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# **Spacetime and Separability: Problems of Identity and Individuation in Fundamental Physics**

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## *1. Introduction: Einstein, Separability, and a “Pauli Program” for Fundamental Physics*

On 15 June 1935, shortly after the appearance of the Einstein-Podolsky-Rosen (EPR) paper (Einstein, Podolsky, and Rosen 1935), and before he knew that Bohr would himself be publishing a reply (Bohr 1935), Wolfgang Pauli wrote a long, worried letter to Werner Heisenberg complaining about the damage the paper might do: “*Einstein* has again expressed himself publicly on quantum mechanics, indeed in the 15 May issue of *Physical Review* (together with Podolsky and Rosen—no good company, by the way). As is well known, every time that happens it is a catastrophe.” Since the EPR paper was published in an American physics journal, there was a danger, Pauli said, of a “confusion” in American public opinion, and he urged Heisenberg to publish a “pedagogical” reply in the same journal.<sup>1</sup> Pauli was most concerned that the reply elucidate what he, Pauli, took to be the major issue in the debate:

[Einstein] now understands this much, that one cannot simultaneously measure two quantities corresponding to non-commuting operators and that one cannot simultaneously ascribe numerical values to them. But where he runs into trouble in this connection is the way in which, in quantum mechanics, two systems are joined to form a composite system.

...

A pedagogical reply to [this] train of thought must, I believe, clarify the following concepts. The difference between the following statements:

a) Two systems 1 and 2 are not in interaction with one another (= absence of any interaction energy).

*Definition.* This is the case if, after a maximal observation on 1, the expectation values of all quantities of 1 have *the same temporal evolution* as if 2 were not present. (NB. Anyhow, for sufficiently short times the concept of an interaction plays no role.)

b) The composite system is in a state where the subsystems 1 and 2 are *independent*. (Decomposition of the eigenfunction into a product.)

*Definition.* This is the case if, after a measurement of an arbitrary quantity  $F_2$  is carried out on 2, with a *known* result  $F_2 = (F_2)_0$  (number), the expectation values of the quantities  $F_1$  of 1 remain the same as without a measurement on 2 having been carried out.

Quite independently of *Einstein*, it appears to me that, in providing a systematic foundation for quantum mechanics, one should *start* more from the composition and separation of systems than has until now (with Dirac, e.g.) been the case. — This is indeed—as Einstein has *correctly* felt—a very fundamental point in quantum mechanics, which has, moreover, a direct connection with your reflections about the *cut* and the possibility of its being shifted to an arbitrary place. (Pauli 1985, pp. 402-404)

This is a striking letter in many respects. Let me single out just three.

First, Pauli displays real prescience with his highlighting of the concept of “independence” in point b), for this is essentially the concept of independence that we now know to lie at the heart of the puzzles over quantum nonlocality and nonseparability that have come to the fore in the literature on the Bell argument and the Bell experiments of the last decade. More specifically, the concept of independence highlighted by Pauli is essentially the same as the concept of “outcome independence” that Abner Shimony introduced in his mid-1980s papers on the interpretation of the Bell experiments (see Shimony 1984, 1986), which concept is, in turn, equivalent to the condition that Jon Jarrett dubbed “completeness” in his 1983 University of Chicago dissertation (Jarrett 1983) and subsequent papers (Jarrett 1984, Ballentine and Jarrett 1987), which were the inspiration for Shimony’s own mid-1980s papers. It is, therefore, also equivalent to what I have called “separability” in my own papers on the subject (Howard 1984, 1989, 1990, 1993).

As I will explain in more detail below, outcome independence or separability is to be distinguished from the logically independent concept of “parameter independence,” or “locality,” in my version of the argument (and Jon Jarrett’s original version), the conjunction of the two being logically equivalent to the Bell locality condition, and hence sufficient for the derivation of the Bell inequality, so that when the quantum mechanically predicted violations of the Bell inequality are

confirmed in the laboratory we can trace the cause to a failure of either outcome independence (separability), or parameter independence (locality), or both. Thus isolating the cause of the violations is helpful, because parameter independence arguably encapsulates the relevant physical content of special relativistic prohibitions on superluminal signaling in the context of the Bell experiments (hence Jarrett's and my dubbing it "locality" *simpliciter*), leaving a failure of outcome independence or separability as the likely culprit if, as reasonable physicists, we do not too hastily abandon special relativity. What the physical significance of the outcome independence or separability condition might be is something of an open question, although, as I will argue, the recognition that outcome independence is equivalent to a separability condition opens one window on the understanding of its possible physical significance.

The second striking feature of Pauli's last-quoted paragraph is that it points backward to what was by 1935 an *old* debate over the nonseparable manner in which quantum mechanics describes interacting systems. The fact that this was the central issue in the pre-1935 debate over the adequacy of the quantum theory disappeared from the collective memory of the physics community after EPR. But this aspect of the quantum had bothered Einstein since his own first paper on the quantum hypothesis in 1905, and from the time in 1925 when his work on Bose-Einstein statistics convinced him that it would be an unavoidable feature of any eventual quantum formalism, Einstein had been trying in every which way to convince his colleagues that this was sufficient reason to abandon the quantum path. Einstein believed that the spatio-temporal mode of individuating physical systems, and hence the assumption of the separability of spatio-temporally separated systems, had virtually the status of a priori truths, that they are conditions for the very possibility of scientific knowledge, and that a nonseparable quantum mechanics was, therefore, at

a very deep level, something fundamentally incoherent. Showing this was the point of his famous thought experiments at the Solvay meeting as well as the various proto-EPR arguments of the early 1930s. Showing this was also the point of the EPR paper itself, although as a result of Podolsky's having written the paper, this point got "buried by the erudition," as Einstein put it in a letter to Erwin Schrödinger right after the paper's publication. But it was not just Einstein who worried about quantum nonseparability in the years before 1935. It was also at the forefront of the thinking of Bohr and Schrödinger, and understanding better the quantitative phenomena associated with quantum nonseparability was also the point of what might be called the "proto-Bell" correlation experiments carried out by Walter Bothe and others during the 1920s.

Curiously, this pre-1935 debate about quantum nonseparability came to an end after EPR, largely, of course, because Bohr's reply to EPR was thought by most to have settled the question. A shift of interest to nuclear physics and quantum field theory no doubt also helped to divert attention from the problem of quantum nonseparability, as did the personal, professional, and political dislocations of the late 1930s and the absorption of the physics community into the war effort. When Einstein himself tried to revive the question in the late 1940s, he found no audience. Only in the wake of Bell's work have we once again realized that there is still something to be learned in connection with quantum nonseparability.

The third point of interest in the last-quoted paragraph of Pauli's June 1935 letter to Heisenberg—perhaps the most important point of all—is the fact that it points forward to a program for fundamental physics quite different from the then ascendent quantum field theory program. Pauli's reasoning is not spelled out in any detail, unfortunately, nor are the details of the alternative program he envisions. But recognizing that quantum nonseparability is the chief ontological novelty

of the quantum theory, and the chief point of difference between it and “classical” theories, including general relativity, Pauli seems to be suggesting that rather than pursue a program of field quantization—the concept of the field having been introduced, after all, only as device for explaining interactions—we should instead start from the quantum mechanical description of interactions.

In what follows, I will follow out all three of the leads indicated by Pauli. I will first briefly sketch the pre-1935 history of debates over quantum nonseparability, emphasizing Einstein’s worries about the failure of separability and his careful disentangling of the relevant conception of separability from the properly special relativistic concept of locality. I will then just as briefly review the current state of the discussion of quantum nonseparability in light the recent literature on Bell’s theorem and the Bell experiments. But this is really only by way of setting the context for taking up Pauli’s last suggestion about an alternative program for fundamental physics. For the real aim of this paper is to ask what a program of fundamental physics would look like that took seriously the challenge of “start[ing] more from the composition and separation of systems,” a program that I will call a “Pauli program.”

Proceeding in this fashion—history first, then contemporary theory and experiment, then speculative extrapolation—is determined by more than just the accident of Pauli’s having organized his letter to Heisenberg in this fashion. It is determined as well by the conviction that, with fundamental physics in the state it is in today, with ever more frequent bold claims that we are on the verge of a final unification, only a few small problems needing to be cleared up, the time is right for a revival of the Machian historical-critical analysis of concepts and theories in physics. My intuition is that we are a lot farther from final unification than many think, that what look like small

problems in quantum field theory, quantum gravity, cosmology, and other areas, may well be symptoms of deeper ills in our fundamental physics, much like the problems of black-body radiation and anomalous specific heats at the end of the last century were symptoms of deep problems in mechanics and electromagnetism. Mach's historical-critical analysis of the received concepts and theories in areas like mechanics and the theory of heat helped to prepare the ground for the quantum and relativity revolutions; indeed, it was this part of Mach's legacy, not his positivism, that Einstein praised in his 1916 obituary of Mach.

