter that local complete theories are precisely those which satisfy strong locality.

CHAPTER VII

STRONG LOCALITY

A theory will be said to be strongly local (or to satisfy strong locality) just in case the probability functions it specifies meet the following constraint:

For all $\lambda \varepsilon \Lambda$, $d_L, d_R \varepsilon \Delta$, $\bar{\mu}_L \varepsilon M_L^{d_L}$, $\bar{\mu}_R \varepsilon M_R^{d_R}$, $\bar{\mu}_L^\circ \varepsilon M_L^0$, $\bar{\mu}_R^\circ \varepsilon M_R^0$, and $x_L, x_R \varepsilon X$,

$$\pi_\lambda^{\mu_L,\mu_R}(d_L,x_L;d_R,x_R) = \pi_\lambda^{\mu_L,\mu_R^\circ}(d_L,x_L;0,0) \cdot \pi_\lambda^{\mu_L^\circ,\mu_R}(0,0;d_R,x_R).$$

It is not difficult to show that strong locality is logically equivalent to the conjunction of locality and completeness. Consider an arbitrary strongly local theory. Then, for all $\lambda \varepsilon \Lambda$, $d_L, d_R \varepsilon \Delta$, $\bar{\mu}_L \varepsilon M_L^{d_L}$, $\bar{\mu}_R \varepsilon M_R^{d_R}$, $\bar{\mu}_L^\circ \varepsilon M_L^0$, $\bar{\mu}_R^\circ \varepsilon M_R^0$, and $x_L, x_R \varepsilon X$,

$$\sum_{x_R' \varepsilon X} \pi_\lambda^{\mu_L,\mu_R}(d_L,x_L;d_R,x_R') = \pi_\lambda^{\mu_L,\mu_R^\circ}(d_L,x_L;0,0)\left[\sum_{x_R' \varepsilon X} \pi_\lambda^{\mu_L^\circ \cdot \mu_R}(0,0;d_R,x_R')\right]$$

$$= \pi_\lambda^{\mu_L,\mu_R^\circ}(d_L,x_L;0,0) \qquad \text{by equation (2);}$$

and

$$\sum_{x_L' \varepsilon X} \pi_\lambda^{\mu_L,\mu_R}(d_L,x_L';d_R,x_R) = \left[\sum_{x_L' \varepsilon X} \pi_\lambda^{\mu_L \cdot \mu_R^\circ}(d_L,x_L';0,0)\right] \pi_\lambda^{\mu_L^\circ,\mu_R}(0,0;d_R,x_R),$$

$$= \pi_\lambda^{\mu_L^\circ,\mu_R}(0,0;d_R,x_R) \qquad \text{by equation (1).}$$

Therefore, all strongly local theories are local.

Also, if strong locality holds, then for all $\lambda \varepsilon \Lambda$, $d_L, d_R \varepsilon \Delta$, $\bar{\mu}_L \varepsilon M_L^{d_L}$, $\bar{\mu}_R \varepsilon M_R^{d_R}$, $\bar{\mu}_L^\circ \varepsilon M_L^0$, $\bar{\mu}_R^\circ \varepsilon M_R^0$, and $x_L, x_R \varepsilon X$,

$$\sum_{x_R' \varepsilon X} \pi_\lambda^{\mu_L,\mu_R}(d_L,x_L;d_R,x_R') \sum_{x_L' \varepsilon X} \pi_\lambda^{\mu_L,\mu_R}(d_L,x_L';d_R,x_R)$$

$$= \pi_\lambda^{\mu_L,\mu_R^\circ}(d_L,x_L;0,0)\left[\sum_{x_R' \varepsilon X} \pi_\lambda^{\mu_L,\mu_R^\circ}(d_L,x_L';0,0)\right]$$

$$\cdot \left[\sum_{x_L' \varepsilon X} \pi_\lambda^{\mu_L,\mu_R^\circ}(d_L,x_L';0,0)\right]\pi_\lambda^{\mu_L^\circ,\mu_R}(0,0;d_R,x_R)$$

$$= \pi_\lambda^{\mu_L,\mu_R^\circ}(d_L,x_L;0,0) \cdot \pi_\lambda^{\mu_L^\circ,\mu_R}(0,0;d_R,x_R) \qquad \text{by (1) and (2)}$$

$$= \pi_\lambda^{\mu_L,\mu_R}(d_L,x_L;d_R,x_R).$$

Therefore, all strongly local theories are complete.

It remains only to show that complete local theories are strongly local. Assume locality and completeness. Then, for all $\lambda \varepsilon \Lambda$, $d_L, d_R \varepsilon \Delta$, $\bar{\mu}_L \varepsilon M_L^{d_L}$, $\bar{\mu}_R \varepsilon M_R^{d_R}$, $\bar{\mu}_L^\circ \varepsilon M_L^0$, $\bar{\mu}_R^\circ \varepsilon M_R^0$, and $x_L, x_R \varepsilon X$,

$$\pi_\lambda^{\mu_L,\mu_R}(d_L,x_L;d_R,x_R) = \sum_{x_R' \varepsilon X} \pi_\lambda^{\mu_L,\mu_R}(d_L,x_L;d_R,x_R') \sum_{x_L' \varepsilon X} \pi_\lambda^{\mu_L,\mu_R}(d_L,x_L';d_R,x_R) \qquad \text{by equation (12)}$$

$$= \pi_\lambda^{\mu_L,\mu_R^\circ}(d_L,x_L;0,0) \cdot \pi_\lambda^{\mu_L^\circ,\mu_R}(0,0;d_R,x_R)$$

by equations (4) and (5).

This conclusion the proof.[1]

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[1]Therefore, for complete theories, locality is logically equivalent to strong locality. Since all deterministic theories are complete (as was

Strong locality is the assumption used in the derivation of generalized Bell-type inequalities.[1] As the conjunction of locality and completeness, strong locality requires of the probability for an outcome of a measurement on particle L(R) that it be independent of the outcome of any corresponding measurement on particle R(L) and that it be independent of the state of the R(L) measuring device. The local realistic character of strong locality derives directly from that of locality and completeness.

It is important to be clear about the distinction between completeness and strong locality. Although all strongly local theories are complete, complete theories need not be strongly local. This is so for complete theories which violate locality. The predictions of such a theory have the following character: The probability for the outcome of the L(R) measurement depends not only on the state of the two-particle system and the state of measuring device L(R), but it depends also on the state of the (other) measuring device R(L). (This is the "non-locality.") Nevertheless, that probability is invariant under conditionalization on the outcome of

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shown in the previous chapter), it follows that for deterministic theories, locality is logically equivalent to strong locality. (See Hellman [21] for a different treatment of essentially the same point.)

[1]In other treatments of the subject, one finds the assumption from which generalized Bell-type inequalities may be derived called by a variety of names. These include "local causality" (e.g., Bell [6]), "factorizability" (e.g., Fine [18]), "conditional stochastic independence" (e.g., Hellman [22]), and a number of others. Since the argument is set up differently in different presentations, these conditions cannot be regarded as logically equivalent in any strict sense. However, it will be generally acknowledged that each "does the same work" in that version of the argument in which it occurs and that their content is essentially that of strong locality. It should be noted that Clauser and Shimony ([12]: 1891-93) do employ a locality condition which is tantamount to strong locality, but their discussion of that condition may be a bit misleading; they appear to suggest that their condition is no more than the relativistic locality constraint (i.e., the condition herein called "locality") for indeterministic theories.

the R(L) measurement (if any); the three state descriptions, taken together, include a full record of all predictively relevant physical properties of the systems. (This is the "completeness.")

