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EINSTEIN ON LOCALITY AND SEPARABILITY

. . . all things which are different must be distinguished in some way, and in the case of real things position alone is not a sufficient means of distinction. This overthrows the whole of purely corpuscularian philosophy.

Leibniz, *'On the Principle of Indiscernibles'* (1696)

Introduction

EINSTEIN is famous for his enduring doubts about the quantum theory's claim to be a complete theory of nature. The 1935 Einstein, Podolsky and Rosen (EPR) paper, 'Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?', is a landmark in the history of twentieth-century physics.[1] The puzzle first posed there about a *Gedankenexperiment* involving previously interacting systems, the puzzle ever since referred to as the 'EPR paradox', has been the focus of intense and sustained philosophical debate about the relation between theory and reality.[2] The *Gedankenexperiment* itself has recently been turned into an actual experiment, the results of which appear to refute the possibility of local hidden-variable theories, which is an ironic turn of events, since local hidden-variable theories have been regarded by many as an attractive

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[1] *Physical Review* 47 (1935), 777 – 80.

[2] Bohr is, of course, the most famous respondent. See both his immediate reply, 'Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?' *Physical Review* 48 (1935), 696 – 702, and his later summary of his years-long controversy with Einstein, 'Discussion with Einstein on Epistemological Problems in Atomic Physics' in *Albert Einstein: Philosopher – Scientist*, P. A. Schilpp (ed.) (Evanston, Illinois: The Library of Living Philosophers, 1949), pp. 199 – 241. A helpful, thorough discussion of the Bohr – Einstein controversy can be found in C. A. Hooker's 'The Nature of Quantum Mechanical Reality: Einstein Versus Bohr', in *Paradigms and Paradoxes: The Philosophical Challenge of the Quantum Domain*, R. G. Colodny (ed.) (Pittsburgh: University of Pittsburgh Press, 1972), pp. 67 – 302.

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way to repair the defect in quantum mechanics which so troubled Einstein.[3]

However, Einstein's doubts about quantum mechanics were never enough to shake the faith of the defenders of quantum mechanical orthodoxy. Indeed, too many physicists and philosophers have been inclined to dismiss Einstein's misgivings as being a result of the naive but forgivable failures of understanding of an old man still clinging to an outmoded, deterministic metaphysics. I do not share Einstein's doubts about quantum mechanics, but I think it a mistake so easily to dismiss his arguments. In fact, I believe that his views on incompleteness have never been understood properly, precisely because we have not taken them seriously enough to study them with the care they deserve. And I believe that the loss to our understanding of physics is considerable. In what follows, I will argue that Einstein's real reasons for believing quantum mechanics incomplete were quite different from those attributed to him on the basis of casual readings of the EPR paper. And I will argue that his reflections on the quantum mechanical challenge to classical ways of thinking are far more insightful than we have taken them to be. Should we be surprised to learn that Einstein was a profound, not a superficial critic of the quantum theory?

More specifically, I will defend four claims:

(1) Einstein did not write the Einstein, Podolsky and Rosen pa and he was unhappy with the way it turned out, arguing that the 'main point' was 'buried by the erudition'.[4]

(2) Shortly after the appearance of the EPR paper, Einstein set forth his own incompleteness argument, which over the course of the next fifteen years he presented at least three times in print and at least twice in his correspondence. The paradoxical behavior of previously interacting systems is still the key, but Einstein's argument differs from the EPR argument in both its logical form and the assumptions upon which it rests. In brief, what Einstein argues is that the incompleteness of quantum mechanics follows from the conjunction of two

[3]For a survey of all of the experimental results through 1978, as well as an analysis of Bell's theorem, which was the starting point for this line of investigation, see J. F. Clauser and A. Shimony, 'Bell's Theorem: Experimental Tests and Implications', *Reports on Progress in Physics* **41** (1978), 1881 – 1927. Bell first presented his theorem in 'On the Einstein, Podolsky and Rosen Paradox', *Physics* **1** (1964), 195 – 200. The most recent experimental results, which provide the strongest evidence to date against local hidden-variable theories, are reported in A. Aspect, P. Grangier and G. Roger, 'Experimental Realization of Einstein – Podolsky – Rosen – Bohm *Gedankenexperiment*: A New Violation of Bell's Inequalities', *Physical Review Letters* **49** (1982), 91 – 94, and A. Aspect, J. Dalibard and G. Roger, 'Experimental Test of Bell's Inequalities Using Time-Varying Analyzers', *Physical Review Letters* **49** (1982), 1804 – 7. In what follows, I refer to the various experimental tests of local hidden-variable theories, collectively, as the 'Bell experiments'.