Received concepts and theories, what we learn from our textbooks and teachers, come to us with a certain fixity, almost an air of inevitability or necessity, not wholly the result of their empirical credentials or their logical power. They seem so inevitable in large part simply because they are so *old* and well-entrenched. But they often must be removed before a new understanding of nature can emerge, as was the case with the quantum and relativity theories, and one of the best ways—not the only way—to facilitate their removal is through a Machian historical-critical analysis that seeks to discover how and why those concepts and theories became entrenched in the first place. Concepts and theories have histories, such that understanding those concepts and theories *requires* understanding those histories. So, for that reason, a proper critique of those concepts and theories also requires a critical approach to their histories.

## *2. Einstein's Life-long Worries about Quantum Nonseparability*

Already in this first paper on the light quantum hypothesis in 1905, Einstein realized that the full story of the quantum would involve some compromise with classical notions of particle independence. He remarked explicitly that the assumption that light quanta behave like independent

particle-like carriers of electromagnetic energy would yield only the Wien end of the black-body spectrum. As to what kind of independence assumption we were making here, he was quite explicit. We are assuming, he said, that Boltzmann's principle is valid for a collection of light quanta, meaning, as he again said explicitly, that the joint probability for two light quanta to occupy two specific cells of phase space factorizes as a product of the separate occupation probabilities (Einstein 1905, pp. 139-142). But again, this assumption yields only the Wien end of the black-body spectrum, to get the other end, by implication, we will have to assume something else.<sup>2</sup>

Einstein returned to this issue in 1909, in his masterful survey paper, "Zum gegenwärtigen Stand des Strahlungsproblems" (Einstein 1909a), in correspondence with H.A. Lorentz, and in his influential address to the Salzburg *Naturforscherversammlung*, "Über die Entwicklung unserer Anschauungen über das Wesen und die Konstitution der Strahlung" (Einstein 1909b). It was in these papers and this correspondence that the notion of wave-particle duality first made its appearance (as well as the idea of "ghost" or "guiding fields"), the wave conception being introduced by contrast with particles as a device for conceptualizing the interference effects and failure of classical assumptions about particle independence that are manifest in those aspects of the interaction between matter and radiation that give rise both to the full black-body spectrum and to the mean-square fluctuations in energy and radiation pressure in black-body radiation. The issue came to the fore again in 1914 in some papers by Mieczysław Wolfke (1914a, 1914b) reporting the contents of conversations with Einstein about the kind of independence assumption made in the 1905 light quantum paper, where Wolfke repeats the point about the factorizability of the probabilities being the essence of the independence assumption.



It was only in late 1924 and early 1925 that the issue became acute for Einstein, namely, when his own work on Bose-Einstein statistics convinced him that the failure of classical notions of particle independence would have to be a fundamental and ineliminable feature of the physics of the quantum. The nature of this non-independence was still quite obscure to Einstein at that time, as is evident from a letter to Schrödinger of 28 February 1925, where he writes:

In the Bose statistics employed by me, the quanta or molecules are not treated as being *independent of one another*. . . . A complexion is characterized through giving the number of molecules that are present in each individual cell. The number of the complexions so defined should determine the entropy. According to this procedure, the molecules do not appear as being localized independently of one another, but rather they have a preference to sit together with another molecule in the same cell. One can easily picture this in the case of small numbers. [In particular] 2 quanta, 2 cells:

	Bose-statistics		independent molecules		
	1st cell	2nd cell		1st cell	2nd cell
1st case	■ ■	—	1st case	I II	—
2nd case	■	■	2nd case	I	II
3rd case	—	■ ■	3rd case	II	I
4th case			4th case	—	I II

According to Bose the molecules stack together relatively more often than according to the hypothesis of the statistical independence of the molecules. (EA 22-002)

In a postscript, Einstein remarks that Bose-Einstein are not incompatible with the statistics employed in his 1916 papers on transition probabilities, where the standard Maxwell-Boltzmann distribution was employed (Einstein 1916a, 1916b), because it is really only in relatively dense gases where the difference between the statistics of independent particles and the Bose-Einstein statistics

will be noticeable: “There the interaction between the molecules makes itself felt,— the interaction which, for the present, is accounted for statistically, but whose physical nature remains veiled.”

The problem here is simple. Boltzmann statistics, the statistics of independent particles, assume that, owing to their being spatially separated, the systems in question have separate, trackable identities, sufficient to enable us to name or label them and thus distinguish cases 2 and 3, which are collapsed into a single case in Bose-Einstein statistics because the systems in question there lack separate, trackable identities. If we still assume equal prior probabilities for the distinguishable cases, then the probability of two particles occupying the same cell of phase space, and hence of being spatially close together, is  $2/3$  in Bose-Einstein statistics, whereas it was only  $1/2$  in Boltzmann statistics, so that the failure of the classical mode of particle individuation manifests itself as a tendency for the systems in question to clump together. But how to explain this tendency to clump and this failure of the classical scheme of individuation is not at all clear to Einstein at this time.

The puzzles about the non-independence of systems obeying Bose-Einstein statistics became a prime focus of Einstein’s attention over the next few years, especially in his correspondence with Schrödinger. At every step, one finds Einstein worrying Schrödinger with the question whether or not the new wave-mechanical formalism he is developing will entail the same non-classical way of individuating interacting systems evinced in Bose-Einstein statistics. Indeed, when looked at through the window of his correspondence with Einstein in 1925-1926, Schrödinger’s development of wave mechanics can be described as being aimed primarily at constructing a formalism within which the Bose-Einstein scheme of individuation of interacting particles can be explained. The key move, of course, was Schrödinger’s realization that wave mechanics had to be done in configuration

space, not physical space, because only in configuration space can we find an ontology of joint states rich enough to include non-factorizable joint states of the kind that are necessary for explaining why two bosons cannot be endowed with separate, trackable identities.

As late as the spring of 1927, Einstein was still trying to salvage separability in quantum mechanics by means of a hidden variables model of the theory, as he explained in a lecture to the Prussian Academy on 5 May of that year, “Bestimmt Schrödingers Wellenmechanik die Bewegung eines Systems vollständig oder nur im Sinne der Statistik?” But he discovered to his dismay, after the manuscript of the talk was already in proofs, that even his own hidden variables model suffered from the same nonseparability that afflicted wave mechanics. This is the point of the following remark added in proof:

I have found that the schema does not satisfy a general requirement that must be imposed on a general law of motion for systems.

Consider, in particular, a system  $\Sigma$  that consists of two energetically independent subsystems,  $\Sigma_1$  and  $\Sigma_2$ ; this means that the potential energy as well as the kinetic energy is additively composed of two parts, the first of which contains quantities referring only to  $\Sigma_1$ , the second quantities referring only to  $\Sigma_2$ . It is then well known that

$$\Psi = \Psi_1 \cdot \Psi_2,$$

where  $\Psi_1$  depends only on the coordinates of  $\Sigma_1$ ,  $\Psi_2$  only on the coordinates of  $\Sigma_2$ . In this case we must demand that the motions of the composite system be combinations of possible motions of the subsystems.

The indicated scheme does not satisfy this requirement. In particular, let  $\mu$  be an index belonging to a coordinate of  $\Sigma_1$ ,  $\nu$  an index belonging to a coordinate of  $\Sigma_2$ . Then  $\Psi_{\mu\nu}$  does not vanish. (EA 2-100)

The paper was never published. One might conjecture that it was because of the nonseparability of the model.<sup>3</sup>

From this time on Einstein ceased making positive contributions to the development of the quantum theory. He turned his own constructive efforts back to unified field theory, which did satisfy a spatiotemporal principle of separability. His future remarks on the quantum theory were

to be confined to attempts to elucidate the paradoxical consequences of quantum nonseparability, chiefly via the series of thought experiments that began to develop at the Solvay meeting in September 1927. This was, for example, the real aim of the famous photon-box thought experiment that was the focus of Einstein's debate with Bohr at the 1930 Solvay meeting. Contrary to Bohr's reading of this as an attempt to disprove the indeterminacy relations (Bohr 1949, pp. 224-228), what Einstein was actually trying to show was that, if one assumed the separability of the box and the emitted photon, then one could ascribe two *different* state functions to the photon, even though, by virtue of the separation between the box and the photon, there would be one and only one reality pertaining to the photon. One of these states would correspond to a definite energy or color, if one chose to weigh the box, and one would correspond to a definite time of return, after reflection at a distant mirror, if one chose to read the clock controlling the shutter. That means that each of the two different state functions must be at best only an incomplete description of the unitary real state of the photon.<sup>4</sup>

It was no doubt this long history of Einstein's wrangles with Schrödinger, Bohr, and others that Pauli had in mind when he alluded to Einstein's concern with the way we do the physics of interacting systems in the above-quoted letter to Heisenberg. The issue of separability is not obviously thematized in the published EPR paper, but Pauli had been arguing about separability with Einstein and other colleagues for long enough to know what Einstein intended.