Each of these three conditions, locality, completeness, and strong locality, is a requirement that certain probabilities be invariant under some form of conditionalization. With the help of a bit more notation, this similarity of form may be exhibited more explicitly. The required definitions will now be introduced.

For all $\lambda \varepsilon \Lambda$, $d_L, d_R \varepsilon \Delta$, $\bar{\mu}_L \varepsilon M_L^{d_L}$, $\bar{\mu}_R \varepsilon M_R^{d_R}$, and $x_L, x_R \varepsilon X$,

$$\pi_\lambda^{\mu_L,\mu_R}(d_L,x_L;d_R,-) \equiv \sum_{x_R' \varepsilon X} \pi_\lambda^{\mu_L,\mu_R}(d_L,x_L;d_R,x_R')$$

and

$$\pi_\lambda^{\mu_L,\mu_R}(d_L,-;d_R,x_R) \equiv \sum_{x_L' \varepsilon X} \pi_\lambda^{\mu_L,\mu_R}(d_L,x_L';d_R,x_R).$$

For all $\lambda \varepsilon \Lambda$, $d_L \varepsilon \Delta$, $\bar{\mu}_L \varepsilon M_L^{d_L}$, and $\bar{\mu}_R \varepsilon M_R^0$,

$$\pi_\lambda^{\mu_L,\mu_R}(d_L,-;0,0) \equiv \sum_{x_L \varepsilon X} \pi_\lambda^{\mu_L,\mu_R}(d_L,x_L;0,0) = 1$$

(by equation (1))

For all $\lambda \varepsilon \Lambda$, $\bar{\mu}_L \varepsilon M_L^0$, $d_R \varepsilon \Delta$, and $\bar{\mu}_R \varepsilon M_R^{d_R}$,

$$\pi_\lambda^{\mu_L,\mu_R}(0,0;d_R,-) \equiv \sum_{x_R \varepsilon X} \pi_\lambda^{\mu_L,\mu_R}(0,0;d_R,x_R) = 1$$

(by equation (2))

Now each of the three conditions can be expressed in the same form: For all $\lambda \varepsilon \Lambda$, $\bar{\mu}_L \varepsilon M_L^{d_L}$, $\bar{\mu}_R \varepsilon M_R^{d_R}$, $\bar{\mu}_L' \varepsilon M_L^{d_L'}$, and $\bar{\mu}_R' \varepsilon M_R^{d_R'}$,

$$\pi_\lambda^{\mu_L,\mu_R}(d_L,x_L;d_R,x_R) = \pi_\lambda^{\mu_L,\mu_R'}(d_L,x_L;d_R',-) \cdot \pi_\lambda^{\mu_L',\mu_R}(d_L',-;d_R,x_R),$$

provided that certain of the variables are restricted to the appropriate sets, as specified in the table below:

TABLE 4

RESTRICTIONS IN THE RANGES OF VARIABLES NEEDED TO EXPRESS COMPLETENESS, LOCALITY, AND STRONG LOCALITY IN THE SAME FORM

		d_L	d_R	x_L	x_R	$\bar{\mu}_L'$	$\bar{\mu}_R'$
COMPLETENESS		Δ	Δ	X	X	$\{\bar{\mu}_L\}$	$\{\bar{\mu}_R\}$
LOCALITY	(4)	Δ	$\{0\}$	X	$\{0\}$	M_L^Δ	M_R^Δ
	(5)	$\{0\}$	Δ	$\{0\}$	X	M_L^Δ	M_R^Δ
STRONG LOCALITY*		$\Delta \cup \{0\}$	$\Delta \cup \{0\}$	$X \cup \{0\}$	$X \cup \{0\}$	M_L^Δ	M_R^Δ

*Of course, those cases for which the π_λ's are undefined are to be excluded. (See p. 17.)

In view of this table, the equivalence between strong locality and the conjunction of locality and completeness is obvious. The table also provides a convenient place to pause for a brief summary:

(i) Completeness requires that the probability for the outcome of an L(R) measurement be invariant under conditionalization on the <u>outcome</u> of the distant R(L) measurement; it may depend on the state of the two-particle system and on <u>both</u> measuring device states. The outcome of the L(R) measurement must be stochastically independent of the outcome of the R(L) measurement.

(ii) Locality requires that the probability for the outcome of an L(R) measurement be invariant under conditionalization on the <u>state</u> of the distant measuring device R(L); it may depend on the state of the two-particle system and on the state of the L(R) measuring device. The outcome of the L(R) measurement need not be stochastically independent of the outcome of the R(L) measurement.

(iii) Finally, as the conjunction of locality and completeness, strong locality requires that the probability for the outcome of an L(R) measurement be invariant under conditionalization on the <u>state</u> of the distant measuring device R(L) <u>and</u> that it be invariant under conditionalization on the <u>outcome</u> of the R(L) measurement. The outcome of the L(R) measurement must be stochastically independent of the outcome of the R(L) measurement.

CHAPTER VIII

A BELL-TYPE INEQUALITY FOR INDETERMINISTIC THEORIES

All of the components are now in place to give the second stage of the argument; i.e., to show that any indeterministic theory which satisfies strong locality and conservation must satisfy the Bell-Wigner inequality. Recall that the derivation in Chapter V of that inequality for deterministic theories relied only on the assumptions of locality and conservation. Since it has been established that locality is a consequence of strong locality, it suffices to show that any theory which satisfies strong locality and conservation is deterministic (in the sense that all probabilities are 0 or 1).[1]

That any strongly local indeterministic theory must run afoul of conservation can readily be seen. Once the outcome x_R of a measurement of some spin component of particle R is given, conservation fixes (with probability 1) the outcome x_L of a measurement of the same spin component of particle L. Since strong locality requires that the probability assigned to outcome x_L be invariant under conditionalization on the outcome x_R, the outcome $x_L = -x_R$ must be assigned unit probability. In this way, conservation forces any strongly local theory to be deterministic.

[1]This strategy for reducing the indeterministic case to the deterministic case is based on Skyrms [29]. It parallels the reasoning in the original EPR argument. See Fine [19] for a proof which establishes connections of a different sort between the deterministic and indeterministic cases.

