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On the Physical Significance of the Locality Conditions in the Bell Arguments

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Bell's Theorem establishes the incompatibility of quantum mechanics and certain "local realistic" premises. Any theory satisfying these premises makes certain predictions (concerning the results of a special class of experiments) which are constrained by the theorem to satisfy the Bell inequalities. Since the results of actual experiments—results which accord well with the predictions of quantum mechanics—do violate the Bell inequalities, there is evidence that one or more of the "local realistic" premises of the theorem is false.[1]

Actually, there are different versions of Bell's Theorem, corresponding to different sets of premises from which a Bell-type inequality is derived. Each version of the theorem, however, employs as such a premise some "locality" condition; i.e. some formulation or other of a constraint to be interpreted as a prohibition against "action-at-a-distance." Some versions of Bell's Theorem, namely those which are restricted to consideration of deterministic local realistic theories, employ as a locality condition the requirement warranted by relativity theory, that superluminal signals not be possible.[2]

More general versions of Bell's Theorem, those which apply to stochastic local realistic theories, make use of a stronger locality condition. The physical content of this latter condition and the precise relationship between the relativistic locality constraint and this stronger locality condition are rather controversial topics about which there seems to be considerable confusion.

This article is an attempt to provide some clarification of these matters by calling attention to a certain "decomposition" of the stronger locality condition into two weaker conditions, each of which has a rather straightforward physical interpretation. More specifically, it will be shown that the stronger locality condition is logically equivalent to the conjunction of the relativistic locality constraint and another condition which has to do with the "completeness" of the state descriptions allowed by the theory.

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I. NOTATION[3]

Consider a system of two spin-½ particles in the singlet state, together with two spin measurement devices (each consisting of a Stern-Gerlach apparatus and particle detectors), all arranged in a standard EPR-Bohm-Bell setup.[4] In what follows, the labels 'L' and 'R' will be used promiscuously to refer to the left-hand member and right-hand member, respectively, of the pair of measuring devices, of a pair of particles, of a pair of measuring device-particle subsystems, etc. In each case, the intended use will be made clear by the context.

Let Λ be the set of states of the two-particle system and let $M_L(M_R)$ be the set of states of measuring device L(R). (For the sake of generality, the exact form of these state descriptions is deliberately left unspecified.) It will be useful to distinguish certain subsets of M_L and M_R as follows: Let d be an arbitrary unit vector in space; then M_L^d is the set containing just those states $\bar{\mu}_L \in M_L$ which could be occupied by an L measuring device prepared to measure the d-component of spin. M_R^d is defined analogously. Since the macroscopic parameter of primary interest in these state descriptions is the direction along which the Stern-Gerlach magnets are aligned (i.e. the component of spin to be measured), let us represent measuring device states $\bar{\mu}_i \in M_i^d$ as ordered pairs $\bar{\mu}_i = (\mu_i, d_i)$, where the subscript i may assume the values 'L' and 'R'. This notation merely collects together into μ_i whatever information in excess of the direction d_i is required to fully specify the state of measuring device i.

It will also be useful to allow the variable d to assume the value 0 (corresponding to no direction at all) in order to denote those sets of states, M_L^0 and M_R^0, which could be occupied by measuring devices unprepared to measure *any* component of spin. If $\triangle$ is the set of unit vectors in space and if $M_i^\triangle$ is defined as the set $\bigcup_{d_i \in \triangle} M_i^{d_i}$, then, for each i, $M_i = M_i^0 \cup M_i^\triangle$ and $M_i^0 \cap M_i^\triangle = \phi$.

Attention will be restricted to theories which employ state descriptions for the two-particle system and measuring devices which determine unique values for the probabilities of the various possible (pairs of) outcomes of joint spin measurements. That is to say, given a state $\lambda \in \Lambda$ of the two-particle system and states $\bar{\mu}_L \in M_L^{d_L}$ and $\bar{\mu}_R \in M_R^{d_R}$ of the measuring devices (where $d_L \in \triangle$ and $d_R \in \triangle$), and given a pair x_L and x_R of possible measurement outcomes, the theory must uniquely specify a number $\pi_\lambda^{\mu_L,\mu_R}(d_L,x_L;d_R,x_R)$ which is to be interpreted as the probability[5] that the outcomes of a measurement of the d_L-component of the spin of particle L and a measurement of the d_R-component of the spin of particle R are x_L and x_R respectively.[6]

For any given direction, the spin up and spin down measurement outcomes will be represented here by the values $+1$ and -1 respectively. Let us also incorporate into the formalism the trivial measurement "outcome," represented by the value 0, which obtains whenever no measurement is performed (i.e. whenever $\bar{\mu}_L \in M_L^0$ or $\bar{\mu}_R \in M_R^0$). Letting $X = \{+1, -1\}$, the condition described above can be expressed as follows: For each state $\lambda \in \Lambda$, there is a function $\pi_\lambda : D_\pi \rightarrow [0, 1]$, where

$$\begin{aligned} D_\pi = \; & \{(\bar{\mu}_L, x_L, \bar{\mu}_R, x_R) \mid \bar{\mu}_L \in M_L^\triangle,\ x_L \in X,\ \bar{\mu}_R \in M_R^\triangle,\ x_R \in X\} \\ & \cup \{(\bar{\mu}_L, x_L, \bar{\mu}_R^\circ, 0) \mid \bar{\mu}_L \in M_L^\triangle,\ x_L \in X,\ \bar{\mu}_R^\circ \in M_R^0\} \\ & \cup \{(\bar{\mu}_L^\circ, 0, \bar{\mu}_R, x_R) \mid \bar{\mu}_L^\circ \in M_L^0,\ \bar{\mu}_R \in M_R^\triangle,\ x_R \in X\}, \end{aligned}$$

satisfying the following conditions:[7]

(1) For all $d_L \in \triangle$, $\bar{\mu}_L \in M_L^{d_L}$, and $\bar{\mu}_R^\circ \in M_R^0$,

$$\sum_{x_L \in X} \pi_\lambda^{\mu_L, \mu_R^\circ}(d_L, x_L; 0, 0) = 1$$

(2) For all $\bar{\mu}_L^\circ \in M_L^0$, $d_R \in \triangle$, and $\bar{\mu}_R \in M_R^{d_R}$,

$$\sum_{x_R \in X} \pi_\lambda^{\mu_L^\circ, \mu_R}(0, 0; d_R, x_R) = 1$$

(3) For all d_L, $d_R \in \triangle$, $\bar{\mu}_L \in M_L^{d_L}$, and $\bar{\mu}_R \in M_R^{d_R}$,

$$\sum_{x_L, x_R \in X} \pi_\lambda^{\mu_L, \mu_R}(d_L, x_L; d_R, x_R) = 1$$

The domain of these probability functions includes pairs of measuring device states corresponding to cases in which one (but not both) of the measuring devices is in a non-measurement state. Conditions (1) - (3) above merely insure that the relevant probabilities summed over the possible outcomes add up to unity. Conditions (1) and (2) express this requirement for those cases in which only one of the measuring devices is prepared to perform a spin measurement and condition (3) expresses it for cases in which both measuring devices are so prepared.