[4]Einstein, letter to Schrödinger, 19 June 1935. The contents of this letter are discussed in some detail below; see sections 1 and 2. Arthur Fine first drew attention to the importance of this letter in his 'Einstein's Critique of Quantum Theory: The Roots and Significance of EPR', in *After Einstein: Proceedings of the Einstein Centennial Celebration at Memphis State University, 14 – 16 March 1979*, P. Barker and C. G. Shugart (eds.) (Memphis, Tennessee: Memphis State University Press, 1981), pp. 147 – 58. However, I disagree in important ways with Fine's analysis of the incompleteness argument found in the letter. In particular, Fine denies that one can find in the letter a distinction of the kind I want to draw between separability and locality. See notes, 23 and 41.

assumptions. The first, which I call the 'separability principle', asserts that any two spatially separated systems possess their own separate real states. The second, the 'locality principle' asserts that all physical effects are propagated with finite, subluminal velocities, so that no effects can be communicated between systems separated by a space-like interval.

(3) Far better than the EPR argument, Einstein's incompleteness argument points up the issues at stake in the choice between quantum and classical theories of nature. In particular, the difference is seen to rest on the quantum theory's denial of the separability principle, which Einstein regards as the essential foundation of all classical, realistic theories. Einstein's incompleteness argument also thus clarifies the link between his doubts about quantum mechanics and his commitment to realism by making clear the *physical* assumptions upon which his realism is grounded.

(4) Einstein's own approach to incompleteness suggests a new way of understanding Bell's theorem and the physical implications of the experimental tests of local hidden-variable theories. For it can be argued that the Bell inequality is itself a consequence of the separability and locality principles, and thus it can be claimed that the Bell experiments demonstrate not the existence of a peculiar kind of 'quantum non-locality', but instead the existence of quantum non-separability. And Einstein's analysis implies that therein lie the seeds of a conflict not with special-relativistic locality constraints, but with assumptions fundamental to general relativity and any other field theory.

Given their importance in what follows, the separability and locality principles should be clearly distinguished. To repeat: separability says that spatially separated systems possess separate real states; locality adds that the state of a system can be changed only by local effects, effects propagated with finite, subluminal velocities. There is no necessary connection between the two principles, though they are frequently stated as if they were one. Some theories conform to both principles, general relativity being an example of such a separable, local theory. Other theories conform to just one or the other. Quantum mechanics is, on my interpretation, a non-separable, local theory. Examples of the opposite sort, namely, of separable, non-local theories, are to be found among the non-local hidden-variable theories. Most importantly, it should be understood that the separability of two systems is not the same thing as the absence of an interaction between them, nor is the presence of an interaction the mark of their non-separability. The separability principle operates on a more basic level as, in effect, a principle of individuation for physical systems, a principle whereby we determine whether in a given situation we have only one system or two. If two systems are not separable, then there can be no interaction between them, because they are not really *two* systems at all.

1. Einstein's Attitude Toward the EPR Argument

The argument developed in the Einstein, Podolsky and Rosen paper is what I call a *direct* argument for incompleteness. It proceeds by first establishing a 'condition of completeness' and then seeking to show that in at least one special case quantum mechanics fails to satisfy this necessary condition. The completeness condition is straightforward: 'every element of the physical reality must have a counterpart in the physical theory'.[5] But for the completeness condition to be applied, at least a sufficient condition for the existence of elements of physical reality is required. Thus, the EPR paper presents the famous criterion of physical reality:

> If, without in any way disturbing a system, we can predict with certainty (i.e. with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.[6]

The remainder of the EPR argument consists of a detailed examination of a special case wherein the value of either of two conjugate parameters of one system can be predicted with certainty on the basis of a measurement of the corresponding parameter of a second system which had previously interacted with the first, the interaction having established correlations between the parameters of the two systems. If the two systems are far enough apart, the measurement on the second system entails no physical disturbance of the first system, and so, the conditions in the reality criterion being satisfied, it is claimed that elements of physical reality exist corresponding to each of the two conjugate parameters of the first system. But quantum mechanics prohibits the simultaneous definiteness of conjugate parameters. Thus, the EPR argument concludes, quantum mechanics is incomplete.