Einstein himself explained his intentions in another series of letters with Schrödinger, initiated by a letter from Schrödinger to Einstein of 7 June 1935 congratulating Einstein on the EPR paper. Einstein wrote back on 19 June, explaining to Schrödinger that he, Einstein, had not written the paper and that he did not like the way it turned out: "I was very pleased with your detailed letter,

which speaks about the little essay. For reasons of language, this was written by Podolsky after many discussions. But still it has not come out as well as I really wanted; on the contrary, the main point was, so to speak, buried by the erudition” (EA 22-047). Here goes on in this letter to explain what he had intended. In principle, the argument is the same as that sketched above in connection with the photon box thought experiment. Consider the two previously interacting but now widely spatially separated systems,  $A$  and  $B$ , in the EPR-type thought experiment. Depending upon the type of measurement we choose to perform on  $A$ , quantum mechanics has us assigning different theoretical states (different  $\psi$ -functions) to  $B$ . Einstein says that if we assume a “separation principle” [“Trennungsprinzip”], according to which the separate real state of  $B$  cannot be influenced by events in the vicinity of  $A$ , then there will be one and only one reality pertaining to  $B$ , regardless of what we do in the vicinity of  $A$ . But that means that quantum mechanics has us assigning *different* theoretical states to the *same* real state. According to Einstein, it follows that these theoretical states must therefore be incomplete.<sup>5</sup>

The “separation principle” of the 19 June 1935 letter to Schrödinger conflates two issues, namely, the two systems possessing separate real physical states by virtue of their spatial separation and one of these state’s not being capable of being influenced by events in the vicinity of the other system. The first, which is what will now be designated more carefully than heretofore a “separability” principle, seems to require only a spatial separation between the two systems. The latter, which will be dubbed a “locality” condition, seems to require, beyond that, a spacelike separation between the measurements we perform on the two systems, since it is obviously motivated by special relativistic locality concerns. It is important that separability and locality be distinguished. Separability is denied by the quantum mechanical theory of interactions, which

ascribes a non-factorizable joint state to previously interacting systems. It is invalid, indeed, for any theory that would incorporate the individuation schema implicit in Bose-Einstein statistics. Quantum mechanics does not, however, violate locality.

It was only in the late 1940s that Einstein clearly disentangled separability from locality. One of the most interesting discussions that evinces this clearer understanding is his 1948 essay, “Quanten-mechanik und Wirklichkeit,” which appeared in a special issue of the Swiss journal, *Dialectica*, that was edited by Pauli and entirely devoted to the interpretation of quantum mechanics. Here is the crucial passage:

If one asks what is characteristic of the realm of physical ideas independently of the quantum-theory, then above all the following attracts our attention: the concepts of physics refer to a real external world, *i.e.*, ideas are posited of things that claim a “real existence” independent of the perceiving subject (bodies, fields, *etc.*), and these ideas are, on the other hand, brought into as secure a relationship as possible with sense impressions. Moreover, it is characteristic of these physical things that they are conceived of as being arranged in a space-time continuum. Further, it appears to be essential for this arrangement of the things introduced in physics that, at a specific time, these things claim an existence independent of one another, insofar as these things “lie in different parts of space.” Without such an assumption of the mutually independent existence (the “being-thus”) of spatially distant things, an assumption that originates in everyday thought, physical thought in the sense familiar to us would not be possible. Nor does one see how physical laws could be formulated and tested without such a clean separation. Field theory has carried out this principle to the extreme, in that it localizes within infinitely small (four-dimensional) space elements the elementary things existing independently of one another that it takes as basic, as well as the elementary laws it postulates for them.

For the relative independence of spatially distant things (*A* and *B*), this idea is characteristic: an external influence on *A* has no *immediate* effect on *B*; this is known as the “principle of local action,” which is applied consistently only in field theory. The complete suspension of this basic principle would make impossible the idea of the existence of (quasi-) closed systems and, thereby, the establishment of empirically testable laws in the sense familiar to us. (Einstein 1948, pp. 321-322)

What Einstein here calls the “mutually independent existence of spatially distant things” is what I call separability. His “principle of local action” is what I call locality. Both are deemed necessary

for the doing of science. Without separability, says Einstein, “physical thought in the sense familiar to us would not be possible.” Denying locality would make impossible “the establishment of empirically testable laws in the sense familiar to us.” Why locality is necessary to secure the existence of closed systems is clear. Why separability is necessary for physical thought is not, at least not immediately. I will return to this question in a moment.

What is most important in this passage for our purposes is what Einstein says about field theories and separability: “Field theory has carried out this principle to the extreme, in that it localizes within infinitely small (four-dimensional) space elements the elementary things existing independently of one another that it takes as basic, as well as the elementary laws it postulates for them.” What Einstein seems to be saying is that, in effect, a classical field theory like general relativity provides an extreme embodiment of the separability principle because it treats each point-event in the spacetime manifold as a separate physical system endowed with its own separate physical reality in the form of, presumably, the value of the metric tensor at that point.

If field theories like general relativity are privileged by their thus providing extreme embodiments of the separability principle that quantum mechanics denies, then it is all the more important that we understand why separability is thought to be necessary for “physical thought in the sense familiar to us.” An important clue is contained in comments that Einstein wrote in the margins of the manuscript version of Max Born’s 1949 Waynflete lectures (Born 1949), Born having sent the manuscript to Einstein early in 1948 to get his reaction to Born’s account of their debates over the foundations of quantum mechanics. Einstein returned his comments in March of 1948. At the end he added the following long remark:

I just want to explain what I mean when I say that we should try to hold on to physical reality. We are, to be sure, all of us aware of the situation regarding what will turn out to be the basic foundational concepts in physics: the point-mass or the particle is surely not among them; the field, in the Faraday—Maxwell sense, might be, but not with certainty. But that which we conceive as existing (“actual”) should somehow be localized in time and space. That is, the real in one part of space, *A*, should (in theory) somehow “exist” independently of that which is thought of as real in another part of space, *B*. If a physical system stretches over the parts of space *A and B*, then what is present in *B* should somehow have an existence independent of what is present in *A*. What is actually present in *B* should thus not depend upon the type of measurement carried out in the part of space, *A*; it should also be independent of whether or not, after all, a measurement is made in *A*.

If one adheres to this program, then one can hardly view the quantum-theoretical description as a *complete* representation of the physically real. If one attempts, nevertheless, so to view it, then one must assume that the physically real in *B* undergoes a sudden change because of a measurement in *A*. My physical instincts bristle at that suggestion.

However, if one renounces the assumption that what is present in different parts of space has an independent, real existence, then I do not at all see what physics is supposed to describe. For what is thought to be a “system” is, after all, just conventional, and I do not see how one is supposed to divide up the world objectively so that one can make statements about the parts. (Born 1969, pp. 223-224)

So separability is the key to understanding Einstein’s realism! But what is the argument for its necessity? The argument is found in the last paragraph. Give up separability and then “I do not at all see what physics is supposed to describe.” Why? Because “what is thought to be a ‘system’ is, after all, just conventional, and [without separability] I do not see how one is supposed to divide up the world objectively so that one can make statements about the parts.”

Separability seems to be necessary for the purpose of dividing the world up into parts, this being necessary in order for us to be able to make statements about the parts. We need some conventional specification of what is to count as a system, and Einstein can see no objective way to do this other than via the scheme of individuation implicit in the separability principle. Why?

In a recent paper, I have explored the broadly Kantian background to this aspect of Einstein’s thinking about separability, arguing that it was Einstein’s reading of Schopenhauer on space and



time as the *principium individuationis* that probably taught Einstein this lesson and that by virtue of his adopting this view he stands in an old tradition regarding space as a ground for the individuation of physical systems, a tradition reaching back at least to Newton, if not beyond that to Pythagoras (see Howard 1996). But for now let us think about this question more from a physical than from an historical point of view.

The key word in Einstein's succinct argument for the necessity of separability is surely "objective." Without separability Einstein can see no "objective" way of dividing the world up into parts. In what sense does separability give an objective principle of individuation? This brings us back to the previously-quoted remark about field theories like general relativity providing an extreme embodiment of the separability principle.

As noted, a field theory like relativity gives embodiment to the separability principle by treating each point of the spacetime manifold as a separate physical system: "Field theory has carried out this principle to the extreme, in that it localizes within infinitely small (four-dimensional) space elements the elementary things existing independently of one another that it takes as basic, as well as the elementary laws it postulates for them." On one reading of this passage, it would be the existence of a non-null spatiotemporal (metric) interval between two spacetime events that serves as a sufficient principle for individuating those events. If we understand the separability condition in this way, then we might have our answer to the question why it provides the only imaginable objective scheme of individuation, for the metric interval is the fundamental invariant of the general relativistic group, and, being an invariant, is objective in a way that most other features of general relativistic spacetime, like simple spatial separation, are not. So even though Einstein sometimes speaks, loosely, of the independence of "spatially distant things,"

perhaps we should, nevertheless, interpret him as assuming that the relevant objective notion of separation is that of non-null spatiotemporal separation.