Before proceeding, a final preliminary point requires attention. Consider that, in general, the state descriptions admitted by a theory may be deterministic (i.e. at each moment, the states may determine the outcome of every possible spin measurement) or they may be indeterministic (i.e. at each moment, the states may determine only the probability for each possible outcome of every possible spin measurement). However, in each of these cases, the states of the system may exhibit time-dependence; they may undergo temporal evolution. Moreover, that temporal evolution itself may be either deterministic or indeterministic (stochastic). (I.e., the law(s) governing the evolution either may or may not be such that the state, at every moment, is sufficient to determine the state at every future moment.)

Thus, the probabilistic or statistical character of theoretical predictions may be associated with states of various types: (1) time-independent indeterministic states; (2) stochastically-evolving deterministic states; (3) determinininistically-evolving indeterministic states; and (4) stochastically-evolving indeterministic states. Similarly, deterministic theoretical predictions may be associated with time-independent deterministic states as well as deterministically-evolving deterministic states.

There is experimental evidence, however, which tells against theories which admit two-particle states which are relevantly[8] time-dependent. For the two-particle systems under consideration (i.e. pairs of spin-½ particles in the singlet state), it is observed that—within experimental uncertainties—joint measurements of any single component of spin always yield opposite outcomes; one particle is measured to have spin up and the other is measured to have spin down, each relative to the same specified axis.

Nevertheless, since perfectly efficient detectors and perfectly functioning analyzers (e.g. Stern-Gerlach devices) are idealizations not to be found among real laboratory equipment, there exists a class of "local realistic" theories which exploit the uncertainties in the experimental data in order to evade the verdict of Bell-type arguments.[9] Theories in this class (including those which admit time-dependent states) typically do not predict anti-correlated outcomes for every joint measurement of the same component of spin.

Given the character of these experiments, the failure to confirm this anti-correlation in particular pairs of measurements is most plausibly regarded as completely spurious. In any case, since standard formulations of Bell-type arguments do not neatly handle theories which admit time-dependent states,[10] the present discussion will be restricted to theories which admit only time-independent states. All of the main points may be extended to include the more general theories, but not without complicating the exposition in ways that could obscure the issues of primary concern here.

II. LOCALITY

For purposes of the present discussion, a theory will be said to be *local* (or to satisfy *locality*) just in case the probability functions it specifies meet the following constraints: For all $\lambda \in \Lambda$, $d_L, d_R \in \triangle$, $\bar{\mu}_L \in M_L^{d_L}, \bar{\mu}_R \in M_R^{d_R}$, $\bar{\mu}_L^\circ \in M_L{}^0$, $\bar{\mu}_R^\circ \in M_R{}^0$, and $x_L, x_R \in X$,

$$(4) \quad \pi_\lambda{}^{\mu_L,\mu_R^\circ}(d_L,x_L;0,0) = \sum_{x_R' \in X} \pi_\lambda{}^{\mu_L,\mu_R}(d_L,x_L;d_R,x_R')$$

and

$$(5) \quad \pi_\lambda{}^{\mu_L^\circ,\mu_R}(0,0;d_R,x_R) = \sum_{x_L' \in X} \pi_\lambda{}^{\mu_L,\mu_R}(d_L,x_L';d_R,x_R).$$

Locality requires that the probability for the outcome $x_i \in X$ of a spin measurement on particle i be determined "locally"; i.e. that it depend only on the state $\lambda \in \Lambda$ of the two-particle system and on the state $\bar{\mu}_i \in M_i{}^\triangle$ of the i measuring device. In particular, that probability must be independent of which (if any) component of spin the distant measuring device is set to measure.

It should be noted that locality does *not* exclude the possibility of acquiring information about subsystem R(L)—information bearing on the outcome of a subsequent R(L) measurement—by performing a measurement on particle L(R). Since λ presumably contains information about both particles, a measurement on one particle, from the outcome of which partial knowledge of λ may be inferred, may therefore yield information about the other particle as well. Strictly correlated pairs of measurement outcomes represent the extreme situation of this sort. Such correlations are entirely compatible with locality.

What locality *does* exclude, however, is the possibility that the preparation of either measuring device in some particular state can exert a causal influence on the other subsystem so as to affect the probabilities for the possible outcomes of measurements performed on that other subsystem. Since these two events, the preparation of one measuring device in a given state and the measurement executed by the other measuring device, can be space-like related, locality is a requirement of relativity theory.[11]

In order to establish the relativistic basis for locality, it suffices to show that if enough control over the state preparations is assumed, violations of the locality condition provide (at least in principle) the means for superluminal signal transmission.[12] For this purpose, it will be convenient to treat deterministic theories and indeterministic theories separately.

Consider first an arbitrary deterministic theory. Suppose this theory violates (4). (One could just as well suppose instead the theory violates (5) and give a similar argument). Then for some $\lambda \in \Lambda$, $d_L, d_R \in \triangle$, $\bar{\mu}_L \in M_L^{d_L}$, $\bar{\mu}_R \in M_R^{d_R}$, $\bar{\mu}_R^\circ \in M_R{}^0$, and $x_L \in X$,

$$\pi_\lambda{}^{\mu_L,\mu_R^\circ}(d_L,x_L;0,0) \neq \sum_{x_R \in X} \pi_\lambda{}^{\mu_L,\mu_R}(d_L,x_L;d_R,x_R).$$

Since, under the assumption of determinism, all probabilities are either 0 or 1, there are only two cases to consider:

Case (i) $\pi_\lambda^{\mu_L,\mu_R^\circ}(d_L,x_L;0,0) = 0$

and

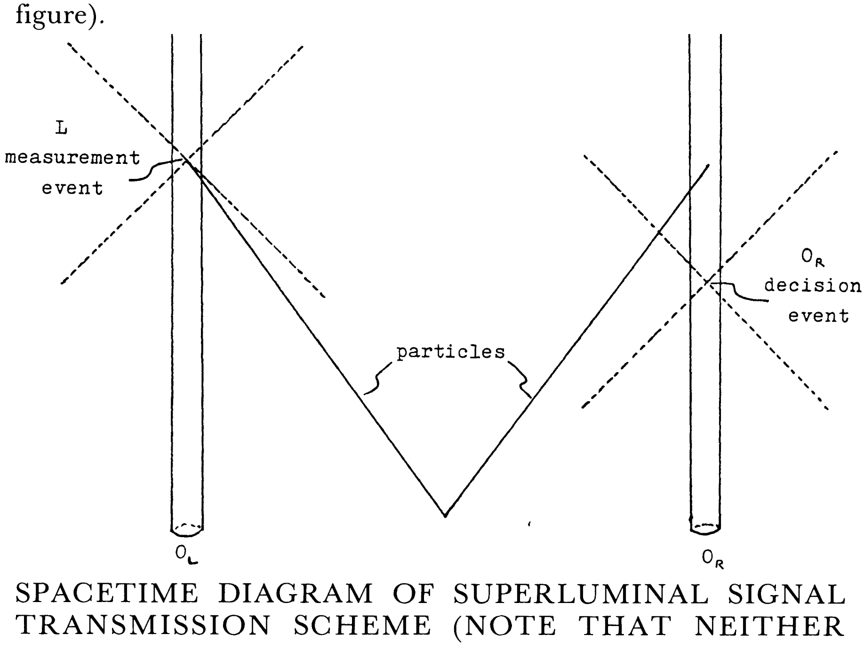
$\pi_\lambda^{\mu_L,\mu_R}(d_L,x_L;d_R,+1) = 1$ or $\pi_\lambda^{\mu_L,\mu_R}(d_L,x_L;d_R,-1) = 1$

Case (ii) $\pi_\lambda^{\mu_L,\mu_R^\circ}(d_L,x_L;0,0) = 1$

and

$$\pi_\lambda^{\mu_L,\mu_R}(d_L,x_L;d_R,+1) = \pi_\lambda^{\mu_L,\mu_R}(d_L,x_L;d_R,-1) = 0$$

Suppose a pair of particles is prepared in the state λ.[13] Suppose also that experimenters 0_L and 0_R arrange to perform measurements of the d_L and d_R components (respectively) of the spin of the L and R particles (respectively). Let it be agreed beforehand that 0_L is to perform his measurement by preparing his measuring device in the state $\bar{\mu}_L = (\mu_L, d_L)$, but that 0_R has the choice of performing his measurement by preparing his measuring device in the state $\bar{\mu}_R = (\mu_R, d_R)$ or he can perform no measurement at all by preparing his measuring device in the state $\bar{\mu}_R^\circ = (\mu_R^\circ, 0)$.[13] Moreover, 0_R is not to make his decision until such time as the making of that decision and the measurement by 0_L are space-like related events. (See the figure).