Many commentators have been troubled by gaps in the EPR argument. The most significant lacuna is encountered at the place where the *simultaneous* reality of two conjugate parameters of the first system is inferred from the possibility of measuring *either* of the corresponding parameters of the second system. In general, the relevant parameters of the second system are themselves conjugate and so cannot be measured simultaneously. Additional assumptions are therefore required to license the inference from the possibility of measuring either parameter of the second system, to the actual simultaneous reality of both conjugate parameters of the first.

One way to repair the defect is just to assume that all physical systems at all times possess definite, observer-independent properties which are revealed to us by observation, including the kind of indirect observations which lie at

[5] *Op. cit.*, note 1, p. 777.
[6] *Ibid.*

the heart of the EPR *Gedankenexperiment*.[7] But this common maneuver is objectionable on two counts. First, it reduces the EPR argument to a *petitio principii*. The aim of the EPR argument is to *prove*, via the reality criterion, the existence of elements of physical reality not represented in quantum mechanics, and thereby to prove quantum mechanics incomplete. However, the suggestion under consideration is that we simply *assume* the existence of these (and many other) elements of physical reality. Second, and more importantly, this proposal naively and tacitly presupposes that we know what counts as a physical system, and that we know what is meant by talk of the 'independence' of a system's properties. Thus the proposal obscures what I will argue is the most important issue at stake in the debate over the completeness of quantum mechanics. For the genius of Einstein's own argument lies precisely in its demonstration that the incompleteness of quantum mechanics can be inferred only if one posits an explicit principle of individuation for physical systems, thereby grounding the independence of systems in a prior assumption of the existence of distinct real states of affairs. This principle of individuation turns out to be the aforementioned principle of separability.

What was Einstein's opinion of the EPR argument? The surprising answer is found in his correspondence with Erwin Schrödinger from the summer of 1935. The EPR paper appeared in the 15 May 1935 issue of *Physical Review*. Shortly thereafter, on 7 June, Schrödinger wrote Einstein a long letter which begins thus:

> I was very pleased that in the work which just appeared in *Phys. Rev.* you openly seized dogmatic quantum mechanics by the scruff of the neck, something we had already discussed so much in Berlin.[8]

Einstein replied on 19 June, and the beginning of his letter is interesting:

> I was very pleased with your detailed letter, which speaks about the little essay. For reasons of language, this was written by Podolsky after many discussions. But still it has not come out as well as I really wanted; on the contrary, the main point was, so to speak, buried by the erudition.[9]

[7]Essentially the same prosthesis is recommended in Clauser and Shimony, 'Bell's Theorem', p. 1885; their survey opens with a discussion of EPR.

[8]Schrödinger to Einstein, 7 June 1935: 'ich hab mich sehr gefreut, dass Du in der eben erschienenen Arbeit in *Phys. Rev.* die dogmatische Quantenmechanik auch öffentlich bei dem Schlafittchen erwischt hast, über das wir in Berlin schon so viel discutiert hatten.' Schrödinger and Einstein had been colleagues in Berlin from 1927 through 1932, Schrödinger as Planck's successor in the Chair of Theoretical Physics at the University of Berlin, Einstein as the Director of the Kaiser Wilhelm Institute.

[9]Einstein to Schrödinger, 19 June 1935: 'Ich habe mich sehr gefreut mit Deinem ausführlichen Briefe, der über die kleine Abhandlung handelt. Diese ist aus Sprachgründen von Podolsky geschrieben nach vielen Diskussionen. Es ist aber doch nicht so gut herausgekommen, was ich eigentlich wollte; sondern die Hauptsache ist sozusagen durch Gelehrsamkeit verschüttet.'