One might object that this could not have been Einstein's intention, since if we take non-null spatiotemporal separation as a sufficient condition for individuation, that would mean that we would have no basis for designating two light-like related events as separable, such as two stages in the career of a photon or two events like the emission of a photon at one location and its absorption or reflection at another location. Surely, one might argue, Einstein did not intend to have us deny the independence of two such events. Thus, in Einstein's version of the photon-box thought experiment, mentioned above, would not the emission, reflection, and eventual detection of the reflected photon have to count as independent events? Frankly, I do not know what Einstein's intention was in this respect; nowhere, to my knowledge, did he comment on it specifically. But in point of fact, it is not two or more events in the career of a single photon that have to be accounted independent by terms of the logic of the photon-box thought experiment or Einstein's own intended version of the incompleteness argument. All that is required is the independence of typically spacelike related events such as, in the case of the photon-box thought experiment, the reflection of the photon at a distant mirror and the choice either to weigh the box or to check the time of emission on the clock.

What are the alternatives to taking non-null spatiotemporal separation as sufficient for individuation? Surely it is not enough to stipulate just spacelike separation as a sufficient condition for individuation, for while that is an invariant, and thus objective notion, we certainly want to designate as independent all manner of timelike related events. Indeed, outside of the context of the quantum mechanical account of interactions, it would be causally related, which is to say

timelike related events (think of the collisions of molecules) that an Einstein would need most to mark as independent for the purpose of securing the applicability of the Boltzmann principle,  $S = k \cdot \log W$ , which is to say, the factorizing of the cell-occupation probabilities, in classical statistical mechanics.

Yet another invariant, objective notion is the existence of a non-null spatial interval *in at least one frame*. That would comprise spacelike, timelike, and lightlike related events, thus marking any two events anywhere in spacetime as separable, which, as I suggested above, was meaning of Einstein's characterizing classical field theories as providing the most extreme embodiment of the separability principle. Moreover, this is not at all an implausible reading of Einstein's talk of the mutual independence of "spatially distant things."

On the other hand, it was that same Albert Einstein who recognized, already at the time of his 1905 paper on the photon hypothesis, that the full Planck distribution formula for black-body radiation (as opposed to just the Wien end of the formula), could be derived *only* if one denied to the constituents of a photon gas the kind of independence possessed by molecules as described in classical, Boltzmann statistical mechanics. And it was the same Einstein who came to understand in 1924, inspired by the work of Satyendra Nath Bose, precisely how the denial of classical assumptions of particle independence in the case of photons could be turned into a positive program for the derivation of the Planck formula. Finally, as shall be explained in more detail shortly, it was this same Einstein who in 1920 invented what is today, sadly, a too-little-known argument to the effect that general relativistic covariance considerations actually entail the indistinguishability and hence the non-independence of particles, like photons, that act as the carriers of a field. On the face of it, of course, it is one thing to deny the mutual independence of two photons and quite another

to deny the independence of two events in the career of a single photon. Yet a closer consideration of Einstein's mentioned argument from covariance to indistinguishability will show that there is less of a difference between these two denials of independence than might at first appear to be the case.

Thus, the first proposed reading of Einstein's conception of separability, namely, as resting on the presence of a non-null spatiotemporal interval—rather than the existence of a non-null spatial interval in at least one frame—may well not be too far from the mark, even with its possibly paradoxical implication that two events in the career of a single photon are not independent of one another. In any case, even if one wants to argue for the weaker, more inclusive condition of the existence of a non-null *spatial* separation *in at least one frame*, the stronger condition of separation by a non-null *spatiotemporal* interval still works as a *sufficient* condition for individuation. The disagreement would be over whether or not it should also be a *necessary* condition. In section 4, below, this feature of the spatiotemporal interpretation of Einstein's separability condition will be turned to advantage in my sketch of a "Pauli program" for fundamental physics.

It should now be possible to understand better why Einstein attacked the quantum theory with such conviction and tenacity. Separability is necessary because, in its infinitesimal, four-dimensional, classical field theoretic embodiment, it provides the only objective criterion of individuation. But quantum mechanics denies separability. Quantum mechanics is therefore not merely incomplete, it is downright incoherent for Einstein. All the more unfortunate for Einstein's case then that precisely this feature of the quantum theory seems to be gain in credibility thanks to Bell's theorem and the results of the Bell experiments, to which I now turn.

3. *Shimony, Jarrett and Bell: The Experimental Evidence for Quantum Nonseparability.*

One of the reasons why Einstein's disentangling the respective roles of the separability and locality assumptions was so important is because there is a way of telling the story of Bell's theorem and the Bell experiments that places essentially those two principles at center stage. The crucial step toward this new understanding of Bell's theorem was taken by Jon Jarrett in his 1983 University of Chicago dissertation, written under the direction of Howard Stein and David Malament (Jarrett 1983). Let me summarize what I am now taking the separability and locality principles to assert, and then let me briefly outline what I like to call the Jarrett decomposition theorem.

*Separability Principle:* Regardless of their history of interaction, any two systems,  $A$  and  $B$ , separated by a non-null spatiotemporal interval possess their own separate real states, of such kind that the joint state is completely determined by the separate states.

*Locality Principle:* Given any two space-like separated systems,  $A$  and  $B$ , the separate real state of  $B$ , say, cannot be influenced by events (such as the choice of an observable to measure, say by rotating a Stern-Gerlach apparatus plus detectors) in the vicinity of system  $A$ .

(In the statement of both the separability and locality conditions, reference to separations between systems should be taken as shorthand for separation between the relevant events in which those systems figure, such as two measurement events or the "events" of the two systems' merely *being* at specific spacetime points. )



**Figure 1** Schematic of Bell Experiment

To understand what Jarrett proved with his decomposition theorem, first recall the original Bell locality condition, which may be written in this form:<sup>6</sup>

$$p_{\lambda}^{AB}(x,y|i,j) = p_{\lambda}^A(x|i) \cdot p_{\lambda}^B(y|j). \quad (\text{BL})$$

The superscripts  $A$  and  $B$  refer to the two wings of the typical Bell experiment, as illustrated schematically in Figure 1. The joint state of the composite system is represented by  $\lambda$ . The symbols  $i$  and  $x$  represent, respectively, the observable to be measured in the  $A$  wing and its possible values, while  $j$  and  $y$  represent the observable measured in the  $B$  wing and its possible values. In this form, then, the Bell locality condition says that for a composite system in joint state  $\lambda$  the joint probability of outcomes  $x$  and  $y$  for measurements of observables  $i$  and  $j$  in the  $A$  and  $B$  wings, respectively, is equal to the product of the separate probabilities.

Jarrett proved that the Bell locality condition is actually a conjunction of two logically independent conditions (Jarrett 1983, 1984; Ballentine and Jarrett 1987; Shimony 1986). Jarrett calls one of these the locality condition:<sup>7</sup>

$$p_{\lambda}^A(x|i,j) = p_{\lambda}^A(x|i), \quad (\text{JL-A})$$

$$p_{\lambda}^B(y|i,j) = p_{\lambda}^B(y|j). \quad (\text{JL-B})$$

This condition says that the outcome in  $A$  wing is statistically independent of the choice of a parameter to measure in the  $B$  wing and, conversely, that the outcome in the  $B$  wing is independent of the choice of a parameter to measure in the  $A$  wing. Thus, Shimony call this condition “parameter independence.” Jarrett calls this the locality condition because it is supposed to capture the physical content of special relativistic locality constraints more precisely than the original Bell locality condition, and, indeed, special relativity would seem to require that the act of setting the

apparatus in , say, the  $B$  wing, if it were spacelike separated from a measurement event in the  $A$  wing, should not affect the outcome of that measurement.

Jarrett calls the other condition the completeness condition:

$$p_{\lambda}^A(x|ij,y) = p_{\lambda}^A(x|ij), \quad (\text{COM-A})$$

$$p_{\lambda}^B(y|ij,x) = p_{\lambda}^B(y|ij). \quad (\text{COM-B})$$

From a mathematical point of view, the content of this condition is clear. It says that the outcome in each wing should be statistically independent of the outcome in the other wing, which is why Shimony calls it “outcome independence.” The physical content is much less clear, until we see that it is, essentially, identical to the separability condition.

The proof that the Bell locality condition is equivalent to the conjunction of Jarrett’s locality and completeness conditions is quite straightforward; for details, the reader is referred to standard presentations (see especially Jarrett 1984 and Ballantine and Jarrett 1987). Our interest is in the relation between Jarrett’s completeness condition and separability.