SPACETIME DIAGRAM OF SUPERLUMINAL SIGNAL TRANSMISSION SCHEME (NOTE THAT NEITHER LABELED EVENT LIES INSIDE THE FUTURE LIGHT CONE OF THE OTHER)

Now 0_L can use the outcome of his measurements to determine whether or not 0_R decided to perform his measurement.[14] If case (i) above obtains and 0_L finds outcome x_L, then he knows that 0_R must also have performed a measurement. On the other hand, if 0_L finds outcome $-x_L$, then (by equations (1) and (3)) he can be certain that 0_R did not perform his measurement. Similarly, if case (ii) above obtains and 0_L finds outcome x_L, he can infer that 0_R did not perform his measurement. If 0_L finds outcome $-x_L$, however, he can infer that 0_R did perform his measurement. (Naturally, in order for 0_L to make the correct inference, he would have to know whether case (i) or case (ii) obtained, but nothing in principle precludes this.) In this way, 0_R could communicate his decision to 0_L in a time less than that in which light would traverse the distance between them.

Thus, deterministic theories which violate locality are quite difficult to reconcile with relativity.[15] An analogous argument (actually a generalization of the one just given for deterministic theories) can be given for indeterministic theories, but because of complications owing to the statistical character of the predictions, the situation here is a bit more delicate. Consider an arbitrary indeterministic theory which violates (4). Then, as before, for some $\lambda \in \Lambda$, d_L, $d_R \in \triangle$, $\bar{\mu}_L \in M_L^{d_L}$, $\bar{\mu}_R \in M_R^{d_R}$, $\bar{\mu}_R^{\circ} \in M_R^{0}$, and $x_L \in X$,

$$\pi_\lambda^{\mu_L,\mu_R^{\circ}}(d_L,x_L;0,0) \neq \sum_{x_R \in X} \pi_\lambda^{\mu_L,\mu_R}(d_L,x_L;d_R,x_R).$$

Suppose that each pair of a large ensemble of pairs of particles is prepared in the state λ.[13] Suppose also that experimenters 0_L and 0_R arrange to perform measurements of the d_L and d_R components (respectively) of the L and R particles (respectively) of each pair in the ensemble by means of corresponding ensembles of L and R measuring devices. Let it be agreed beforehand that 0_L is to perform his measurements by preparing each L measuring device in the state $\bar{\mu}_L = (\mu_L, d_L)$, but that 0_R has the choice of performing his measurements by preparing each measuring device in the state $\bar{\mu}_R = (\mu_R, d_R)$ or he can perform no measurements at all by preparing each R measuring device in the state $\bar{\mu}_R^{\circ} = (\mu_R, 0)$.[13] As before, 0_R is not to make his decision until such time as the making of that decision is space-like related to each of the L measurements.

If the ensembles are sufficiently large, we can expect the relative frequency of the outcome x_L in the L measurements to be insensitive to the actual number of systems in the ensembles (at least approximately). If 0_R decides not to perform his measurements, then this relative frequency "ought" to equal $\pi_\lambda^{\mu_L,\mu_R^{\circ}}(d_L,x_L;0,0)$ approximately. If 0_R does decide to perform his measurements, then it "ought" to equal $\sum_{x_R \in X} \pi_\lambda^{\mu_L,\mu_R}(d_L,x_L;d_R,x_R)$ approximately.

Of course, there is no guarantee of this; finite sequences (as well as infinite ones, for that matter) of measurement outcomes need not contain specific outcomes distributed precisely so that the relative frequencies equal or even approximate the corresponding single-case probabilities. For this reason, 0_L cannot infer with certainty 0_R's decision. However, this fact is without significance for the point at issue. Either 0_R performs the measurements or he does not, and the probabilities for the possible outcomes of 0_L's measurements depend on which of these is the case. That one might never have conclusive evidence for the value of a probability[16] does not subvert the relativistic basis for the locality condition.

Thus, locality is (if not strictly, at least nearly enough) a consequence of the relativistic prohibition against superluminal signals. It can also be shown that locality entails the impossibility of sending a superluminal signal with a Bell-type experimental setup. Therefore, in the context of such Bell-Type experimental setups, locality is (to all intents and purposes) fully equivalent to the impossibility of superluminal signals.[17] In order to complete the demonstration of this equivalence, it will now be shown that if a Bell-type experimental setup could be used to send a superluminal signal, then any theory which adequately described the phenomena would violate locality.

Once again, it will be convenient to treat deterministic theories and indeterministic theories separately. Consider the deterministic case first. Suppose experimenter 0_R could use a Bell-type setup to send a superluminal signal to 0_L, telling him whether or not measuring device R was set to measure some (not necessarily specified) component of the spin of particle R; i.e. suppose that by performing a measurement, e.g., of the d_L-component of the spin of particle L, 0_L could infer whether or not 0_R performs his measurement on particle R.[18] For this to be possible, a one-to-one correspondence must hold between the outcome of 0_L's measurement and 0_R's message (i.e. whether or not an R measurement is performed). For example:

OUTCOME OF MEASUREMENT OF d_L-COMPONENT OF SPIN OF PARTICLE L	0_R's MESSAGE
+1	"NO MEASUREMENT ON PARTICLE R"
−1	"MEASUREMENT ON PARTICLE R"

In this example, 0_L finds outcome $-1(+1)$ of his measurement just in case 0_R does (does not) perform his measurement.[19] It follows that for some chosen $\lambda \in \Lambda$, $\bar{\mu}_L \in M_L^{d_L}$, and $\bar{\mu}_R^\circ \in M_R{}^0$,

$$(6) \quad \pi_\lambda^{\mu_L,\mu_R^\circ}(d_L, +1;0,0) = 1.$$

Otherwise,[14] a $+1$ outcome of the measurement on particle L would indicate that a measurement had occurred at R. Also, for λ and $\bar{\mu}_L$ the same as above and for some chosen $d_R \in \triangle$ and $\bar{\mu}_R \in M_R^{d_R}$,

$$(7) \quad \pi_\lambda^{\mu_L,\mu_R}(d_L, +1;d_R, +1) = \pi_\lambda^{\mu_L,\mu_R}(d_L, +1;d_R, -1) = 0.$$

Otherwise, a $+1$ outcome of the measurement on particle L would be compatible with a measurement of some component of the spin of particle R. But locality (see eqn. (4)) requires that for all $\lambda \in \Lambda$, $d_L,d_R \in \triangle$, $\bar{\mu}_L \in M_L^{d_L}$, $\bar{\mu}_R \in M_R^{d_R}$, and $\bar{\mu}_R^\circ \in M_R{}^0$,

$$\pi_\lambda^{\mu_L,\mu_R^\circ}(d_L, +1;0,0) = \pi_\lambda^{\mu_L,\mu_R}(d_L, +1;d_R, +1) + \pi_\lambda^{\mu_L,\mu_R}(d_L, +1;d_R, -1).$$

Obviously this cannot be true if both (6) and (7) are true. This completes the argument for the deterministic case.