Einstein proceeds immediately to sketch his own incompleteness argument, which I will examine shortly. But first we should savor the import of the just quoted remark. Einstein says that he did not write the EPR paper, and that he was unhappy with the way it turned out! It is thus a mistake to read the EPR argument as if it accurately represented Einstein's views. It is even more of a mistake to read between the lines of the EPR argument for evidence of Einstein's deeper philosophical assumptions. There are gaps in the EPR argument, and they can be filled by additional assumptions, as we have just seen. But the gaps are not Einstein's gaps, and so neither are the assumptions with which we fill them.

In particular, we should not attribute to Einstein the kind of uncritical or unreflective realism which was considered two paragraphs back as a remedy for the defects in the EPR argument. Einstein did believe that all physical systems at all times possess definite, observer-independent properties which are revealed to us by observation. But he did not just *assume* this. Instead, Einstein *grounded* his realism about physical systems and their properties in the deeper assumption of separability, which is important, because the latter assumption is susceptible to revealing kinds of physical and philosophical scrutiny that cannot touch an unanalyzed postulate of physical realism.

2. Einstein's Own Incompleteness Argument

What then was Einstein's own argument for the incompleteness of quantum mechanics? It was presented first in the just-mentioned letter to Schrödinger of 19 June 1935. It was set before the public for the first time in 1936 in Einstein's essay, 'Physik und Realität',[10] and it was given a more careful statement when, in 1946 or 1947, Einstein drafted the 'Autobiographical Notes' section for the volume, *Albert Einstein: Philosopher–Scientist.*[11] The most detailed and thoughtful discussion of the argument is found in a paper Einstein wrote in 1948 for the journal, *Dialectica*, a paper entitled 'Quanten-Mechanik und Wirklichkeit'.[12] There is one final presentation of it in a letter Einstein wrote to Max Born on 18 March 1948.[13] Essentially the same argument was given on

[10] *Journal of the Franklin Institute* 221 (1936), 313–47. An English translation of this essay, under the title 'Physics and Reality', was published immediately following the German original in the same issue of the *Journal of the Franklin Institute*, pp. 349–82. A different English translation was later published in Einstein's *Ideas and Opinions* (New York: Bonanza Books, 1954), pp. 290–323. Unfortunately, both translations are flawed, often in serious ways. I have therefore retranslated all of the passages quoted here.

[11] 'Autobiographisches', in *Albert Einstein: Philosopher–Scientist*, pp. 2–94. The volume as a whole was published only in 1949; however, Einstein begins his 'Autobiographical Notes' by writing, 'Here I sit in order to write, at the age of 67, something like my own obituary'; this dates his composition of the 'Notes' between 14 March 1946 and 14 March 1947.

[12] *Dialectica* 2 (1948), 320–24.

[13] M. Born (ed.), *Albert Einstein — Hedwig und Max Born: Briefwechsel, 1916–1955* (Munich: Nymphenburger, 1969), pp. 223–24.

each occasion, but there is a development in the direction of greater clarity about the two basic assumptions which I claim underlie the argument. Each of these statements of the argument will be considered below, beginning with that in the 19 June 1935 letter to Schrödinger.

Most of Schrödinger's letter to Einstein of 7 June 1935 is a critical discussion of the mathematics involved in the quantum theoretical treatment of interacting systems, Schrödinger being concerned that the special case treated in the EPR paper is so special that the incompleteness argument fails of complete generality. But near the end of Schrödinger's letter one finds the following remark:

> Until now, the 'story' which I just made up about the whole matter was this. We possess no quantum mechanics which takes relativity theory into account, that is, which takes into account the finite velocity with which all effects are propagated. In the whole scheme of things, we possess only the analogue of the old, absolute mechanics. At best, that can apply to infinitely small systems, in which the light intervals which come into consideration are to be neglected. If we want to *separate* two systems, then indeed long before their interaction disappears it will cease to be approximable through absolute Coulomb laws and the like. And that's the end of the 'story'. The process of separation is not at all to be conceived according to the orthodox schema.[14]