In order to prove to prove that outcome independence or completeness (COM) is equivalent to a separability condition, we must be careful about the state concept that we employ. I define a state  $\lambda$  as a conditional probability measure  $p_{\lambda}(x|m)$  assigning a probability to a measurement result  $x$ , conditional upon the presence of a measurement context  $m$ . This is, by intention, a very general state concept. Designed to facilitate the comparison of the ontologies of diverse theories, it is supposed to comprise the state concepts of both the quantum theory and general relativity. Thus, a ray in a Hilbert space can be made to generate such a probability measure, as can the metric tensor at a point of the spacetime manifold. For our purposes, it makes no difference that, in the latter case, which is a deterministic theory, the probabilities are all zero or one. I might also add that

while it is not essential for the sake of the proof to follow, it is nevertheless natural, from a physical point of view, to interpret states as I have defined them here as corresponding to physical propensities, which is to say, real dispositions of physical systems to display certain properties in certain contexts.<sup>8</sup>

Systematic reference to the measurement context,  $m$ , is a crucial feature of the concept of a physical state. Failure to “contextualize” the state concept has historically, I think, obscured the difference between separability and locality conditions as well as the connection between a separability condition and outcome independence. Think of the context, for the moment as consisting of the whole universe in which a measurement event is located, including events outside of the measurement’s forward and backward light cones, events spacelike separated from the measurement event. Conditionalizing thus on the context allows for the possibility that measurement results might be influenced by events that are spacelike separated from the measurement, this disposition to respond differentially to events in spacelike separated regions of the universe being thus a *part of* the very reality that the state in question is supposed to represent, as opposed to being seen only as an influence *on* that reality. Omitting explicit reference to the context from the state concept, say by defining states simply as probability measures assigning probabilities to measurement outcomes, means that we are implicitly taking all states to be local states, which in turn makes it hard, both conceptually and mathematically, to distinguish the different varieties of possible influences on measurement outcomes. Make the context part of the state concept and then one can distinguish failures of locality or parameter independence from failures of outcome independence or separability.



For our immediate purposes, we can think of the context as including only the observables to be measured in each wing. Thus, one should read an expression like  $p_\lambda(x|i,j)$  as representing a state  $\lambda$  that assigns a probability to measurement result  $x$ , in a context in which observable  $i$  is measured in the  $A$  wing, and observable  $j$  is measured in the  $B$  wing.<sup>9</sup>

To prove that outcome independence or completeness is equivalent to a separability condition, we must first formulate the latter in an appropriate manner. Thus, two systems  $A$  and  $B$  will be said to be separable if there exist separate states  $\alpha$  and  $\beta$  for these systems, such that

$$p_\lambda^{AB}(x,y|i,j) = p_\alpha^A(x|i,j) \cdot p_\beta^B(y|i,j), \quad (\text{SEP})$$

with  $\lambda$  representing the joint state of  $A$  and  $B$  together. One can easily show that (SEP) is closely related to the ordinary quantum mechanical definition of separable systems, according to which the joint state is a tensor product of the separate states, i.e.,  $\psi_{AB} = \psi_A \otimes \psi_B$ , inasmuch as pairs of systems satisfying this latter condition satisfy (SEP) and pairs of systems that are nonseparable in the quantum mechanical sense violate (SEP). Furthermore, it should be evident that two systems satisfying the *separability principle* defined a few paragraphs earlier, which asserted that two spatiotemporally separated systems possess separate real states of such a kind that their joint is exhaustively determined by the separated states, also satisfy (SEP) and that violating the *separability principle* would violate (SEP), given, that is, the state concept being employed here.

(SEP) is a factorizability condition, but it is not equivalent to Bell-locality (BL), for on the right-hand side of (SEP), unlike the right-hand-side of (BL), we conditionalize on *both*  $i$  and  $j$ . For that reason, (SEP) makes no assumption about whether or not Jarrett locality holds.

To prove that (SEP) is equivalent to (COM), one first introduces the following identifications:

$$p_a^A(x|i,j) = p_\lambda^A(x|i,j), \quad (1)$$

$$p_\beta^B(y|i,j) = p_\lambda^B(y|i,j). \quad (2)$$

From the definitions of joint and conditional probability we have

$$p_\lambda^{AB}(x,y|i,j) = p_\lambda^A(x|i,j,y) \cdot p_\lambda^B(y|i,j), \quad (3)$$

which, for the case where  $p_\lambda^B(y|i,j) \neq 0$ , can be rewritten in the form:

$$p_\lambda^A(x|i,j,y) = p_\lambda^{AB}(x,y|i,j) / p_\lambda^B(y|i,j). \quad (4)$$

From equation (4) and (SEP) one gets immediately

$$p_\lambda^A(x|i,j,y) = [p_a^A(x|i,j) \cdot p_\beta^B(y|i,j)] / p_\lambda^B(y|i,j). \quad (5)$$

(COM-A) follows from (5) with (1) and (2). An analogous derivation yields (COM-B) from (SEP).

To complete the other half of the proof of equivalence, note that equation (3) and (COM-A) yield

$$p_\lambda^{AB}(x,y|i,j) = p_\lambda^A(x|i,j) \cdot p_\lambda^B(y|i,j). \quad (6)$$

(SEP) follows directly from (6) together with (1) and (2). Since (SEP) and (COM) imply one another, they are logically equivalent.

We have already seen Einstein arguing that both the locality and separability principles, but especially the latter, are virtually a priori conditions for the very possibility of a physical science. Locality was said to be necessary to insure the existence of systems that are “closed,” or at least screened off from extraneous influences, in a manner sufficient to allow us to ascribe measurement results to the systems putatively undergoing measurement. Separability was said to be constitutive of the very concept of physical reality itself. Now we see that their conjunction—upon which Einstein based his incompleteness argument—also entail the Bell locality condition, meaning that the experimental confirmation of quantum-mechanically predicted violations of the Bell inequality now force us to abandon one or the other (or both) of locality and separability.

Quantum mechanics, we now understand, works in the context of the Bell experiments by virtue of its being a local, nonseparable theory. Bohm-type hidden variable theories, which are empirically equivalent to quantum mechanics in the context of the Bell experiments, pull off the same trick by being separable—all systems at all times possess their own, individual states, such that joint states factorize—but nonlocal, the nonlocality being found in the quantum potential.

Which is the right choice? Bohr or Bohm? The answer seems simple: If we do not want to repudiate special relativistic locality constraints, then we must choose the quantum mechanical path, meaning locality but nonseparability. My guess is that even Einstein would have concurred in this choice, given the news about the results of the Bell experiments, because separability partakes more of the character of a constructive model, whereas locality has more the character of those high-level empirical generalizations that Einstein dubbed “principles” and that he thought should serve as regulative principles guiding our search for possible constructive models of the phenomena.<sup>10</sup>

What Einstein would have chosen is not, however, the important issue. We are the ones who must choose and what is important in that connection is our understanding the physical implications of our choice. Foremost among those is that, if we take the quantum mechanical path, then the spatiotemporal mode of individuation for physical systems that Einstein thought essential to the classical field theoretic approach of general relativity must be abandoned. That is to say that, the results of the Bell experiments do not threaten special relativity; they threaten, instead, general relativity.

#### 4. A “Pauli Program” Program for Fundamental Physics

So the situation is this. Einstein rightly believed that field theories like general relativity, which assign a determinate value of the fundamental field parameter—the metric tensor in general relativity—to every point of spacetime, represent an extreme embodiment of spatiotemporal separability, since, in effect, they treat each point of the spacetime manifold as a physical system endowed with its own well-defined physical state, separate from the physical states of all of its neighbors, near and far, excepting perhaps those with which it is lightlike related. Einstein thus takes any nonnull, even infinitesimal spatiotemporal separation as a sufficient condition for regarding two points or regions of spacetime as separate and separable physical systems, this latter possibility being secured by the fact that the infinitesimal metric interval is the fundamental invariant of the general relativistic class of transformations, and hence an objective feature of general relativistic spacetime.

On the other hand, Bell's theorem and the results of the Bell experiments must be read as confirming the quantum mechanical violations of spatiotemporal separability, if, that is, we agree not to admit nonlocalities of the kind prohibited by special relativistic prohibitions on superluminal physical effects. But this means that we have discovered a fundamental incompatibility between general relativity and quantum mechanics at the level of the fundamental ontologies of the two theories: General relativity requires separability, quantum mechanics denies it, and experience, in the form of the Bell experiments, suggests that quantum mechanics is right and general relativity wrong.

The conflict between general relativity and quantum mechanics at the level of their fundamental ontologies suggests that any ontologically naive unification program is bound to fail, that gravitation will not be incorporated into a unified approach to fundamental physics by anything

so simple as quantizing the gravitational field. Another approach is needed, one that begins with the lessons of quantum nonseparability at the level of fundamental ontology. In line with Pauli's suggestion to Heisenberg, it is this alternative approach that I want to call a "Pauli program" for fundamental physics. What would such a program look like?