More formally,[1] consider arbitrary $\lambda \varepsilon \Lambda_o$, $d \varepsilon \Delta$, $\bar{\mu}_L \varepsilon M_L^d$, $\bar{\mu}_R \varepsilon M_R^d$, $\bar{\mu}_L^\circ \varepsilon M_L^0$, $\bar{\mu}_R^\circ \varepsilon M_R^0$, and $x \varepsilon X$. Then, by conservation, $\pi_\lambda^{\mu_L,\mu_R}(d,x;d,x) = 0$. It follows, by strong locality, that $\pi_\lambda^{\mu_L,\mu_R^\circ}(d,x;0,0) \cdot \pi_\lambda^{\mu_L^\circ,\mu_R}(0,0;d,x) = 0$. Therefore,

$$\text{(i)} \quad \pi_\lambda^{\mu_L,\mu_R^\circ}(d,x;0,0) = 0 \quad \text{or} \quad \pi_\lambda^{\mu_L^\circ,\mu_R}(0,0;d,x) = 0.$$

Repeating the above with -x in place of x gives

$$\text{(ii)} \quad \pi_\lambda^{\mu_L^\circ,\mu_R}(d,-x;0,0) = 0 \quad \text{or} \quad \pi_\lambda^{\mu_L^\circ,\mu_R}(0,0;d,-x) = 0.$$

But
$$\begin{aligned}
\pi_\lambda^{\mu_L^\circ,\mu_R}(0,0;d,x) = 0 &\Longrightarrow \pi_\lambda^{\mu_L^\circ,\mu_R}(0,0;d,-x) = 1 && \text{by (2)} \\
&\Longrightarrow \pi_\lambda^{\mu_L,\mu_R^\circ}(d,-x;0,0) = 0 && \text{by (ii)} \\
&\Longrightarrow \pi_\lambda^{\mu_L,\mu_R^\circ}(d,x;0,0) = 1 && \text{by (1)}.
\end{aligned}$$

So, by (i),

$$\text{(iii)} \quad \pi_\lambda^{\mu_L,\mu_R^\circ}(d,x;0,0) = 0 \quad \text{or} \quad \pi_\lambda^{\mu_L,\mu_R^\circ}(d,x;0,0) = 1.$$

Similarly,

$$\begin{aligned}
\pi_\lambda^{\mu_L,\mu_R^\circ}(d,x;0,0) = 0 &\Longrightarrow \pi_\lambda^{\mu_L,\mu_R^\circ}(d,-x;0,0) = 1 && \text{by (1)} \\
&\Longrightarrow \pi_\lambda^{\mu_L^\circ,\mu_R}(0,0;d,-x) = 0 && \text{by (ii)} \\
&\Longrightarrow \pi_\lambda^{\mu_L^\circ,\mu_R}(0,0;d,x) = 1 && \text{by (2)}.
\end{aligned}$$

So, by (i),

$$\text{(iv)} \quad \pi_\lambda^{\mu_L^\circ,\mu_R}(0,0;d,x) = 0 \quad \text{or} \quad \pi_\lambda^{\mu_L^\circ,\mu_R}(0,0;d,x) = 1.$$

---

[1] This form of the argument is based on a suggestion by Howard Stein.

Since (iii) and (iv) hold for arbitrary $\lambda \varepsilon \Lambda_o$, $d \varepsilon \Delta$, $\bar{\mu}_L \varepsilon M_L^d$, $\bar{\mu}_R \varepsilon M_R^d$, $\bar{\mu}_L^\circ \varepsilon M_L^0$, $\bar{\mu}_R^\circ \varepsilon M_R^0$, and $x \varepsilon X$, it now follows immediately from strong locality that for all $\lambda \varepsilon \Lambda_o$, $d_L, d_R \varepsilon \Delta$, $\bar{\mu}_L \varepsilon M_L^{d_L}$, $\bar{\mu}_R \varepsilon M_R^{d_R}$, and $x_L, x_R \varepsilon X$,

$$\pi_\lambda^{\mu_L,\mu_R}(d_L,x_L;d_R,x_R) = 0 \text{ or } \pi_\lambda^{\mu_L,\mu_R}(d_L,x_L;d_R,x_R) = 1.$$

Therefore, any strongly local theory which satisfies conservation is deterministic (in the sense that all probabilities are 0 or 1) and the Bell-Wigner inequality follows by the previous argument for the deterministic case.

As discussed briefly in Chapter IV, relaxing the requirement of conservation opens the way for a local realistic account of Bell-type phenomena by means of theories which admit time-dependent states. Even though conservation is not required for the derivation of generalized Bell-type inequalities, strong locality is. Since the local realistic basis for completeness collapses in the context of theories which admit time-dependent states, neither is strong locality acceptable as a local realistic constraint on such theories. The next chapter will take up the problem of extending the Bell-type arguments so as to cover local realistic theories which admit time-dependent states.

CHAPTER IX

TIME-DEPENDENT STATES

It is the purpose of this chapter to explore the status of the class of local realistic theories which admit time-dependent states. The claim to be defended is two-fold:

(i) The theories of this class need not satisfy the crucial premise (as it has been expressed heretofore) of the Bell-type arguments, but

(ii) a suitable reformulation of that crucial premise leads to an extension of the Bell-type arguments which does constrain these theories to obey the standard Bell-type inequalities, though not without employing additional premises, the plausibility of which may be disputed.

So as not to appear misleading, however, it should be acknowledged at the outset that, the difficulties in adapting Bell-type arguments to cover this class of theories notwithstanding, further considerations render the prospects for discovering among that class a respectable theory whose predictions for the correlated spin phenomena coincide with those of quantum mechanics exceedingly poor.

In this chapter, it will be convenient to adopt a more widely used notation[1] and to express the condition of interest, strong locality, as follows[2]:

(13) $$p_{12}(\lambda,a,b) = p_1(\lambda,a)\cdot p_2(\lambda,b),$$

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[1]See, for example, Clauser and Horne [11].

[2]As a further simplification, here and throughout this chapter the obvious universal quantification clauses which belong in the full statement of such conditions are suppressed.

where $p_1(\lambda,a)$ is the probability of a spin up outcome of a measurement of the a-component of the spin of particle 1, $p_2(\lambda,b)$ is the probability of a spin up outcome of a measurement of the b-component of the spin of particle 2, and $p_{12}(\lambda,a,b)$ is the probability of joint spin up outcomes when both measurements are performed, all for two-particle systems in the state $\lambda \varepsilon \Lambda$. Further specification of the states of the measuring devices is suppressed in this formalism, but whatever additional information is deemed appropriate may be encoded in the parameters a and b, where these parameters are allowed to have however many components as are necessary to fully specify the measuring device states. Note that locality has been built into this notation by the assumption that p_1 and p_2, as given above, are well-defined; i.e., that $p_1(\lambda,a)$ is independent of the state of measuring device 2 and that $p_2(\lambda,b)$ is independent of the state of measuring device 1. The remarks to follow apply <u>mutatis mutandis</u> to those counterparts to (13) which play the role of strong locality in any Bell-type argument.