Now consider the indeterministic case. Suppose experimenter 0_R could use a large ensemble of Bell-type setups to send a superluminal signal to 0_L, telling him whether or not all of the R measuring devices were prepared to measure some (not necessarily specified) component of the spin of each particle R; i.e. suppose that by performing a measurement, e.g., of the d_L-component of the spin of each particle L, 0_L could infer whether or not 0_R performs his measurement on each particle R.[18] For this to be possible, a suitable correspondence must hold between the probabilities for the possible outcomes of 0_L's measurements (as approximated by the observed relative frequencies) and 0_L's message (i.e. whether or not the R measurements are performed). For example:

$\lvert p - x \rvert$	0_R's MESSAGE
≈ 0	"NO MEASUREMENT ON PARTICLE R"
$>> 0$	"MEASUREMENT ON PARTICLE R"

Here p is the fraction of the systems in which 0_L's measurement of the d_L-component of the spin of particle L yields the outcome $+1$ and x is some previously agreed upon value in the interval from 0 to 1.

In this example, 0_L performs his measurements and finds p significantly different from x just in case 0_R does perform his measurements.[19] It follows that for some chosen $\lambda \in \Lambda$, $\bar{\mu}_L \in M_L^{d_L}$, and $\bar{\mu}_R^\circ \in M_R{}^0$,

$$(8)\quad \pi_\lambda^{\bar{\mu}_L,\mu_R^0}(d_L,\ +1;0,0) \approx x.$$

Otherwise, values of p significantly different from x would be compatible[20] with there being no measurements at R. Also, for λ and $\bar{\mu}_L$ the same as above and for some chosen $d_R \epsilon \triangle$ and $\bar{\mu}_R \epsilon M_R^{d_R}$,

$$(9)\quad \left| \pi_\lambda^{\bar{\mu}_L,\bar{\mu}_R}(d_L,\ +1;d_R,\ +1) + \pi_\lambda^{\bar{\mu}_L,\bar{\mu}_R}(d_L,\ +1;d_R,\ -1) - x \right| >> 0.$$

Otherwise, values of p approximately equal to x would be compatible[20] with measurements of some component of the spin of the R particles. It remains only to point out that (8) and (9), taken in conjunction, are incompatible with locality (see eqn. (4)).

III. COMPLETENESS

A theory will be said to be *complete* (or to satisfy *completeness*) just in case the probability functions it specifies meet the following constraint:

For all $\lambda \epsilon \Lambda, d_L, d_R \epsilon \triangle, \bar{\mu}_L \epsilon M_L^{d_L}, \bar{\mu}_R \epsilon M_R^{d_R}, x_L, x_R \epsilon X$,

$$(10)\quad \pi_\lambda^{\bar{\mu}_L,\bar{\mu}_R}(d_L,x_L;d_R,x_R) = \sum_{x_R' \epsilon X} \pi_\lambda^{\bar{\mu}_L,\bar{\mu}_R}(d_L,x_L;d_R,x_R') \cdot \sum_{x_L' \epsilon X} \pi_\lambda^{\bar{\mu}_L,\bar{\mu}_R}(d_L,x_L';d_R,x_R).$$

Completeness asserts the stochastic independence of the two outcomes in each pair of spin measurements.[21] This can be interpreted as a natural requirement for theories which represent observable phenomena as the effects of interacting (but otherwise independently existing) entities whose physical state may be exhaustively characterized by the specification of some (not necessarily unique) set of definite, well-defined properties.[22]

For such theories, given the state of the two-particle system and the states of *both* measuring devices, the probability for a given outcome of the spin measurement at L(R) is independent of the outcome of the other spin measurement at R(L); that is to say, the probability for the given outcome at L(R) is invariant under conditionalization on the outcome of the measurement at R(L). However, unlike local theories, the probability for that outcome of the L(R) measurement may very well exhibit a dependence on the state of the distant measuring device R(L). By including in the state descriptions of the two-particle system and the measuring devices all those properties of the systems (i.e. precise numerical values of whatever physical quantities are appropriate for the types of entities posited by the theory) in virtue of which measurement interactions yield each possible outcome with its assigned probability, such state descriptions automatically "screen off" any correlation of the pairs of measurement outcomes which might arise from the omission from the state descriptions of predictively relevant information.

To be sure, knowledge of the outcome x_R of an R measurement may (depending on the character of the laws governing the interactions) ground inferences about the properties of particle L. In this way, one might learn from the outcome x_R that particle L is characterized by such-and-such values of such-and-such quantities with such-and-such probabilities; but *the very most* that one could infer from the outcome x_R is that particle L definitely possessed certain properties. If all such information about particle L is antecedently included in the state description, then knowledge of the outcome x_R provides only redundant information about the L subsystem.[23] Hence, the conditional probability of the outcome x_L of the L measurement given the outcome x_R of the R measurement must be equal to the unconditioned probability for x_L.[24] In other words, the two outcomes are stochastically independent.

A simple (and incorrect) model for the Bell-type correlated spin phenomena may serve as a useful illustration.[25] Suppose, purely for the sake of this illustration, that spin is correctly represented as an ordinary classical angular momentum. Suppose further that when a pair of particles is prepared in the singlet state, the spin vectors for the two particles are aligned exactly anti-parallel to each other. Moreover, given an ensemble of such two-particle systems, suppose that each direction in space is equally likely to be the direction of alignment for an arbitrarily selected member of the ensemble. Finally, if unit vector d gives the direction along which the axis of the Stern-Gerlach apparatus is aligned (i.e. the d-component of the spin is to be measured), and if s is the spin vector of the particle (with units chosen so that s has unit magnitude), then the outcome of that measurement is $+1$ if $d \cdot s > 0$ and -1 if $d \cdot s < 0$.[26]

According to this model, then, the probability for a spin up (or $+1$) outcome of a measurement of any component of the spin of either particle of a randomly chosen pair of particles in the singlet state is ½. For cases in which measurements are performed on both particles of a pair in the singlet state (denoted below by 'σ'), this may be expressed as follows:

For all $d_L, d_R \in \triangle, \bar{\mu}_L \in M_L^{d_L}$, $\bar{\mu}_R \in M_R^{d_R}$, and $x_L, x_R \in X$

$$\sum_{x_R' \in X} \pi_\sigma{}^{\mu_L,\mu_R}(d_L,x_L;d_R,x_R') = \tfrac{1}{2} = \sum_{x_L' \in X} \pi_\sigma{}^{\mu_L,\mu_R}(d_L,x_L';d_R,x_R).$$

However, if a d-component spin measurement is performed on each particle of a pair, the probability of finding the same outcome for both of the measurements must be 0. So, for all d ϵ $\triangle$, $\bar{\mu}_L \, \epsilon \, M_L{}^d$, $\bar{\mu}_R \, \epsilon \, M_R{}^d$, and x ϵ X,

$$\pi_\sigma{}^{\mu_L,\mu_R}(d,x;d,x) = 0\cdot$$

In this case, then, completeness is clearly violated. (See equation (10)).