The orthodox schema mentioned by Schrödinger is simply standard, non-relativistic quantum mechanics. The problem to which he is pointing is that this orthodox schema permits us to represent interactions only by the device of an interaction potential, such as a Coulomb potential. And any such representation tacitly assumes that ~~all~~ effects are propagated instantaneously, whereas relativity theory teaches us that all effects are propagated with a finite velocity. Moreover, as the EPR *Gedankenexperiment* makes clear, at least in theory, the correlations established by an interaction persist long after the interaction potential becomes negligible. The ultimate issue of Schrödinger's worry was his development of the now standard quantum mechanical interaction formalism.[15] But in the

[14]Schrödinger to Einstein, 7 June 1935: 'Der Vers, den ich nur auf die ganze Sache bisher machte, war dieser. Wir besitzen keine Quantenmechanik, welche der Relativitätstheorie d.h. unter anderem, welche der endlichen Ausbreitungsgeschwindigkeit aller Wirkung Rechnung trägt. Wir besitzen in dem ganzen Schema nur das Analogon der alten absoluten Mechanik. Das kann bestenfalls für unendlichkleine Systeme gelten, in denen die in Betracht kommenden Lichtzeiten zu vernachlässigen sind. Wenn wir zwei Systeme *trennen* wollen, so wird ihre Wechselwirkung schon lange bevor sie verschwindet, aufhören durch absolute Coulomb -u. dgl. Gesetze approximierbar zu sein. Und da endet unser Latein. Der Trennungsvorgang ist gar nicht nach dem orthodoxen Schema zu erfassen.' Schrödinger goes on in this letter to register his doubts about Dirac's relativistic quantum mechanics. One year later, Einstein echoes Schrödinger's worries about the 'orthodox schema' when he writes: 'In the Schrödinger equation absolute time, or potential energy, plays a decisive role though these concepts have been recognized through the relativity theory to be inadmissable in principle. If one wants to escape this problem, then he must take fields and field laws as basic, instead of interaction forces.' See Einstein's 'Physik und Realität' (note 10, above), pp. 342–43. (My translation — D.H.)

[15]E. Schrödinger, 'Discussion of Probability Relations Between Separated Systems', *Proceedings of the Cambridge Philosophical Society* **31** (1935), 555–62; and 'Probability Relations Between Separated Systems', *Proceedings of the Cambridge Philosophical Society* **32** (1936), 446–52.

meantime, his concerns about our failure to understand the process of separation elicited from Einstein a very interesting reaction.

Einstein opens his 19 June letter to Schrödinger with the above-quoted disclaimer to the effect that Podolsky wrote the EPR paper and that he, Einstein, was unhappy with how it turned out. He then proceeds immediately to set out his own thoughts on incompleteness, starting with a simple, non-quantum illustration of 'incompleteness'. He asks Schrödinger to consider two boxes and a single ball which is always found in one or the other of the boxes. We make an 'observation' on a box simply by lifting the lid and looking inside. Einstein then asks whether a certain state description — 'The probability that the ball is in the first box is ½' — is a complete description. He says that an adherent of the 'Born' interpretation would answer, 'No', on the grounds that a complete description would have to take the form of a categorical assertion to the effect that the ball *is* in the first box (or not). However, a follower of the 'Schrödinger' interpretation, says Einstein, would answer, 'Yes', arguing that before an observation is made the ball is not really in either box, and that this 'being in a definite box' only comes about through an observation. According to the 'Schrödinger' interpretation, the state of the first box before an observation is performed is completely described by the probability ½. There is one additional alternative, according to Einstein. He says:

> The talmudic philosopher doesn't give a hoot for 'reality', which he regards as a hobgoblin of the naive, and he declares that the two points of view differ only as to their mode of expression.[16]

The 'talmudist', we learn from a later letter, is Bohr[17]

What is Einstein's own opinion? He explains:

> *My* way of thinking is now this: properly considered, one cannot get at the talmudist if one does not make use of a supplementary principle: the 'separation principle'. That is to say: 'the second box, along with everything having to do with its contents, is independent of what happens with regard to the first box (separated partial systems).' If one adheres to the separation principle, then one thereby excludes the second

[16]Einstein to Schrödinger, 19 June 1935: 'Der talmudistische Philosoph aber pfeift auf die "Wirklichkeit" als auf einen Popanz der Naivität und erklärt beide Auffassungen als nur der Ausdruckweise nach verschieden.'