λ is generally taken to be the state into which the two-particle system has been "prepared." It is just that "hidden" state in which the particles have been left by their prior interactions. The probabilities in (13) are associated with the outcomes of possible <u>future</u> measurements. In the context of theories for which the state of the two-particle system evolves (perhaps, but not necessarily, stochastically) during the interval between the state preparations and the measurements, if λ in (13) is taken to be the "initial" state of the two-particle system, then it makes sense only to regard the probability functions in (13) as <u>expectations</u> of the following kind, where it is assumed for the moment that the subsequent measurement interactions are instantaneous events that are simultaneous in the laboratory frame and that they always occur at some fixed time τ after the preparation of the initial state:

$$(14)\qquad \begin{cases} p_1(\lambda,a) = \int_\Lambda \bar{p}_1(\lambda',a)dP(\lambda'|\lambda) \\ p_2(\lambda,b) = \int_\Lambda \bar{p}_2(\lambda',b)dP(\lambda'|\lambda) \\ p_{12}(\lambda,a,b) = \int_\Lambda \bar{p}_{12}(\lambda',a,b)dP(\lambda'|\lambda). \end{cases}$$

Here, P is a conditional probability measure. $P(S|\lambda)$ is the probability that a system initially in state λ will have evolved into a state which lies in the set S at the end of time τ. $\bar{p}_1$, $\bar{p}_2$, and $\bar{p}_{12}$ represent probabilities (most naturally regarded as propensities of some sort) for measurement outcomes relative to the state of the system at the instant of measurement (i.e., immediately prior to measurement). p_1, p_2, and p_{12} represent second-order probabilistic quantities--that is to say, these probabilities have as their origin not only the presumedly irreducible indeterminism of the measurement outcomes which would obtain even if the state at the time of measurement, λ_τ, were known (which is just the indeterminism represented by the probabilities $\bar{p}_1$, $\bar{p}_2$, and $\bar{p}_{12}$), but also the uncertainty (relative to the initial state λ) associated with the identity of that future state λ_τ; only the probability that any particular state in Λ will actually be the state into which λ evolves after time τ is given. Accordingly, p_1, p_2, and p_{12} are weighted averages of the "true" probabilities $\bar{p}_1$, $\bar{p}_2$, and $\bar{p}_{12}$ respectively. The conditional probability measure P and the probability functions $\bar{p}_1$, $\bar{p}_2$, and $\bar{p}_{12}$ are to be specified by the theory.

Considerations of completeness suggest that something like (13) may appropriately be imposed on the "true" probability functions evaluated at the moment of measurement;[1] i.e.,

(15) $$\bar{p}_{12}(\lambda_\tau, a, b) = \bar{p}_1(\lambda_\tau, a)\cdot\bar{p}_2(\lambda_\tau, b),$$

but that (13) itself, where the probabilities are of the form given by (14), is not an appropriate constraint on local realistic theories. Consider (15) in the following form:

(15') $$\bar{p}_1(\lambda_\tau, a) = \frac{\bar{p}_{12}(\lambda_\tau, a, b)}{\bar{p}_2(\lambda_\tau, b)}, \quad \text{for } \bar{p}_2(\lambda_\tau, b) \neq 0.$$

The expression on the right-hand side of (15') is the conditional probability[2] for the outcome +1 (i.e., spin up) of an a-component spin measurement on particle 1 given that the outcome of a b-component spin measurement on particle 2 is also +1, for a pair of particles in the state λ_τ at the time of the measurements. (15)' asserts the equality of this conditional probability and the unconditioned probability for the +1 outcome of the

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[1]Actually, the proper condition would have to be stronger than (15). (See n.2, p. 26.) Since λ_τ (it is presumed) must include information about spacetime regions which are spacelike with respect to the measurement on particle 1, a fully relativistic locality condition would require that $\bar{p}_1(\lambda_\tau, a)$ be equal to $\bar{p}_1(\lambda'_\tau, a)$ for all λ_τ and λ'_τ which differ at most on what they say about regions of spacetime which lie outside the backward light cone of the measurement event. A completeness-type condition would then be conjoined with this requirement (and analogous ones for $\bar{p}_2$ and $\bar{p}_{12}$) to form the desired constraint. For purposes of the present discussion, (15) will suffice.

[2]If $\bar{p}_1$, $\bar{p}_2$, and $\bar{p}_{12}$ are regarded as propensities, this expression need be regarded as a "conditional propensity" only in a trivial sense. The point under discussion in this section is that in the context of suitably complete state descriptions, the idea of a genuinely conditional propensity (i.e., one that does not reduce to an ordinary "unconditioned" propensity) cannot be maintained.

measurement on particle 1 (this latter probability being the left-hand side of (15)').

The local realistic basis for this stochastic independence requirement was discussed in Chapter VI. If λ_τ is a suitably complete description of the state of the two-particle system at the time of measurement, then the measurement on particle 2 cannot reveal any predictively relevant information about particle 1 not antecedently deducible from λ_τ. The attempt to provide a parallel account in support of (13), however, fails.

Consider (13) in the following form:

$$(13') \qquad p_1(\lambda,a) = \frac{p_{12}(\lambda,a,b)}{p_2(\lambda,b)}, \quad \text{for } p_2(\lambda,b) \neq 0.$$

The expression on the right-hand side of (13') is the conditional probability, relative to the initial state λ, for the outcome +1 of an a-component spin measurement on particle 1 at time τ, given that the outcome of a simultaneous b-component spin measurement on particle 2 is also +1.[1] (13') asserts the equality of this conditional probability and the unconditioned probability for the +1 outcome of the measurement on particle 1.

To appreciate why it is that (13') need not hold for theories which are nonetheless local realistic, consider that the outcome of the measurement on particle 2 at time τ might very well yield information about particle 1 at time τ which was not included in the initial state λ. It may, for instance, provide some further clue to the identity of λ_τ. Indeed,

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[1]One might have supposed that, instead of the expression on the right-hand side of (13'), which is a ratio of weighted averages of certain probabilities, the correct expression for the conditional probability expressed (in words) above ought to be an appropriately weighted average of the ratio of the probabilities. In fact, the latter expression reduces to that on the right-hand side of (13').

that outcome might even be incompatible with certain possible final states. At any rate, the conditional probability, relative to λ, for some outcome of the measurement on particle 1 at time τ, given some particular outcome of the simultaneous measurement on particle 2, certainly may differ from the unconditioned probability, relative to λ, for the specified outcome of the measurement on particle 1.

A simple example will illustrate the point. Let λ be the initial state of the two-particle system. Suppose that only two possible final states, λ_1 and λ_2, are accessible from λ. Suppose also that the theory makes the following probability assignments:

$$\bar{p}_1(\lambda_1,a) = \frac{1}{2} \qquad \bar{p}_2(\lambda_1,b) = \frac{1}{2}$$

$$\bar{p}_1(\lambda_2,a) = \frac{1}{3} \qquad \bar{p}_2(\lambda_2,b) = 0$$

$$\bar{p}_{12}(\lambda_1,a,b) = \frac{1}{4}$$

$$\bar{p}_{12}(\lambda_2,a,b) = 0.$$

(These probabilities satisfy the strong locality condition expressed by (15).) Finally, suppose that λ evolves stochastically into λ_1 or λ_2 after time τ in accordance with $P(\lambda_1|\lambda) = \frac{1}{4}$ and $P(\lambda_2|\lambda) = \frac{3}{4}$. From (14) it follows that

$$p_1(\lambda,a) = \frac{1}{4}\cdot\frac{1}{2} + \frac{3}{4}\cdot\frac{1}{3} = \frac{3}{8},$$

$$p_2(\lambda,b) = \frac{1}{4}\cdot\frac{1}{2} + \frac{3}{4}\cdot 0 = \frac{1}{8},$$

and

$$p_{12}(\lambda,a,b) = \frac{1}{4}\cdot\frac{1}{4} + \frac{3}{4}\cdot 0 = \frac{1}{16} \neq \frac{3}{8}\cdot\frac{1}{8}.$$

This theory therefore violates (13). In terms of the previous discussion, it can be seen that knowledge of a +1 outcome of the measurement on particle 2 reveals predictively relevant information about particle 1 which was not entailed by λ--namely, that $\lambda_\tau = \lambda_1$.[1] The correct[2] conditional probability (relative to λ) for the +1 outcome of the measurement on particle 1, given that the outcome of the simultaneous measurement on particle 2 is also +1, is therefore ½.

$$\frac{1}{2} = \bar{p}_1(\lambda_1,a) = \frac{p_{12}(\lambda,a,b)}{p_2(\lambda,b)} \neq p_1(\lambda,a).$$

In general, the proper completeness condition requires the two measurement outcomes to be stochastically independent relative to the final state description λ_τ. This is what (15) expresses. Relative to the initial state λ, however, the two measurement outcomes may be correlated in ways that can be traced to the incompleteness of the initial state λ for making "true" probability assignments to the outcomes of measurements at the future time τ.