The probabilities specified for this model are grounded in a blatantly incomplete description of the two-particle state. In the context of this model, if the theory assigns probabilities only on the basis of the occupancy by the two-particle system of the singlet state, then conditioning on the outcome, let us say, of a d-component spin measurement on particle R(L) may well yield a different probability for the outcome of a spin measurement on particle L(R) than would have been given by the corresponding unconditioned probability (i.e., ½). This is so because, if the outcome of the measurement on particle R is +1, then it may be inferred that $d \cdot s_L < 0$ (with probability 1), where s_L is the spin of particle L. Similarly, if the outcome is −1, then it may be inferred that $d \cdot s_L > 0$ (with probability 1). The outcome of the measurement on particle R provides information about particle L which was not included in the incomplete state description σ.

For a theory which assigns probabilities on the basis of the complete state description (for this model, such a state description would entail exact values for s_L and $s_R = - s_L$), any information about particle L(R) which may be inferred from the outcome of a measurement on particle R(L) is clearly redundant. In general, the very most that one could infer about an arbitrary spin measurement on particle L(R) from the outcome of a spin measurement on particle R(L) is considerably less than the knowledge of the L(R) measurement outcome; but given a complete state description, that outcome is determined (with probability 1), antecedently by s_L (s_R) and the state of the L(R) measuring device (i.e. the component of spin to be measured).

Indeed, all deterministic theories satisfy the completeness condition. This holds not only for models of the sort just described, but is true quite generally. To see this, consider the table below. It lists in each column one of the four possible sets of numerical assignments (those satisfying equation (3)) to the terms occurring in equation (10) for deterministic theories.

$\pi_\lambda^{\mu_L,\mu_R}(d_L, +1; d_R, +1)$	1	0	0	0
$\pi_\lambda^{\mu_L,\mu_R}(d_L, +1; d_R, -1)$	0	1	0	0
$\pi_\lambda^{\mu_L,\mu_R}(d_L, -1; d_R, +1)$	0	0	1	0
$\pi_\lambda^{\mu_L,\mu_R}(d_L, -1; d_R, -1)$	0	0	0	1
$\sum_{x_R' \epsilon X} \pi_\lambda^{\mu_L,\mu_R}(d_L, +1; d_R, x_R')$	1	1	0	0
$\sum_{x_R' \epsilon X} \pi_\lambda^{\mu_L,\mu_R}(d_L, -1; d_R, x_R')$	0	0	1	1
$\sum_{x_L' \epsilon X} \pi_\lambda^{\mu_L,\mu_R}(d_L, x_L'; d_R, +1)$	1	0	1	0
$\sum_{x_L' \epsilon X} \pi_\lambda^{\mu_L,\mu_R}(d_L, x_L'; d_R, -1)$	0	1	0	1

Inspection of this table reveals that for every instantiation of numerical values for x_L and x_R in (10), the values in each column do satisfy (10).[27]

This accords well with the interpretation of the completeness condition just put forward. If the state descriptions go so far as to *determine* measurement outcomes (and not just the probabilities for such), then those state descriptions include *a fortiori* (at least implicitly) a full account of whatever features of the physical situation contribute to the probabilities for the possible measurement outcomes.

The underlying deterministic character exhibited by the model discussed in this section is by no means essential for completeness; genuinely indeterministic theories certainly may satisfy (10). However, it is true that models which attribute strictly anti-correlated spins to the particles can be adequately described by a local complete theory only if that theory is deterministic (at least in the sense that it makes probability assignments of 0 and 1 only).[28] The proof that local complete indeterministic theories[29] are incompatible with such models is straightforward and will not be given here.

IV. STRONG LOCALITY

A theory will be said to be *strongly local* (or to satisfy *strong locality*) just in case the probability functions it specifies meet the following constraint:

For all $\lambda \epsilon \Lambda$, $d_L, d_R \epsilon \triangle$, $\bar{\mu}_L \epsilon M_L^{d_L}$, $\bar{\mu}_R \epsilon M_R^{d_R}$, $\bar{\mu}_L^o \epsilon M_L^0$, $\bar{\mu}_R^o \epsilon M_R^0$, and $x_L, x_R \epsilon X$,

$$\pi_\lambda^{\mu_L,\mu_R}(d_L,x_L;d_R,x_R) = \pi_\lambda^{\mu_L,\mu_R^\circ}(d_L,x_L;0,0) \cdot \pi_\lambda^{\mu_L^\circ,\mu_R}(0,0;d_R,x_R) \cdot$$

It is not difficult to show that strong locality is logically equivalent to the conjunction of locality and completeness. Consider an arbitrary strongly local theory. Then, for all $\lambda \in \Lambda$, $d_L, d_R \in \triangle$, $\bar{\mu}_L \in M_L^{d_L}$, $\bar{\mu}_R \in M_R^{d_R}$, $\bar{\mu}_L^\circ \in M_L{}^0$, $\bar{\mu}_R^\circ \in M_R{}^0$, and $x_L, x_R \in X$,

$$\begin{aligned}\sum_{x_R' \in X} \pi_\lambda^{\mu_L,\mu_R}(d_L,x_L;d_R,x_R') &= \pi_\lambda^{\mu_L,\mu_R^\circ}(d_L,x_L;0,0) \cdot \left[\sum_{x_R' \in X} \pi_\lambda^{\mu_L^\circ,\mu_R}(0,0;d_R,x_R')\right] \\ &= \pi_\lambda^{\mu_L,\mu_R^\circ}(d_L,x_L;0,0) \qquad \text{by equation (2);}\end{aligned}$$

and

$$\begin{aligned}\sum_{x_L' \in X} \pi_\lambda^{\mu_L,\mu_R}(d_L,x_L';d_R,x_R) &= \left[\sum_{x_L' \in X} \pi_\lambda^{\mu_L,\mu_R^\circ}(d_L,x_L';0,0)\right] \cdot \pi_\lambda^{\mu_L^\circ,\mu_R}(0,0;d_R,x_R) \cdot \\ &= \pi_\lambda^{\mu_L^\circ,\mu_R}(0,0;d_R,x_R) \qquad \text{by equation (1).}\end{aligned}$$

Therefore, all strongly local theories are local.

Also, if strong locality holds, then for all $\lambda \in \Lambda$, $d_L, d_R \in \triangle$, $\bar{\mu}_L \in M_L^{d_L}$, $\bar{\mu}_R \in M_R^{d_R}$, $\bar{\mu}_L^\circ \in M_L{}^0$, $\bar{\mu}_R^\circ \in M_R{}^0$, and $x_L, x_R \in X$,

$$\begin{aligned}&\sum_{x_R' \in X} \pi_\lambda^{\mu_L,\mu_R}(d_L,x_L;d_R,x_R') \cdot \sum_{x_L' \in X} \pi_\lambda^{\mu_L,\mu_R}(d_L,x_L';d_R,x_R) \\ &= \pi_\lambda^{\mu_L,\mu_R^\circ}(d_L,x_L;0,0) \cdot \left[\sum_{x_R' \in X} \pi_\lambda^{\mu_L^\circ,\mu_R}(0,0;d_R,x_R')\right] \\ &\qquad \cdot \left[\sum_{x_L' \in X} \pi_\lambda^{\mu_L,\mu_R^\circ}(d_L,x_L';0,0)\right] \pi_\lambda^{\mu_L^\circ,\mu_R}(0,0;d_R,x_R) \\ &= \pi_\lambda^{\mu_L,\mu_R^\circ}(d_L,x_L;0,0) \cdot \pi_\lambda^{\mu_L^\circ,\mu_R}(0,0;d_R,x_R) \text{ by (1) and (2)} \\ &= \pi_\lambda^{\mu_L,\mu_R}(d_L,x_L;d_R,x_R) \cdot\end{aligned}$$

Therefore, all strongly local theories are complete.