In an earlier paper, I deliberately urged an unblushingly metaphysical approach to the question of how quantum nonseparability might be incorporated into a fundamental understanding of nature, with special reference to the task of imagining alternatives to general relativistic spacetime as the context in which such a fundamental theory should be located. I toyed there with ideas like higher-dimensional spaces, where systems separated in ordinary four-dimensional spacetime would be immediately adjacent in one or more higher dimensions, or miniature wormholes providing backdoor connections in spacetime (Howard 1989). Here, again, I want to focus on questions of spacetime structure, but I now want to pursue a different, more "Einsteinian" approach, and this in two respects.

First, Einstein was famous for arguing that progress is more likely to be made in a new theoretical domain not by seeking, too early, a constructive model of the phenomena, but instead by identifying a few well-confirmed, high-level empirical generalizations, like the light and relativity principles in special relativity or the Boltzmann principle in statistical physics, that can serve as regulative principles guiding the search for a constructive model (Einstein 1919). Newton advocated a similar approach, especially in his early work on the constitution of light, where he cautioned his critics, like Hooke, not to "mingle conjectures with certainties," "conjecture" being hypothetical models for the constitution of light, "certainties" being high-level empirical generalizations about the constitution of light of the kind confirmed in Newton's prism experiments

(see Howard 1991). What would be the corresponding “principles” or “certainties” that might govern the search for conjectural, constructive models of a nonseparable quantum spacetime?

The obvious answer is nonseparability itself, understood first and foremost as a restriction on the kinds of statistical correlations between previously interacting systems that a Pauli program can admit, and by extension both as a constraint on the kind of joint state functions that are to be assigned to previously interacting systems, and as a constraint on the basic ontology itself, at the level of the method whereby we individuate the physical systems that are the possessors of those physical states. What kind of constraint on the joint state functions? If states are taken to be assignments of joint probabilities to ordered pairs of contexts and outcomes, as explained in section 3 above, then the joint state functions must not be decomposable into separate state functions. What kind of constraint on the basic ontology itself? At least by way of example, non-null spatiotemporal separation may not be taken as a principle or ground for the individuation of otherwise identical fundamental physical systems.

The spacetime event ontology of general relativity does not satisfy these constraints. Let us pause to reflect, for a moment, on how it was that we nevertheless thought our way to a spacetime theory whose fundamental ontology now proves to be unacceptable. Newton invented spacetime as the ground against which systems interacting gravitationally and their states were to be individuated, space individuating the systems and time individuating the states of those systems. The gravitational field, which was a metaphysical puzzle for Newton, was invented as a device for explaining gravitational interactions without assuming either action-at-a-distance or some kind of mysterious, Leibnizian pre-established harmony.

Spacetime and the gravitational field were married by Einstein when he elevated the equivalence of inertial and gravitational mass to the level of principle, and thus was effected a considerable ontological economy, since, once the two were conjoined, we had one and the same structure doing both the mediating of gravitational interactions in a local fashion and the individuating of the interacting systems in a separable manner. Not only that, but with the relativistic merger of space and time into spacetime, we now had just one structure, spacetime, individuating both systems and their states, which is to say that the distinction between systems and states of systems also loses its fundamental significance in the event ontology of relativity. Further simplification came via Einstein's choice to enlarge the relevant group from the Lorentz group to the general relativistic group, a move that was not only possible, but in a sense natural, since neither mediation nor individuation required the invariance of finite spatiotemporal intervals, the invariance of the infinitesimal metric interval in general relativity being all that was needed for either purpose. Electricity and magnetism had already been given a home in the relativistic spacetime framework when, with the development of special relativity, the Galilean group was replaced by the Lorentz group, the latter being the implicit invariance group for Maxwell's equations.

Quantum mechanics was born out of the effort to understand the peculiar non-Boltzmannian statistics that emerge when we examine the interaction of electromagnetic radiation with ponderable matter, that is with systems having non-null gravitational mass, the photon eventually being invented as the massless carrier of electromagnetic energy. As long as we considered either electromagnetic interactions between charged particles (where inertial mass still plays a role in parameterizing a system's resistance to changes in its state of motion due to electromagnetic forces) or purely gravitational interactions (with inertial mass here parameterizing a system's resistance to changes

in its state of motion due to gravitational forces), no quantum theory was needed. Again, quantum phenomena came to the fore *only* when we asked about what might be termed “mixed” interactions: electromagnetic effects on bodies capable also of feeling gravitational effects thanks to their gravitational mass. And what was found for those situations was that the non-Boltzmannian statistics were most naturally generated by adopting a non-classical scheme for the individuation of the relevant physical systems, with non-null spatiotemporal separation no longer being a sufficient condition for endowing two (or more) interacting systems with trackable identities, herein lying the germ of quantum nonseparability.

Quite apart from the nonseparability that emerges in quantum mechanics, there were already, as mentioned before, some puzzling features about the way the photon, the carrier of electromagnetic energy, lives in relativistic spacetime. Most importantly, even though the infinitesimal metric interval is the fundamental invariant of general relativity, its objectivity thereby securing the possibility of the spatiotemporal mode of individuation of physical systems so prized by Einstein, nevertheless, there is a null separation between any two lightlike related events and hence between any two stages in the career of a single photon. The same will be true for any massless particle “traveling” at the speed of light. This is a consequence of our doing physics on a spacetime manifold with Lorentz signature.

Let’s stop and think for a moment about what this feature of photon trajectories entails. In particular, let’s think more systematically about the consequences of interpreting the existence of a non-null spatiotemporal separation as a *necessary* condition for individuation. Historically, of course, non-null spatiotemporal separation was read as both a sufficient and a necessary condition for individuation. Its role as a necessary condition for individuation is implicit in the classical



definition of matter or a material body, according to which no two pieces of material substance could occupy the same place at the same time. For, in effect, this definition says that if two pieces of material substance do occupy the same place at the same time, that is, if they are separated by a null spatiotemporal interval, then they are not two, but one. Moreover, the idea that non-null spatiotemporal separation is a necessary condition for individuation also played, historically, a crucial, if not always clearly articulated role in our thinking about the distinguishability of particles. Because if we permitted two different pieces of matter to occupy the same portion of space at the same time, then two otherwise identical particles could interpenetrate and exchange identities in a “collision,” the only kind of purely mechanical interaction in classical mechanics. But if particle identities could be exchanged in a collision, we would no longer have trackable particle identities and so, for example, the Boltzmann principle would no longer be valid in statistical mechanics.

What was never contemplated classically was the existence of a non-null spatiotemporal interval between lightlike related events in spacetimes with Lorentz signature. If non-null spatiotemporal separation is regarded as a necessary condition for individuating two systems, then two events lying on a lightlike trajectory will not be endowed with separate identities. However widely separated in space or time two such events may appear to some observer, if they represent two points on the world-line of a photon they would not then qualify as separable systems.

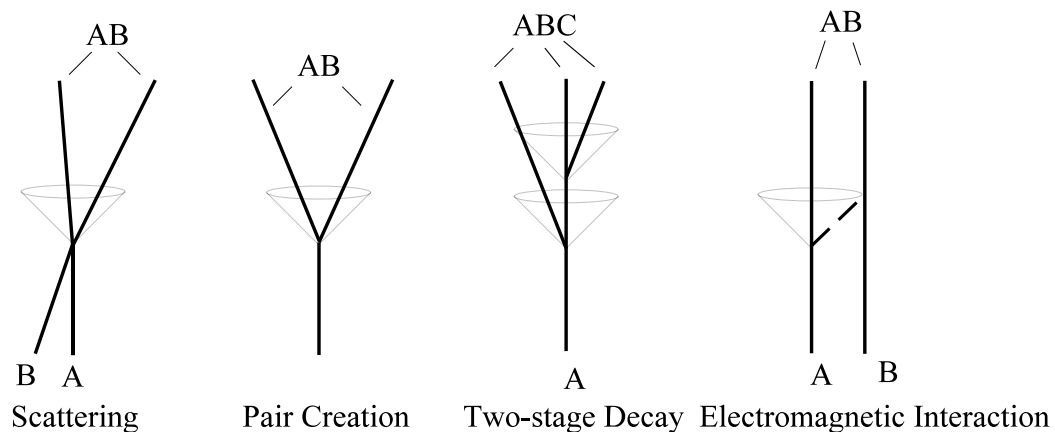
In general relativistic spacetime, there is one other circumstance under which two systems find themselves separated by a null interval, namely, when their worldlines intersect, as when a neutral particle scatters from another particle, a particle-antiparticle pair is produced by pair creation, or a particle decays, yielding two or more offspring particles as decay products. So here too, if the presence of a non-null spatiotemporal interval is supposed to be a necessary condition for

individuating physical systems, two systems whose world-lines intersect would not be capable of being individuated, at least at the point where their world-lines intersect. But that means that if we discount purely gravitational interactions (and ignore, for the moment, strong and weak nuclear interactions), general relativistic spacetime already has within it structure that to some extent mimics the pattern of individuation typical of quantum mechanics for all interacting systems.