If (13) need not hold for local realistic theories, then the possibility exists that (15) may be satisfied by some theory which reproduces the predictions of quantum mechanics. This possibility will now be investigated.

Let ρ be the probability measure on Λ_o associated with the distri-

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[1]Actually, it could be inferred only that $\lambda_\tau = \lambda_1$ with probability 1. In the general case, one could not expect to learn (even in this restricted sense) from the outcome of the measurement on particle 2 the identity of λ_τ, but one would expect the outcome of the measurement on particle 2 to differentially favor the various possible final states--i.e., to alter the probability distribution for the final states--and this is sufficient to establish the point. See also n. 1, p. 44.

[2]This is the conditional probability which ought to be reflected in the observed relative frequencies of measurement outcomes.

bution of initial states. The observable predictions for the theory are obtained by integrating over the set of "hidden" states as follows:[1]

$$
(16)\quad \begin{cases}
p_1(a) = \int_{\Lambda_o} d\rho(\lambda) \int_{\Lambda_o} dP(\lambda'|\lambda)\bar{p}_1(\lambda',a) \\[1em]
\qquad = \int_{\Lambda_o} d\rho'(\lambda')\bar{p}_1(\lambda',a), \\[1em]
\text{where the new measure } \rho' \text{ is given by} \\[1em]
\rho'(S) = \iint_{S\times\Lambda_o} dP(\lambda'|\lambda)d\rho(\lambda) = \int_{\Lambda_o} d\rho(\lambda)P(S|\lambda). \\[1em]
\text{Similarly, } p_2(b) = \int_{\Lambda_o} d\rho'(\lambda)\bar{p}_2(\lambda,b) \\[1em]
\qquad \text{and } p_{12}(a,b) = \int_{\Lambda_o} d\rho'(\lambda)\bar{p}_{12}(\lambda,a,b).
\end{cases}
$$

For the theory to succeed, it would have to provide probability functions $\bar{p}_1$, $\bar{p}_2$, and $\bar{p}_{12}$, as well as probability measures ρ and P such that the quantities $p_1(a)$, $p_2(b)$, and $p_{12}(a,b)$ given by (16) come out to be equal to the corresponding quantum mechanical probabilities.[2] In particular, that would require:

$$
(17)\quad \begin{cases}
p_1(a) = p_2(b) = \tfrac{1}{2} \\[1em]
p_{12}(a,b) = \tfrac{1}{2}\sin^2 \frac{\measuredangle(a,b)}{2}.
\end{cases}
$$

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[1] Integration over the microstates of the measuring devices has been suppressed here. Note that in Chapter V the assumption of conservation obviated this step.

[2] See the Appendix.

However, a little reflection reveals that (17) cannot be satisfied by any theory which satisfies (15) and (16). Any such theory has the same form as a theory with time-<u>independent</u> states distributed in accordance with the probability measure ρ'. Since the foregoing considerations have no relevance for theories which admit only time-independent states, the standard arguments may be followed to produce Bell-type inequalities for theories which satisfy (15) and (16).[1]

In retrospect, this result should not be too surprising. Bell-type arguments apply to theories which specify a distribution of "hidden" states and underlying probabilities satisfying (13). Whether one regards that distribution as referring to the initial states (the "prepared" states) or to the final states (i.e., after time τ) is without significance as far as the structure of the derivation is concerned. The two cases are formally identical.

Summarizing, it has been found that considerations of completeness in the context of time-dependent states provide reasons for challenging (13), at least as it is applied in standard versions of the Bell arguments. It has also been found that these considerations nevertheless apparently fail to block the derivation of Bell-type inequalities for theories which admit time-dependent states.

However, this conclusion ought not to be drawn too hastily. Recall the simplifying background assumption according to which pairs of measurements are taken to be simultaneous in the laboratory frame and to occur at a single fixed time τ in that frame. Following the standard Bell-type arguments, then, using the distribution of states at the specific time τ,

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[1]Arthur Fine has reached a similar conclusion in the context of his discussion of "random devices in harmony." See his [18].

the Bell-type inequalities which emerge should explicitly exhibit this built-in time-dependence; for example,[1]

$$(18) \qquad -1 \leq p_{12}(a,b;\tau) - p_{12}(a,b';\tau) + p_{12}(a',b;\tau) + p_{12}(a',b';\tau) - p_1(a',\tau) - p_2(b,\tau) \leq 0.$$

While this might be a satisfactory result if the background assumptions were true, it cannot be expected to hold--even as an approximation--if those assumptions are false. The foregoing analysis relied crucially on their holding exactly. However nearly they may hold in actual experiments, their failure to hold exactly would necessitate a complete reexamination; and, even though it may be true to a very good approximation that pairs of measurements occur simultaneously in the laboratory frame and that they occur at the same fixed time for each trial, these assumptions are undoubtedly false. (Consider that, because real measuring devices must present a non-zero surface area to the particles if they are to interact at all, actual experiments will involve measurements on particles which traverse a range of distances from the source, so that even photons will not all take exactly the same amount of time to pass from the source to the point of measurement. Moreover, there are practical, if not in principle, limits to the precision with which one can position the source and the measuring devices so as to guarantee simultaneity of the measurements. This is also complicated by the fact that actual measurements are not instantaneous but take place over some time interval.)

The problem therefore is this: If it must be allowed that pairs of measurements need neither be simultaneous in the laboratory frame nor occur

[1]This is a version of the Clauser-Horne inequality derived in [11].

at the same pair of different times from one trial to the next, from what strong locality condition (i.e., something like (15)) can Bell-type inequalities which bear on the actual experiments be derived? Under these circumstances, (15) does not even make sense. To be sure, the right-hand side of (15) could be amended to include, in place of τ in the $\bar{p}_1$ and $\bar{p}_2$ terms, the two different times, τ_1 and τ_2, respectively, at which the measurements on particles 1 and 2 are taken to occur; but if $\tau_1 \neq \tau_2$, what value then belongs in the place occupied by τ in the $\bar{p}_{12}$ term?; and even if this question can be answered for the values of τ_1 and τ_2 associated with any given measurement trial, how is account to be taken of the fact that these values may vary from one trial to the next?; i.e., how can the results of a sequence of trials be combined to produce ensemble averages of the terms occurring in (18) which continue to satisfy inequalities of that form?