It remains only to show that complete local theories are strongly local. Assume locality and completeness. Then, for all $\lambda \in \Lambda$, $d_L, d_R \in \triangle$, $\bar{\mu}_L \in M_L^{d_L}$, $\bar{\mu}_R \in M_R^{d_R}$, $\bar{\mu}_L^\circ \in M_L{}^0$, $\bar{\mu}_R^\circ \in M_R{}^0$, and $x_L, x_R \in X$,

$$\begin{aligned}\pi_\lambda^{\mu_L,\mu_R}(d_L,x_L;d_R,x_R) &= \sum_{x_R' \in X} \pi_\lambda^{\mu_L,\mu_R}(d_L,x_L;d_R,x_R') \cdot \sum_{x_L' \in X} \pi_\lambda^{\mu_L,\mu_R}(d_L,x_L';d_R,x_R) \\ &\qquad \text{by equation (10)} \\ &= \pi_\lambda^{\mu_L,\mu_R^\circ}(d_L,x_L;0,0) \cdot \pi_\lambda^{\mu_L^\circ,\mu_R}(0,0;d_R,x_R) \\ &\qquad \text{by equations (4) and (5).}\end{aligned}$$

This concludes the proof.[30]

Strong locality is the assumption used in the derivation of generalized Bell-type inequalities.[31] As the conjunction of locality and completeness, strong locality requires of the probability for an outcome of a measurement on particle L(R) that it be independent of the outcome of any corresponding measurement on particle R(L) *and* that it be independent of the state of the R(L) measuring device. The local realistic character of strong locality derives directly from that of locality and completeness.

It is important to be clear about the distinction between completeness and strong locality. Although all strongly local theories are complete, complete theories need not be strongly local. This is so for complete theories which violate locality. The predictions of such a theory have the following character: The probability for the outcome of the L(R) measurement depends not only on the state of the two-particle system and the state of measuring device L(R), but it depends also on the state of the (other) measuring device R(L). (This is the "non-locality.") Nevertheless, that probability is invariant under conditionalization on the outcome of the R(L) measurement (if any); the three state descriptions, taken together, include a full record of all predictively relevant physical properties of the systems. (This is the "completeness.")

Each of these conditions, locality, completeness, and strong locality, is a requirement that certain probabilities remain invariant under some form of conditionalization. With the help of a bit more notation, this similarity of form may be exhibited more explicitly. The required definitions will now be introduced.

For all $\lambda \in \Lambda$, $d_L,d_R \in \triangle$, $\bar{\mu}_L \in M_L^{d_L}$, $\bar{\mu}_R \in M_R^{d_R}$, and $x_L,x_R \in X$,

$$\pi_\lambda^{\mu_L,\mu_R}(d_L,x_L;d_R,-) \equiv \sum_{x_R' \in X} \pi_\lambda^{\mu_L,\mu_R}(d_L,x_L;d_R,x_R')$$

and

$$\pi_\lambda^{\mu_L,\mu_R}(d_L,-;d_R,x_R) \equiv \sum_{x_L' \in X} \pi_\lambda^{\mu_L,\mu_R}(d_L,x_L';d_R,x_R)$$

For all $\lambda \in \Lambda$, $d_L \in \triangle$, $\bar{\mu}_L \in M_L^{d_L}$, and $\bar{\mu}_R \in M_R^{0}$,

$$\pi_\lambda^{\mu_L,\mu_R}(d_L,-;0,0) \equiv \sum_{x_L \in X} \pi_\lambda^{\mu_L,\mu_R}(d_L,x_L;0,0) = 1$$

(by equation (1))

For all $\lambda \in \Lambda$, $\bar{\mu}_L \in M_L^{0}$, $d_R \in \triangle$, and $\bar{\mu}_R \in M_R^{d_R}$,

$$\pi_\lambda^{\mu_L,\mu_R}(0,0;d_R,-) \equiv \sum_{x_R \in X} \pi_\lambda^{\mu_L,\mu_R}(0,0;d_R,x_R) = 1$$

(by equation (2))

Now each of the three conditions can be expressed in the same form:

For all $\lambda \in \Lambda$, $\bar{\mu}_L \in M_L^{d_L}$, $\bar{\mu}_R \in M_R^{d_R}$, $\bar{\mu}_L' \in M_L^{d_L'}$, and $\bar{\mu}_R' \in M_R^{d_R'}$,

$$\pi_\lambda^{\mu_L,\mu_R}(d_L,x_L;d_R,x_R) = \pi_\lambda^{\mu_L,\mu_R'}(d_L,x_L;d_R',-) \cdot \pi_\lambda^{\mu_L',\mu_R}(d_L',-;d_R,x_R),$$

provided that certain of the variables are restricted to the appropriate sets, as specified in the chart below:

	d_L	d_R	x_L	x_R	$\bar{\mu}_L'$	$\bar{\mu}_R'$
COMPLETENESS	$\triangle$	$\triangle$	X	X	$\{\bar{\mu}_L\}$	$\{\bar{\mu}_R\}$
LOCALITY (4)	$\triangle$	$\{0\}$	X	$\{0\}$	$M_L^{\triangle}$	$M_R^{\triangle}$
LOCALITY (5)	$\{0\}$	$\triangle$	$\{0\}$	X	$M_L^{\triangle}$	$M_R^{\triangle}$
STRONG LOCALITY*	$\triangle \cup \{0\}$	$\triangle \cup \{0\}$	$X \cup \{0\}$	$X \cup \{0\}$	$M_L^{\triangle}$	$M_R^{\triangle}$

*Of course, the case in which $d_L = d_R = 0$ and $x_L = x_R = 0$ must be excluded.

In view of this chart, the equivalence between strong locality and the conjunction of locality and completeness is obvious. The chart also provides a convenient place to pause for a brief summary:

(i) Completeness requires that the probability for the outcome of an L(R) measurement be invariant under conditionalization on the *outcome* of the distant R(L) measurement; it may depend on the state of the two-particle system and on *both* measuring device states. The outcome of the L(R) measurement must be stochastically independent of the outcome of the R(L) measurement.

(ii) Locality requires that the probability for the outcome of an L(R) measurement be invariant under conditionalization on the *state* of the distant measuring device R(L); it may depend on the state of the two-particle system and on the state of the L(R) measuring device. The outcome of the L(R) measurement need not be stochastically independent of the outcome of the R(L) measurement.

(iii) Finally, as the conjunction of locality and completeness, strong locality requires that the probability for the outcome of an L(R) measurement be invariant under conditionalization on the *state* of the distant measuring device R(L) *and* that it be invariant under conditionalization on the *outcome* of the R(L) measurement. The outcome of the L(R) measurement must be stochastically independent of the outcome of the R(L) measurement.