Can we build upon this insight in such a way as to elicit from general relativistic spacetime itself enough structure so as to reproduce the characteristic quantum mechanical pattern of individuation? I think that the answer may be yes, and this is the second respect in which I want to pursue here a more “Einsteinian” approach to spacetime and individuation.

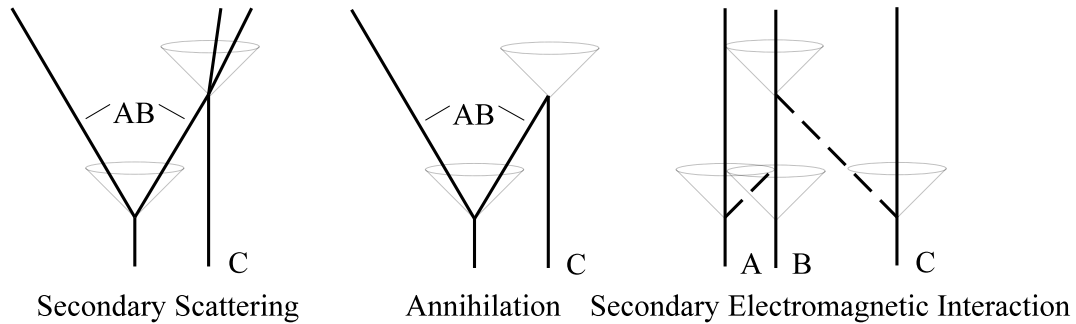
Let me give a provisional formulation of the idea that I have in mind as a principle that I will dub the *QM-GR principle of individuation*:

I. Any two systems,  $A$  and  $B$ , that were anywhere separated by a non-null spatiotemporal interval in their causal pasts are to be regarded as separable systems at all points in their causal futures unless (1) their world-lines intersect, or (2) their world-lines are both intersected by the world-line of a photon. In either of these cases, systems  $A$  and  $B$  are to be regarded as nonseparable at all points in the causal future(s) of the intersection(s), unless condition II obtains.



**Figure 2** Ways of Engendering Nonseparability

II. Any two nonseparable systems,  $A$  and  $B$ , are to be regarded as separable systems at all points in the causal futures of any intersection of either  $A$  or  $B$  with a third system  $C$ , as in I. (1) above, or with a photon emitted by a third system  $C$ , as in I. (2) above.



**Figure 3** Ways of Breaking the Nonseparability

Notice how this principle of individuation differs from Einstein's way of using the point structure of the manifold. For Einstein, any non-null spatiotemporal interval was a sufficient condition for individuating two physical systems. Thus, even if the world lines of two systems intersected, from the moment after their intersection Einstein's spatiotemporal individuation principle would count them as once again as separable systems. The proposal under consideration here would not.

At the root of the difference between Einstein's spatiotemporal principle of individuation and the QM-GR principle of individuation is the fact that Einstein in effect wants to allow the individuation of physical systems in spacetime to be induced by the underlying individuation of the points of the manifold itself, each point corresponding to a separate physical system. By contrast, I am trying to tease a quantum mechanical scheme of individuation out of the worldline structure. That Einstein wanted to proceed as he did should come as no surprise. Newtonian space tacitly individuated systems in the same way, every point of space being, at least potentially, if God wills

it, a point particle.<sup>11</sup> The concept of the topological manifold is the descendent of Newtonian space, its more immediate ancestor being the  $n$ -dimensional Riemannian manifold  $\mathbb{R}^n$ . One can take away from  $\mathbb{R}^n$  all remnants of the idea of individuation via coordinatization, but that still leaves a bare manifold with a built-in notion of the individuation of its points via their separations, albeit not in any metrical sense, from their neighbors. Thus, Einstein's wanting to use the point structure of the spacetime manifold as the basis for his scheme of individuation means that he is just being true to the Newtonian tradition on individuation, which I mentioned above had been passed on to Einstein through Kant and Schopenhauer.

The QM-GR principle of individuation, however, uses not the point structure of the manifold, but the worldline structure. It would therefore be desirable, perhaps, to try to free the formulation of that principle from all vestiges of notions borrowed from the point structure, such as talk of non-null spatiotemporal interval between two systems during their history prior to some interaction of interest, as in clause (I). Though I will not attempt such an explicit reformulation here, there should be no obstacle to our doing so, since all that we need from the point of view of the QM-GR principle of intervention is a more basic notion of non-coincidence. The reference to the metrical notion of the null interval between two events that are lightlike related, which led to our conferring special status on photon worldlines, can be replaced by a definition of photon worldlines that turns upon their role in determining the lightcone structure of general relativistic spacetime. We need the lightcone structure also to ground the notions of causal past and causal future. Thus, we should be able to succeed in the attempt to construct a QM-GR principle of individuation with nothing more than world lines, the concept of the intersection of world lines, and the lightcone structure.

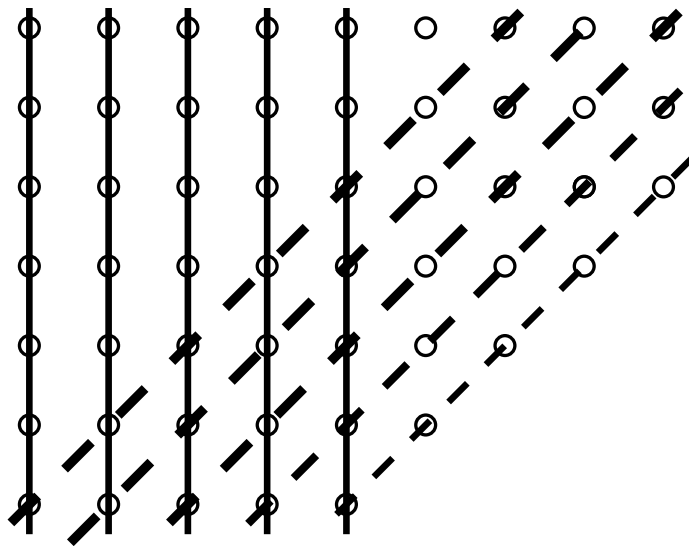
There are independent reasons for trying to free ourselves from reliance on the point structure of the manifold in favor of the world line structure, reasons stemming from Einstein himself. They are to be found in the so-called point-coincidence argument that Einstein developed in the effort to get around the error of the “hole argument,” the fallacious argument that contributed the three-year delay in Einstein’s affirmation of the correction general relativistic field equations. Einstein realized that the problem at the root of the hole argument was a confusion over the individuation of the points of the spacetime manifold. In 1912, at the time of the hole argument, Einstein wrongly believed that a coordinatization of the manifold was sufficient for individuating its points, overlook the fact that coordinatizations are not invariant. When he rejected coordinatization as a principle of individuation in late 1915, he proposed that we instead take the points of the manifold to be defined implicitly, and thus individuated as points of intersection of world lines. This is, to be sure, not the only way around the error of the hole argument, as John Norton has shown using the notion of “Leibniz equivalence” to construct equivalence classes of isomorphic physical spacetimes (see, for example, Norton 1989). On the other hand, John Stachel has taken the central idea of the point-coincidence argument as a reason for urging that we replace the point manifold with fiber bundles as the appropriate and natural framework within which to do general relativity (see Stachel 1986a, 1986b).<sup>12</sup>

Following up Einstein’s idea of defining spacetime events as points of intersection of world lines leads us to another feature of relativistic spacetime that turns out to have peculiar and surprising relevance to our worries about individuation and thus to provide yet another point of purchase for a “Pauli program” for fundamental physics. The central insight is, again, Einstein’s. In his 1920 lecture, *Äther und Relativitäts-Theorie* (Einstein 1920), Einstein came up with what at

the time he believed was a relativistic argument against any residual notion of substance in physics. He argued that the electromagnetic field (and the gravitational field, for that matter) could not be viewed as being carried by a sea of material particles, because such particles, having trackable identities, would therefore possess trajectories that would, in effect, define a privileged frame of reference, somewhat like an ether frame. Thus, covariance considerations ruled out the field's being carried by material field particles.

As best I can tell, Einstein let this idea drop, never returning to it in later publications or correspondence. That he did so is unfortunate, because had he taken the idea up again after the discovery in 1924 of the fundamental significance of the statistics of indistinguishable particles, he might have realized that the argument could be run in a different direction. For one can turn Einstein's original insight around in the following way. One can argue that if there are to be material particles acting as carrier of electromagnetic (or any other species of) field energy, then covariance considerations alone require them to be indistinguishable particles as the only way to guarantee that their presence in spacetime does not pick out a privileged frame of reference. Their being indistinguishable means that there is no one uniquely correct way of connecting up different "photon- events," if you will, into photon trajectories.