Two possible approaches may be suggested[1]:

(i) For each trial, the two measurements are simultaneous in <u>some</u> frame. Let τ_i be the time at which measurements in the i-th trial occur in the frame of simultaneity for that trial. (It is allowed that the frames of simultaneity for different trials may differ; i.e., τ_i need not equal τ_j for $i \neq j$.) The strong locality condition then has this form:

$$\text{(19)} \qquad \text{(For all i)} \quad \bar{p}_{12}(\lambda_{\tau_i},a,b) = \bar{p}_1(\lambda_{\tau_i},a)\cdot\bar{p}_2(\lambda_{\tau_i},b).$$

The problem now consists in getting from (19) to a Bell-type inequality. There might be plausible assumptions (presumably restrictions on the structure of the λ_τ's) which would justify taking ensemble averages which combine

[1] It will continue to be assumed here that measurements may be treated as instantaneous events.

quantities associated with different frames of simultaneity, but as the situation stands, it is not obvious that no theory satisfying strong locality in the sense of (19) is compatible with the predictions of quantum mechanics.

(ii) Alternatively, one might entertain a strong locality condition of the form:

(20) (For each trial) $\bar{p}_{12}(\lambda_{\tau_1},a;\lambda_{\tau_2},b) = \bar{p}_1(\lambda_{\tau_1},a)\cdot\bar{p}_2(\lambda_{\tau_2},b)$.

Even if the standard procedures for deriving Bell-type inequalities are extended in a suitable way to accommodate distributions of both states at time τ_1 and states at time τ_2, it is not immediately clear how to employ these procedures in a manner that would take into account the variation of τ_1 and τ_2 from trial to trial. At least it is not obvious that no theory satisfying (20) is compatible with the predictions of quantum mechanics.

While no attempt will be made here to provide a definitive treatment of these issues, the remainder of this chapter presents one strategy for extending the Bell-type arguments in a way which seeks to resolve these difficulties along the lines of approach (ii) above.[1]

Let $\pi(S_1,S_2|\lambda)$ be the probability that a two-particle system prepared in state λ evolves into a state in set S_1 at the time of the measurement on particle 1 <u>and</u> into a state in set S_2 at the time of the measurement on particle 2. (No assumption about the order of the measurements is being made here.) Let $h_1(t_1)$ be the probability density for the occurrence at time t_1 of the

[1]Note that the states of the measuring devices may also be time-dependent. This complication will not be treated explicitly here, but it is not hard to see how a parallel attempt to extend the Bell-type arguments to cover theories which admit such measuring device states would proceed.

measurement on particle 1. Define $h_2(t_2)$ analogously.[1]

Let $\lambda(t)$ be the state of the evolving two-particle system at time t, where $\lambda = \lambda(0)$ is the initial state.

Let ρ be the probability measure on Λ_o associated with the (time-dependent) distribution of states.

Any candidate theory must also provide a conditional probability density u which is to be interpreted as follows: $u(\lambda(t')|\lambda(t))$ is the probability density for the transition from the state $\lambda(t)$ to the state $\lambda(t')$, where $t' > t$.

Using these definitions, $\pi(S_1,S_2|\lambda)$ may be expressed as

$$\int_0^\infty dt_1 h_1(t_1) \int_0^\infty dt_2 h_2(t_2) \int_{S_i} d\rho(\lambda(t_i))u(\lambda(t_i)|\lambda) \int_{S_j} d\rho(\lambda(t_j))u(\lambda(t_j)|\lambda(t_i)),$$

$$\text{where} \quad i = \begin{cases} 1, & t_1 \leq t_2 \\ 2, & t_1 > t_2 \end{cases} \quad \text{and} \quad j = \begin{cases} 1, & t_1 > t_2 \\ 2, & t_1 \leq t_2 \end{cases}.$$

This expression then becomes

$$\int_0^\infty dt_2 h_2(t_2) \left[\int_0^{t_2} dt_1 h_1(t_1) \int_{S_1} d\rho(\lambda(t_1))u(\lambda(t_1)|\lambda) \int_{S_2} d\rho(\lambda(t_2))u(\lambda(t_2)|\lambda(t_1)) \right.$$
$$\left. + \int_{t_2}^\infty dt_1 h_1(t_1) \int_{S_2} d\rho(\lambda(t_2))u(\lambda(t_2)|\lambda) \int_{S_1} d\rho(\lambda(t_1))u(\lambda(t_1)|\lambda(t_2))\right]$$

Now let $f(S_1,S_2) = \int_{\Lambda_o} d\rho(\lambda)\pi(S_1,S_2|\lambda)$. (Note that this measure is defined in terms of quantities which must be provided by the theory.)

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[1] It will be assumed in what follows (without explicitly restricting h_1 and h_2 so as to insure this) that the pair of measurement events which occur in any given trial are spacelike-related.

Adopt the following as the strong locality condition:

$$\bar{p}_{12}(\lambda_1,a;\lambda_2,b) = \bar{p}_1(\lambda_1,a)\cdot\bar{p}_2(\lambda_2,b),$$

where λ_i is the state at the time of the measurement on particle i. This gives

$$p_{12}(a,b) = \iint_{\Lambda_o \times \Lambda_o} df(\lambda_1,\lambda_2)\bar{p}_{12}(\lambda_1,a;\lambda_2,b)$$

$$= \iint_{\Lambda_o \times \Lambda_o} df(\lambda_1,\lambda_2)\bar{p}_1(\lambda_1,a)\cdot\bar{p}_2(\lambda_2,b)$$

and

$$p_i(c) = \iint_{\Lambda_o \times \Lambda_o} df(\lambda_1,\lambda_2)\bar{p}_i(\lambda_i,c) \qquad (i = 1,2).$$

Following Clauser and Horne [11], for example, yields:

$$\begin{aligned} -1 \le\ & \bar{p}_{12}(\lambda_1,a;\lambda_2,b) - \bar{p}_{12}(\lambda_1,a;\lambda_2,b') + \bar{p}_{12}(\lambda_1,a';\lambda_2,b) \\ & + \bar{p}_{12}(\lambda_1,a';\lambda_2,b') - \bar{p}_1(\lambda_1,a') - \bar{p}_2(\lambda_2,b) \le 0. \end{aligned} \tag{21}$$

Finally, integrating (21) over $\Lambda_o \times \Lambda_o$ with f as the measure gives the Clauser-Horne inequality:

$$-1 \le p_{12}(a,b) - p_{12}(a,b') + p_{12}(a',b) + p_{12}(a',b') - p_1(a') - p_2(b) \le 0. \tag{22}$$

However, there are objections which jeopardize this line of reasoning.[1] In order for this derivation to go through, it is essential that the functions h_1 and h_2 not depend on the component of spin to be

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[1]Note that while this discussion is based on approach (ii), approach (i) is vulnerable to similar problems. (In the latter case, however, there would be only one h function.) As previously noted, approach (i) faces other difficulties having to do with carrying out ensemble averages of quantities associated with different frames of reference.

measured.[1] (Otherwise π, and therefore f, could share this dependence. That would block the move from (21) to (22).) However, h_1 and h_2 cannot be completely insensitive to the particular constitution and configuration of the experimental apparatus. If the distance a particle must travel between the source and the region of measurement varies even minutely, but systematically, with the apparatus configurations associated with measurements of different components of spin, then h_1 and h_2 will not be independent of the component of spin being measured.