V. CONCLUDING REMARKS

The interpretation that strong locality inherits from locality and completeness may be exploited to help to understand the significance of the Bell arguments. Those arguments, together with the relevant experimental results, are generally taken to provide excellent evidence that strong locality cannot be satisfied by any empirically adequate theory. Since locality is contravened only on pain of a serious conflict with relativity theory (which is extraordinarily well-confirmed independently), it is appropriate to assign the blame to the completeness condition.

In terms of the interpretation of completeness presented here, one must conclude that certain phenomena simply cannot be adequately represented by any theory which ascribes properties to the entities it posits[32] in such a way that no measurement on the system may yield information which is both non-redundant (not deducible from the state descriptions) and predictively relevant for distant measurements.[33] That ''information'' is not (neither explicitly nor implicitly) contained in the ''incomplete'' state description.

Despite the general acceptance long ago of the ''orthodox'' interpretation of quantum mechanics, it might be said that only since the Bell-type experiments of recent years has there been solid evidence that the world-view which underlies completeness must be relinquished. The predictions of quantum mechanics, in good agreement with the experimental results, satisfy locality,[34] but violate completeness (and so, too, violate strong locality). Investigation of the quantum-mechanical violation of completeness leads immediately to deep unresolved mysteries at the foundations of quantum mechanics (including the infamous ''measurement problem'').

While this analysis provides no resolution of quantum mysteries, it is hoped that it may help to sharpen the focus on our view of those mysteries. By separating out the relativistic component of the strong locality condition (thereby exposing strong locality as the relativistic locality constraint for complete theories), there emerges a clarification of that class of theories excluded by the Bell arguments: the class of theories which satisfy completeness. Although the term ''incompleteness'' may connote a defect (as if, as was the case for the model discussed in Section III, all incomplete theories may be ''complet*ed*''), incomplete theories (e.g. quantum mechanics) are by no means *ipso facto* defective. On the contrary, when the result of Bell-type experiments are taken into account, the truly remarkable implication of Bell's Theorem is that incompleteness, in some sense, is a genuine feature of the world itself.

REFERENCES

[1] J. S. Bell, ''On the Einstein Podolsky Rosen Paradox'', *Physics* 1 (1964): 196-200.
[2] J. S. Bell, ''The Theory of Local Beables'', *Epistemological Letters* 9 (1976): 11-24.
[3] John F. Clauser and Abner Shimony, ''Bell's Theorem: Experimental Tests and Implications'', *Reports on Progress in Physics* 41 (1978): 1881-1927.

[4] P. H. Eberhard, "Bell's Theorem and the Different Concepts of Locality", *Il Nuovo Cimento* 46B, 2 (1978): 392-419.

[5] Arthur Fine, "Correlations and Physical Locality", *PSA 1980, Proceedings of the 1980 Biennial Meeting of the Philosophy of Science Association, Volume II*, edited by P. D. Asquith and R. N. Giere, E. Lansing, Michigan (1981): 535-562.

[6] G. C. Ghirardi, A. Rimini, and T. Weber, "A General Argument Against Superluminal Transmission through the Quantum Mechanical Measurement Process", *Lettere al Nuovo Cimento* 27, 10 (1980): 293-298.

[7] Geoffrey Hellman, "Einstein and Bell: Strengthening the Case for Microphysical Randomness", *Synthese* 53 (1982): 445-460.

[8] Geoffrey Hellman, "Stochastic Einstein-Locality and the Bell Theorems", *Synthese* 53 (1982): 461-504.

[9] Abner Shimony, "Critique of the Papers of Fine and Suppes", *PSA 1980, Proceedings of the 1980 Biennial Meeting of the Philosophy of Science Association, Volume II*, edited by P. D. Asquith and R. N. Giere, E. Lansing, Michigan (1981): 572-580.

[10] Brian Skyrms, "Counterfactual Definiteness and Local Causation", *Philosophy of Science* 49 (1982): 43-50.

NOTES

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[1] For an excellent review of the literature on this subject, see Clauser and Shimony [3].

[2] In fact, the locality condition usually employed in these versions of the argument is not quite that of relativity theory. (This point will be clarified a bit in section II.) Nevertheless, expressions such as "relativistic locality constraint" will be used loosely to refer to it.

[3] At least a partial justification can be given for the regrettably cumbersome-looking notation introduced in this section. It has become customary to represent the relevant probability functions in a way which builds in a certain locality condition (the one to be discussed in section II). For many purposes, this practice is convenient and entirely appropriate. However, for present purposes, a notation is needed which permits the expression of a certain condition ("completeness"—to be discussed in section III) which is independent of the locality condition and this requires a departure from the customary, more wieldy formalism. This desideratum still could be met without explicitly including full microstates of measuring devices (as will be done here). Nevertheless, since any properly general treatment of Bell's Theorem must give due consideration to the full microstates of measuring devices, notation of this sort does permit some added flexibility.

[4] Although the correlated spin experiments provide the context for the subsequent discussion in this article, the correlated photon polarization experiments would serve equally well.

[5] Probabilities of this sort are quite naturally regarded as single-case propensities.

[6] Following Skyrms [10], the assertions of the theory are to be regarded as subjunctive conditionals of the form: If measuring device L, in state $\bar{\mu}_L \in M_L^{d_L}$, and measuring device R, in state $\bar{\mu}_R \in M_R^{d_R}$, were to measure, respectively, the d_L-component of the spin of particle L and the d_R-component of the spin of particle R, for a pair of particles in the state $\lambda \in \Lambda$, then the probability that the outcome of the L measurement is x_L and the outcome of the R measurement is x_R would be $\pi_\lambda^{\mu_L,\mu_R}(d_L,x_L;d_R,x_R)$.

[7] Note that although the elements of D_π have the form $(\bar{\mu}_L,x_L,\bar{\mu}_R,x_R)$, the associated probability will be expressed as $\pi_\lambda^{\mu_L,\mu_R}(d_L,x_L;d_R,x_R)$, where $\bar{\mu}_L = (\mu_L,d_L)$ and $\bar{\mu}_R = (\mu_R,d_R)$. This is done merely to isolate the four parameters of greatest interest: the two components of spin to be measured and the two measurement outcomes.

[8] A state of the two-particle system is relevantly time-dependent if at least some of its associated probability functions exhibit time-dependence not solely due to that of the measuring device states. Two-particle states whose time-dependence resides exclusively in parameters

which do not contribute to the probabilities of interest, or whose associated probability functions have no time-dependence apart from that which results from time-dependent measuring device states, are, for present purposes, time-independent.

[9]See, for example, Fine [5] and Shimony [9] for more on such theories.

[10]The difficulty may be summarized in this way: If the states of the two-particle system are time-dependent, then the crucial premise of the Bell arguments—which, at least in standard formulations, constrains probabilities which have to do with possible *future* measurements but which are determined by the "initial" state of the two-particle system—loses its "local realistic" warrant and must be modified.

[11]For spacelike-related measurement events, these constraints constitute a condition close to, but somewhat weaker than Hellman's "stochastic Einstein locality" condition discussed in [8]. Hellman's condition requires that probabilities associated with measurement outcomes depend at most on the state of affairs in the backward light cone of the measurement event in question, while (4) and (5) require only that the probabilities not depend on the state of the distant measuring device.

[12]This point was made previously in Eberhard [4]. See Hellman [8] for another account of the relativistic locality condition.