Curiously, this idea was taken up by Hans Reichenbach in his 1928 *Philosophie der Raum-Zeit-Lehre*. Reichenbach also failed to see that Einstein’s insight could be turned into a covariance argument for indistinguishability, or at least that is not how he presented the argument. His concern was to show merely that the existence of “material fields” carried by indistinguishable particles created problems for the concept of genidentity as the appropriate notion of identity for systems characterized by world lines in spacetime. This is because the indistinguishability of the field particles induces an underdetermination in the way we lay world lines into spacetime, an underdetermination that Reichenbach dubbed the “arbitrariness of striations” (“Willkürlichkeit der



**Figure 4** Arbitrariness of Striations  
(After Reichenbach 1928, p. 271)

Zerspaltung”) (Reichenbach 1928, pp. 270-271). Reichenbach illustrated this arbitrariness with a diagram that makes vivid the way indistinguishability blocks any univocal of photon-events into photon world lines.

An argument for the indistinguishability of photons (or the particle-like carriers of any other species of field energy) from covariance considerations alone is remarkable. But for all that it

promises new insights into the relation between spacetime structure and the individuation of physical systems, it also raises many new questions. Foremost among these would have to be the following: If there is no unique way of laying photon trajectories into spacetime, on pain of

privileging an “ether frame,” then what happens to the notion of conformal or light cone structure? It may have been worries such as this that deterred Einstein from pursuing the argument himself. Instead of being deterred, however, we might ask whether or not an underdetermination of the causal structure of spacetime, as required for the purpose of accommodating indistinguishable particles in spacetime in a covariant way, might not itself offer yet deeper glimpses into the connection between spacetime structure and individuation. Thus one idea might be to try to interpret the multiplicity of different possible “striations,” as Reichenbach called them, as corresponding to a set of virtual trajectories.

Rather than follow this daunting line of thinking any further at the moment, let us return to a consideration of the consequences of taking the worldline and lightcone structure, as opposed to the point structure of the manifold, as ontologically basic in fundamental physics. Were we to do this, how might we begin to build the rest of our physics into this framework? Let’s confine our attention for the moment to the quantum mechanics of charged and chargeless, massive and massless particles and their interactions via electromagnetic forces. There is no reason why we could not associate quantum mechanical states with systems thus individuated in standard quantum mechanical fashion. What might be more difficult will be building gravitation back into this framework, the question being how to get something like a metrical notion of curvature in this scheme. One might conjecture that the right approach would be to let something like spatiotemporal probability densities, defined now on the worldlines, do the work formerly done by the stress-energy tensor, since, after all, the former are a kind of measure of the mass-energy content of regions of spacetime. But if we let something like the metrical structure be evolved out of the structure of the probability densities, then, in a sense, we would have a quantum theory of spacetime, rather than



having a quantum theory that naively presupposes a spacetime background that, as we have seen, is not intrinsically congenial to having a quantum theory implanted within it, owing to the problem of the inherently different quantum mechanical and spatiotemporal modes of individuating physical systems.

Mention of the spatiotemporal probability densities brings to mind a possible worry, which is that worldlines as understood in general relativity represent definite trajectories, which would seem to be incompatible with Heisenberg indeterminacy. But if one were to take the worldlines themselves as ontologically basic, without regarding them as one-dimensional sets of spacetime points, then when would not be begging any questions about spatiotemporal definiteness. From the point of view of the worldlines themselves, there is no reason why they might not be taken to represent something that would look like a “fuzzy” path from the point of view of the spacetime structure we eventually evolve from quantum mechanical probability densities. In other words, to say that a worldline represents the career of a system is not, in and of itself, to say that that career corresponds to a definite trajectory in the classical sense.

I am not prepared to say anything at the moment about how the strong and weak nuclear forces would be integrated into this picture, except to observe that the bosonic carriers of these other forces should be expected to play in this newly conceived quantum spacetime a role not unlike that of the photon, their mediation of interactions inducing connections between otherwise separate systems.

Finally, is there any sense in which the scheme being developed here represents a unification of gravity and electromagnetism? Perhaps so if one can get curvature from probability densities. That there *should* be a unification emerging here follows from the fact that we are taking as our

ontological starting point the pattern of individuation that seems necessary to explain the peculiar statistics that emerge in the interaction between electromagnetic radiation and ponderable, that is, gravitational matter. This scheme of individuation was not necessary for either electromagnetism or gravitation considered alone. But it is the salient characteristic of what I called above “mixed interactions.”

### 5. *Conclusion*

Twenty years ago, when talking with me about the puzzles of the quantum, Abner Shimony once ventured the observation that the ultimate solution to those puzzles might well require a radical revision in our understanding of spacetime structure. At the time, I had no idea what he might mean by that conjecture. Perhaps he had no definite idea in mind himself; it may have been just an idle *Bieridee*. But the remark has haunted me ever since. I would like to think that our understanding at least the fundamental difference in the way quantum mechanics and spacetime theories naturally individuate physical systems and states might give us a clue as to where, finally, we might look for new view of spacetime that could solve the quantum puzzles. I doubt that my musings here will lead directly to such a new view, their being, in the end, little more than *Bierideen* themselves. I hope, however, to have persuaded you that there is a deep and important problem to be explored in the cluster of issues surrounding spacetime and individuation.

*Acknowledgments.* In other publications I have recorded many debts to many individuals whose conversation, criticism, friendship, and sometimes just logistical support have left their mark in my work. Many of them deserve mention here. But on this occasion I want to record just one, namely,

to Abner Shimony himself, who has helped my work along in all the mentioned ways but who has also been for me an example of what it is to be a philosopher of science, especially one for whom the quest for understanding is the polestar of one's professional life. In particular, I want to thank Abner for helpful comments on the present essay, while noting that such mistakes as there may be are still all mine.

I wish also to thank the Hebrew University of Jerusalem, which holds the copyright, for permission to quote from Einstein's unpublished writings and correspondence. Quotations from unpublished material in the Einstein Archive are cited by their control number in the Archive, after the fashion: EA xx-xxx.

#### ENDNOTES

1. See Pauli 1985, pp. 402-406. Heisenberg did write the suggested reply, enclosing a copy with his letter to Pauli of 2 July 1935; see Pauli 1985, pp. 409-418. Another copy is in the Einstein Archive, EA 5-207.
2. For further details on the history that is summarized in this section, see Howard 1984 and 1990.
3. For a thorough account of Einstein's "Bestimmt" manuscript, including an interesting account of the historical background to the mathematical methods that it employs, but with a somewhat different reading of its place in the development of Einstein's thinking about the quantum from the one suggested here, see Belousek 1996.
4. That Einstein was trying to make a point about separability and not indeterminacy emerges from a letter from Paul Ehrenfest to Bohr of 9 July 1931, written immediately after Ehrenfest had visited with Bohr in Berlin. For more on the letter and the interpretation of the photon box thought experiment, see Howard 1990, pp. 98-101.
5. For more detail on Einstein's thinking about separability and locality from 1935 to 1955, see Howard 1984.
6. The following discussion recapitulates an argument that I first published, in extenso, in Howard 1992; see also Howard 1989. I borrow the especially lucid notation from James Cushing's introductory essay (Cushing 1989) to the volume in which the latter is contained.

7. Jarrett's original formulation of the "locality" and "completeness" conditions was more complicated than that given here, taking into account the complete state of the analyzers. Only in response to criticism from Shimony (1984, section 2) did Jarrett adopt the present formulation.

8. Interpreting states as physical propensities and thus according propensities a fundamental ontological status is not uncongenial to Shimony's regarding "potentialities" as ontologically fundamental (see, for example, Shimony 1986, sections 2 and 5). But, in my opinion, something more than a simple distinction between "potentiality" and "actuality" is needed to make sense of the curiously complicated way in which, in quantum mechanics, what may or may not be accounted a definite property of a system can be argued to depend upon the experimental context in which the system finds itself; for more on the issue of context dependence, see Howard 1994.

9. In other settings it proves helpful to think of a context as a set of comensurable observables for the joint system. For a discussion of the role played by such a notion of context in Bohr's thinking about the classical/quantum distinction and the concept of a "phenomenon" proper to the quantum domain, see Howard 1994.

10. For a rather different reading of the bearing of this analysis on the choice between orthodox, Copenhagen quantum mechanics and Bohm-type hidden variable theories, see Cushing 1994, which questions the usefulness of the distinction between locality and separability in the context of Bohmian hidden variable theories.

11. As Newton explains in his unpublished manuscript, *De Gravitatione et æquipondio fluidorum* (Hall and Hall 1962, pp. 89-156), God can, by an act of will momentarily render and region of space a bounded atom, continuous motion coming about by his successively so rendering immediately adjacent regions thus so bounded. For more detail on this aspect of Newton's theory of space, see Howard 1993 and the further references found there.

12. See Stachel 1980 for more details on the history of the hole and point-coincidence arguments in Einstein's work on general relativity.

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