One could try to salvage the argument by imposing additional constraints on the theory. For example, if the state of the two-particle system is required to be a sufficiently slowly-varying function of time, then its associated probability assignments will be nearly constant over small enough time intervals.[2] Slight variations of h_1 and h_2 with the component of spin being measured will then have negligible effect on the theoretical predictions.

Such assumptions cannot be justified on purely local realistic grounds. However, even if a local realistic mechanism of this sort could account for the violation of Bell-type inequalities in one particular experiment, it would only raise a new question: If the distribution of measurement outcomes is so sensitive to the precise details of the experimental arrangement, why is there such good agreement among the results of <u>different</u> experiments?; i.e., how is it to be understood that in a variety

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[1]The assumption that the distribution of hidden states does not depend on the measuring device states also was implicit in the derivation presented in Chapter V. For a somewhat speculative discussion of physical grounds for the failure of this assumption, see Bohm and Hiley [8].

[2]An assumption of this sort might also justify treating measurements as instantaneous events.

of experimental situations, these mechanisms which distort the results of individual experiments all manage to do so in such a way that, within experimental uncertainties, not only are the Bell-type inequalities violated, but the specific numerical predictions of quantum mechanics are confirmed?[1] Under these circumstances, it should be clear that any "explanation" of this sort would have it that a fantastic coincidence is behind the similarity of the results of different experiments.

A better treatment of these issues, one which gives relativity its due, might characterize λ as a field on spacetime satisfying a suitable locality constraint and try to do justice to actual measurement interactions. While it might reasonably be hoped to arrive at more clear-cut conclusions from such a treatment, it might also reasonably be suspected that sufficiently highly contrived local realistic theories will dodge any such conclusions. Certain plausibility considerations which apply also to the discussion of preceding chapters (involving theories which admit only time-independent states) are relevant here.[2] One cannot rule out the possibility that somehow, by local realistic processes, the distribution of two-particle states is correlated with the pre-measurement states of the measuring devices, nor that the measuring device states are somehow correlated with each other, precisely so as to fit the entire body of observed data; but if the <u>only</u> reason for concern about that possibility is that it <u>cannot</u> be eliminated, then perhaps it should be of comparable concern

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[1]See the discussion of the experimental results in Clauser and Shimony [12], especially their remarks on the status of anomalous results in two experiments (p. 1919).

[2]See also n. 1, p. 72. These plausibility considerations include dismissing "conspiracies" in the overlap of the backward light cones of the measurement events, as alluded to in n. 1, p. 24.

that all of what is ordinarily taken for conscious experience could be but a dream. Provided one is willing to countenance sufficiently ad hoc hypotheses, surely there is no conceivable data which cannot be "explained."

CHAPTER X

FURTHER DISCUSSION/CONCLUSION

The view has been held by some that since strong locality is not a requirement of relativity theory, it may not be an appropriate constraint on local realistic theories.[1] It has been argued here that strong locality follows from constraints which are essential to one characterization of local realism. Locality is supported by relativity and, if there is to be "realism" in local realism, completeness appears to be unavoidable.

Regardless of how one chooses to use the expression 'local realism,' the interpretation that strong locality inherits from locality and completeness may be exploited to help to understand the significance of the Bell arguments. Those arguments, together with the relevant experimental results, are generally taken to provide excellent evidence that strong locality cannot be satisfied by any empirically adequate theory. Since locality is contravened only on pain of a serious conflict with special relativity (which is extraordinarily well-confirmed independently), completeness is the likely culprit.

In terms of the interpretation of completeness presented here, one must conclude that certain phenomena simply cannot be adequately represented by any theory which ascribes properties to the entities it posits[2]

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[1]See, for example, Fine [18] and Hellman [22].

[2]The point is not (or at least not merely) that some predictively relevant properties attach "globally" to systems as a whole, but not to parts of the systems. It is more, e.g., than that we may assign the value

in such a way that no measurement on the system may yield information which is both non-redundant (not deducible from the state descriptions) and predictively relevant for distant measurements. That "information" is not contained (neither explicitly nor implicitly) in the "incomplete" state descriptions.

Despite the general acceptance long ago of the "orthodox" interpretation of quantum mechanics, it might be said that only since the Bell-type experiments of recent years has there been convincing evidence that the world-view which underlies completeness must be abandoned. The predictions of quantum mechanics, in good agreement with the experimental results, satisfy locality but violate completeness (and so, too, violate strong locality).[1] Investigation of the quantum-mechanical violation of completeness leads immediately to deep, unresolved mysteries at the foundations of quantum mechanics (including the infamous "measurement problem").[2]

While the analysis presented here provides no resolution of quantum mysteries, it is hoped that it may help to sharpen the focus on our view of those mysteries. By separating out the relativistic component of the strong locality condition (thereby exposing strong locality as the

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zero to any component of the spin of the two-particle system, but we cannot, for particles in this state, ascribe a definite value to any component of the spin of either particle. As previously indicated (n. 2, p. 26), state descriptions of this form have not been excluded from consideration.

[1]See the Appendix. Actually, there is a restricted notion of completeness available for states in non-relativistic quantum mechanics, but recent work suggests that in relativistic quantum mechanics, even this notion must be given up. See Aharonov and Albert [1] and [2].

[2]It should not be thought that the failure of local realism, in and of itself, provides any solution to the foundational problems of quantum mechanics. See Stein [30] for a perceptive discussion of the current situation in the philosophy of quantum mechanics.

relativistic locality constraint for complete theories), there emerges a clarification of that class of theories excluded by the Bell arguments: the class of theories which satisfy completeness.

It is important not to be misled about the role of quantum mechanics in the case against local realism. To be sure, issues connected with the interpretation of quantum mechanics provide a natural and historically important backdrop against which to assess local realism; and it happens that the predictions of quantum mechanics are borne out in those experiments which tell against local realism; but the existence and empirical adequacy of quantum mechanics play no logical role in the case against local realism. Even if quantum mechanics had never been developed or even should quantum mechanics itself one day fall, the case against local realism would still stand. Bell's Theorem shows that local realistic theories make certain unambiguous predictions about readily observable phenomena. Some of those predictions appear to be false, refuted by experiment. This is so regardless of how well the predictions of quantum mechanics fare with the results of those or any other experiments.

While various ad hoc maneuvers always will be logically available, the case against local realism is truly formidable. There appears to be no entirely honorable way out for local realism. Perhaps in these correlation phenomena complementarity asserts itself with a vengeance: perhaps objects do not possess properties individually, but somehow "share" them with those objects with which they interact--even if those objects are galaxies apart and have long since ceased their interaction--and do so in such a way that a measurement at one place may "reveal" information about a spatially distant region in which prior to the measurement that information was unavailable, even assuming maximally "complete" state descriptions; perhaps the very notion of a "distinct physical object" is only an approximation; perhaps any adequate conception of causality implies that the world ultimately is not analyzable in terms of distinct, independently existing components, but is an indivisible whole.

The idea of any such world flatly contradicts Einstein's most deeply held physical intuitions, and he never relinquished them. Einstein, however, never surveyed the evidence against local realism which is before us today. Perhaps now, for the first time, there are clear and compelling reasons for accepting some world-view of a sort which Einstein so strongly and so magnificently opposed. In any case, the adequate elucidation of alternatives to local realism remains the grand unfinished task.