[13]While it is here assumed that the experimenters are able to prepare systems in the desired states, the inability to do so in practice does not undermine the argument being made.

[14]For what follows, note that in the context of determinism, probability 1 corresponds to necessity and probability 0 corresponds to impossibility.

[15]Not all such theories are incompatible with relativity. Since the backward light cones of the two events indicated in the figure eventually do overlap, the *possibility* exists that by some unknown—but purely causal—mechanism, 0_R's decision and 0_L's measurement outcome are correlated in precisely the same way they would have been if there actually had been a superluminal physical disturbance. Such "conspiratorial" possibilities cannot be excluded, but there are (as yet) no positive grounds—none whatsoever—for taking them seriously. See Clauser and Shimony [3]: 1920-1, for more on this point. Still, even if these "conspiratorial" possibilities are excluded, it must be acknowledged that because superluminal transport of matter-energy is not actually entailed by the violation of locality, such a violation *need* not correspond to a violation of relativity theory. However, in this case, it is not clear how the violation of locality could be understood at all.

[16]This is a quite general matter. To whatever extent the confirmation of probabilistic assertions is taken to present serious problems, quantum mechanics is subject to those problems as well.

[17]A more precise evaluation of the status of locality with respect to relativity theory requires a further qualification in addition to that already mentioned in n.15 above. There it was indicated that locality is, in at least one respect, a bit stronger than relativity demands. However, it is also, in a different respect, a bit weaker. Since no assumptions have been made about the relationship between the exact structure of the elements of Λ and the probability functions they determine, locality permits the probabilities for outcomes of measurement events to exhibit a dependence on the physical properties of spatially distant parts of the system; such a probability would depend on information associated with events outside the light cone of the measurement event in question. However, in the present context, the exclusion of any dependence on the state of the distant measuring device is the condition which bears on the possibility of sending superluminal signals by performing spin measurements. In any case, since the truly relativistic locality condition is (in this sense) stronger than locality, conclusions deduced with locality as a premise (e.g. Bell-type inequalities) follow *a fortiori* from the truly relativistic locality constraint (except as qualified in n.15).

It should also be mentioned here that, in connection with considerations addressed in n. 11, since Hellman's concern is with superlimunary action more generally (i.e. not necessarily of a sort which could be utilized by an experimenter to send information), the argument to follow does not refute his position in § 5 of his [8].

[18]Note than any superluminal signal must consist of information coded in some such way. The account given here can readily be modified to treat cases in which 0_R may measure either one of two different specified components of the spin of particle R.

[19]Analogous treatment can be given to any other such scheme.

[20]In recognition of the aforementioned general difficulties associated with the confirmation of probabilistic assertions, the word 'compatible' should be understood here in an appropriate special (very strong) sense. Obviously, mere *logical* compatibility will not suffice. Indeed, all probabilistic assertions are *logically* compatible with any observations whatsoever.

[21]If completeness is satisfied, the two outcomes are stochastically independent relative to the given two-particle system state and measuring device states. To see this, notice that the right-hand side of (10) is a product of two terms. The first term is the probability for the x_L outcome of the L measurement (with unspecified outcome of the R measurement) and the second term is the probability for the x_R outcome of the R measurement (with unspecified outcome of the L measurement).

[22]As will become clearer in what follows, the sense of "completeness" expressed by (10), although it applies to indeterministic (as well as to deterministic) states, reflects certain "classical" intuitions. The reader who wishes to reserve the name 'completeness' for some more general condition which is not based on these intuitions, may choose to read 'classical-completeness' in place of 'completeness' in what follows.

[23]This interpretation of (10) breaks down for theories which permit stochastically-evolving states. (See the discussion which concludes section I.) The problem of devising a suitable generalization of the completeness condition which would apply to such theories will not be considered here.

[24]The issue here is not fundamentally an epistemological one. The point is that if the state descriptions are complete in the relevant sense, then conditionalization on the outcome of a measurement on particle R entails no further restrictions on the physical possibilities that would serve to better define the probabilities for the outcomes of possible measurements on particle L. However, it may be helpful to think of this conditionalization in terms of the acquisition of knowledge.

[25]Bell discussed a model of this sort in his 1964 paper ([1]). It is being used here to illustrate that theories which admit state descriptions which are compatible with more fully specified "hidden" states do not satisfy completeness.

[26]There is no need to consider the set of directions d such that $d \cdot s = 0$, since that set has measure zero and will not bear on the claims of interest here.

[27]Consider the following constraint:

(Condition C) For all $\lambda \in \Lambda$, $d_L, d_R \in \triangle$, $\bar{\mu}_L \in M_L{}^{d_L}$, and $\bar{\mu}_R \in M_R{}^{d_R}$, $\pi_\lambda{}^{\mu_L,\mu_R}(d_L, +1; d_R, +1)$ $\pi_\lambda{}^{\mu_L,\mu_R}(d_L, -1; d_R, -1)$

$$= \pi_\lambda{}^{\mu_L,\mu_R}(d_L, +1; d_R, -1) \cdot \pi_\lambda{}^{\mu_L,\mu_R}(d_L, -1; d_R, +1)$$

That deterministic theories must satisfy completeness may also be seen by first showing that completeness is logically equivalent to condition C and then noting that for deterministic theories, of the four probabilities occurring in the statement of condition C, three must vanish (and the other must be 1).

[28]This would not necessarily be the case if measurements did not minimally have to disclose the anti-correlation. For the claim to be strictly true, it must be assumed that if measurements of the same component of the spin of both particles are performed, the probability for identical outcomes of those measurements is zero.

[29]It will be shown in the next section that local complete theories are precisely those which satisfy strong locality.

[30]Therefore, for complete theories, locality is logically equivalent to strong locality. Since all deterministic theories are complete (as was shown in seciton III), it follows that for deterministic theories, locality is logically equivalent to strong locality. (See Hellman [7] for a different treatment of essentially the same point).

[31]In other treatments of the subject, one finds the assumption from which generalized Bell-type inequalities may be derived called by a variety of names. These include "local causality" (e.g. Bell [2]), "factorizability" (e.g. Fine [5]), "conditional stochastic independence (e.g. Hellman [8]), and a number of others. Since the argument is set up differently in different presentations, these conditions cannot be regarded as logically equivalent in any strict sense. However, it will be generally acknowledged that each "does the same work" in that version of the argument in which it occurs and that their content is essentially that

of strong locality. It should be noted that Clauser and Shimony ([3]: 1891-3) do employ a condition which is tantamount to strong locality, but their discussion of that condition may be a bit misleading; they appear to suggest that their condition is no more than the relativistic locality constraint (i.e. the condition herein called "locality") for indeterministic theories.

[32]The point is not (or at least not *merely*) that some predictively relevant properties attach "globally" to systems as a whole, but not to parts of the systems. It is more, for example, than that we may assign the value zero to every component of the spin of the two-particle system, but we cannot, for particles in this state, ascribe a definite value to any component of the spin of either particle.

It should be acknowledged that further analysis might yet reveal that imposition of the completeness condition excludes from consideration state descriptions which do not permit the analysis of each property of the two-particle system in terms of properties which may be ascribed to the two particles individually. If so, however, it is certainly not evident from what has been presented here.

[33]This emphasizes the sense in which completeness, too, has the character of a "locality" condition.

[34]Indeed, it can be shown quite generally that no superluminal signal can be produced by quantum mechanical measurements. See, e.g., Ghirardi, Rimini, and Weber [6] for a proof.