Although the term "incompleteness" may connote a defect (as if, as was the case for the crude spin model discussed in Chapter VI, all incomplete theories may be "complet*ed*"), incomplete theories (e.g., quantum mechanics) are by no means *ipso facto* defective. On the contrary, when the results of Bell-type experiments are taken into account, the truly remarkable implication of Bell's Theorem is that incompleteness, in some sense, is a genuine feature of the world itself.

APPENDIX

THE PREDICTIONS OF QUANTUM MECHANICS

It has been asserted that the predictions of quantum mechanics accord well with the results of experiments which test Bell-type inequalities. It will now be shown that, indeed, some of the probability assignments made by quantum mechanics do violate the Bell-Wigner inequality.

Let $\binom{1}{0}^u$ and $\binom{0}{1}^u$ be the up and down single-particle eigenstates, respectively, corresponding to the operator σ_u associated with the u-component of spin. The two-particle singlet spin state ψ_o then takes the following form (modulo a phase factor):

$$\psi_o = \frac{1}{\sqrt{2}}\left[\binom{1}{0}^u_L \otimes \binom{0}{1}^u_R - \binom{0}{1}^u_L \otimes \binom{1}{0}^u_R\right].$$

(The direction u is arbitrary, since the state is spherically symmetric.) In order to treat measurements of distinct spin components for the two particles, it will be useful to express $\binom{1}{0}^u$ and $\binom{0}{1}^u$ in terms of the eigenstates $\binom{1}{0}^v$ and $\binom{0}{1}^v$ corresponding to the operator σ_v associated with the v-component of spin.

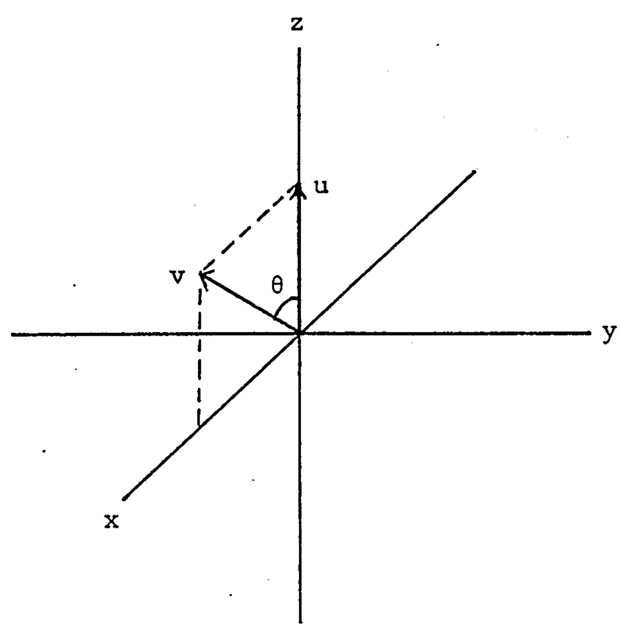

Fig. 6.--Arbitrary Directions u and v

$$\sigma_v = \sigma \cdot v = \sigma_x \sin\theta + \sigma_z \cos\theta$$

$$= \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix},$$

where θ is the angle between the vectors u and v.

It is an easy exercise to verify that the eigenstate of this operator with eigenvalue +1 is $\binom{1}{0}^v = \cos\frac{\theta}{2}\binom{1}{0}^u + \sin\frac{\theta}{2}\binom{0}{1}^u$ and the eigenstate with eigenvalue -1 is $\binom{0}{1}^v = -\sin\frac{\theta}{2}\binom{1}{0}^u + \cos\frac{\theta}{2}\binom{0}{1}^u$. It follows that $\binom{1}{0}^u = \cos\frac{\theta}{2}\binom{1}{0}^v - \sin\frac{\theta}{2}\binom{0}{1}^v$ and

$$\binom{0}{1}^u = \sin\frac{\theta}{2}\binom{1}{0}^v + \cos\frac{\theta}{2}\binom{0}{1}^v.$$

ψ_o can now be expressed conveniently as follows:

$$\psi_o = \frac{1}{\sqrt{2}}\Bigg[\sin\frac{\theta}{2}\binom{1}{0}^u_L \otimes \binom{1}{0}^v_R + \cos\frac{\theta}{2}\binom{1}{0}^u_L \otimes \binom{0}{1}^v_R - \cos\frac{\theta}{2}\binom{0}{1}^u_L \otimes \binom{1}{0}^v_R + \sin\frac{\theta}{2}\binom{0}{1}^u_L \otimes \binom{0}{1}^v_R\Bigg].$$

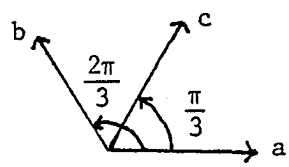

Fig. 7.--One set of directions for which quantum mechanics violates the Bell-Wigner inequality.

Squaring the modulus of the coefficient of $\binom{1}{0}^u_L \otimes \binom{1}{0}^v_R$ in the above expression for ψ_o gives the quantum-mechanical probability that measurements of the u-component of spin of particle L and the v-component of spin of particle R both yield the value +1:

$$P_{QM}(u_L^+ \;\&\; v_R^+) = \frac{1}{2}\sin^2\frac{\theta}{2}.$$

The quantum-mechanical probabilities for the other possible pairs of outcomes are deducible similarly from the above expression for ψ_o.

For the set of coplanar directions shown in Figure 7, $P_{QM}(a_L^+ \;\&\; b_R^+) = \frac{1}{2}\sin^2\frac{\pi}{3} = \frac{3}{8}$ and $P_{QM}(a_L^+ \;\&\; c_R^+) = P_{QM}(c_L^+ \;\&\; b_R^+) = \frac{1}{2}\sin^2\frac{\pi}{6} = \frac{1}{8}$.

Thus (since $\frac{3}{8} > \frac{1}{8} + \frac{1}{8}$), at least for some sets of directions a, b, and c, the predictions of quantum mechanics do violate the Bell-Wigner inequality.[1]

Quantum mechanics does satisfy conservation, since for every direction u, $P_{QM}(u_L^+ \ \& \ u_R^+) = \frac{1}{2} \sin^2 0 = 0$. It follows that quantum mechanics must violate strong locality. (Otherwise, the predictions of quantum mechanics would have to satisfy the Bell-Wigner inequality.) Note first, however, that quantum mechanics does satisfy locality. Let u and v be arbitrary directions. Then

$$P_{QM}(u_L^+) = P_{QM}(u_L^+ \ \& \ v_R^+) + P_{QM}(u_L^+ \ \& \ v_R^-) = \frac{1}{2} \sin^2 \frac{\theta}{2} + \frac{1}{2} \cos^2 \frac{\theta}{2} = \frac{1}{2}$$

$$= P_{QM}(u_L^+ \ \& \ v_R^+) + P_{QM}(u_L^- \ \& \ v_R^+) = P_{QM}(v_R^+).$$

Therefore, unless u and v are orthogonal,

$$P_{QM}(u_L^+ \ \& \ v_R^+) = \frac{1}{2} \sin^2 \frac{\theta}{2} \neq P_{QM}(u_L^+) \cdot P_{QM}(v_R^+).$$

Since quantum mechanics does satisfy locality, its violation of strong locality issues from its violation of completeness.

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[1]See p. 40.

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