[17]That the 'talmudist' is supposed to be Bohr is made clear in a letter from Einstein to Schrödinger of 9 August 1939, where the question of incompleteness is again under discussion. After distinguishing the 'Born' and 'Schrödinger' interpretations of the ψ-function, Einstein says: 'Es gibt auch noch den Mystiker, der ein Fragen nach etwas unabhängiges vom Beobachten Existierenden, . . . überhaupt als unwissenschaftlich verbietet (Bohr). Dann fliessen beide Auffassungen in einen weichen Nebel zusammen, in dem ich mich aber auch nicht besser fühle als in einer der vorgenannten Auffassungen, die zum Realitätsbegriff Stellung nehmen.'

('Schrödinger') point of view, and only the Born point of view remains, according to which the above state description is an *incomplete* description of reality, or of the real states.[18]

The crucial point here is Einstein's claim that one cannot refute the 'talmudist' without invoking the 'separation principle', according to which the contents of the two boxes are independent of one another. This is the principle upon which Einstein's incompleteness argument turns.

For the sake of clarity, I should note right away that what Einstein here calls the 'separation principle' is not exactly the same as what I call the principle of separability. The separability principle is implicit in Einstein's similarly named 'separation principle'; but Einstein has yet to distinguish as clearly as he will between separability, which is the claim that spatially separated systems always possess separate real states, and locality, which is the claim that the separate real states of such separate systems can be changed only by physical effects which are propagated with finite, subluminal velocities. However, what is already clear, indeed, what the 'ball-in-the-box' example is obviously intended to suggest, is that spatially separated systems are characterized by separate real states of affairs. This is the heart of the separability principle.

Let us return to Einstein's argument. After explaining through the 'ball-in-the-box' example what he means in general by 'incompleteness', Einstein turns his attention to the special case of the quantum theory. He first explains what 'completeness' would mean in this context:

> In the quantum theory, one describes a real state of a system through a normalized function, ψ, of the coordinates (of the configuration-space). . . . Now one would like to say the following: ψ is correlated one-to-one with the real state of the real system. . . . If this works, then I speak of a complete description of reality by the theory. But if such an interpretation is not feasible, I call the theoretical description 'incomplete'.[19]

Next, Einstein briefly sketches the EPR *Gedankenexperiment*, stressing as its only important feature the fact that by choosing to measure different observables

[18]Einstein to Schrödinger, 19 June 1935: '*Meine* Denkweise ist nun so: An sich kann man dem Talmudiker nicht beikommen, wenn man kein zusätzliches Prinzip zu Hilfe nimmt: "Trennungsprinzip". Nämlich: "die zweite Schachtel nebst allem, was ihren Inhalt betrifft, ist unabhängig davon, was bezüglich der ersten Schachtel passiert (getrennte Teilsysteme)". Hält man an dem Trennungsprinzip fest, so schliesst man dadurch die zweite ("Schrödinger'sche") Auffassung aus und es bleibt nur die Born'sche, nach welcher aber die obige Beschreibung des Zustandes eine *unvollständige* Beschreibung der Wirklichkeit, bezw. der wirklichen Zustände ist.'

[19]Einstein to Schrödinger, 19 June 1935: 'Man beschreibt in der Quantentheorie einen wirklichen Zustand eines Systems durch eine normierte Funktion ψ der Koordinaten (des Konfigurationsraumes). . . . Man möchte nun gerne folgendes sagen: ψ ist dem wirklichen Zustand des wirklichen Systems ein-eindeutig zugeordnet. . . . Wenn dies geht rede ich von einer vollständigen Beschreibung der Wirklichkeit durch die Theorie. Wenn aber eine solche Interpretation nicht durchführbar ist, nenne ich die theoretische Beschreibung "unvollständig".'

of one system, A, we can attribute different ψ-functions, ψ_B or $\psi_{\underline{B}}$, to the other system, B. (He says that it does not even matter whether ψ_B and $\psi_{\underline{B}}$ are eigenfunctions of observables, as long as they are different.) It is then just a short step to the conclusion that quantum mechanics is incomplete:

> Now what is essential is exclusively that ψ_B and $\psi_{\underline{B}}$ are in general different from one another. I assert that this difference is incompatible with the hypothesis that the ψ description is correlated one-to-one with the physical reality (the real state). After the collision, the real state of (AB) consists precisely of the real state of A and the real state of B, which two states have nothing to do with one another. *The real state of B thus cannot depend upon the kind of measurement I carry out on A.* ('Separation hypothesis' from above.) But then for the same state of B there are two (in general arbitrarily many) equally justified ψ_B, which contradicts the hypothesis of a one-to-one or complete description of the real states.[20]

This argument is quite straightforward and strikingly different from the original EPR argument.

There is, first, a difference in logical structure. The EPR argument seeks to prove quantum mechanics incomplete by proving the existence of elements of physical reality having no counterpart in the theory; it is thus an attempt at a *direct* proof of incompleteness. By contrast, Einstein's own argument does not require the identification of specific elements of physical reality not mirrored in the theory. Einstein's argument seeks, instead, to exhibit a contradiction between the completeness assumption and the consequences of the 'separation principle', which makes it an *indirect* proof of incompleteness. The contradiction is easily derived. The 'separation principle' implies that system B has its own real state which is unaffected by anything we do to the distant system, A. Thus, the different ψ-functions which we assign to B on the basis of different measurements on A must be correlated with one and the same real state of B. But if the ψ-function were to provide a complete description of the real state of B, it would have to be correlated one-to-one with B's real state, and there we have our contradiction with the 'separation principle'.

Behind the difference in logical structure is a significant difference in the necessary conditions for completeness assumed in the two arguments. The EPR argument stipulates that 'every element of the physical reality must have a counterpart in the physical theory'. Einstein demands something sharper and

[20]Einstein to Schrödinger, 19 June 1935: 'Wesentlich ist nun ausschliesslich, dass ψ_B and $\psi_{\underline{B}}$ überhaupt voneinander verschieden sind. Ich behaupte, dass diese Verschiedenheit mit der Hypothese, dass die ψ-Beschreibung ein-eindeutig der physikalischen Wirklichkeit (dem wirklichen Zustande) zugeordnet sei, unvereinbar ist. Nach dem Zusammenstoss besteht der wirkliche Zustand von (AB) nämlich aus dem wirklichen Zustand von A und dem wirklichen Zustand von B, welche beiden Zustände nichts miteinander zu schaffen haben. *Der wirkliche Zustand von B kann nun nicht davon abhängen, was für eine Messung ich an A vornehme.* ("Trennungshypothese" von oben.) Dann aber gibt es zu demselben Zustande von B zwei (überhaupt bel. viele) gleichberechtigte ψ_B, was der Hypothese einer ein-eindeutigen bezw. vollständigen Beschreibung der wirklichen Zustände widerspricht.'

stronger, which is that in a complete theory there must be a one-to-one correlation between the ψ-function and the real state. In effect, the EPR completeness condition amounts to one half of Einstein's completeness condition, for the EPR version already entails that different real states be described by different ψ-functions, since otherwise the 'elements of reality', with respect to which the real states differ, would lack theoretical counterparts — there would be a real difference without a corresponding theoretical distinction. What Einstein adds is the converse requirement, that different ψ-functions must be correlated with different real states, that there must be no theoretical distinction in the absence of a real difference.[21]

Another remarkable difference between Einstein's argument and the EPR argument is that Einstein nowhere makes reference to incompatible observables or to the Heisenberg indeterminacy relations, whereas the EPR argument relies essentially upon quantum mechanical prohibitions on the simultaneous definiteness of incompatible observables. Indeed, as noted just above, Einstein even says that it makes no difference whether the ψ-functions assigned to system B can be regarded as eigenfunctions of observables at all. What is important is simply that these ψ-functions be different.[22] This fact should be savored and its import appreciated, for it suggests that in Einstein's estimation the truly puzzling feature of quantum mechanics, its most objectionable departure from the classical world view, is not its way of treating conjugate parameters, but its way of treating spatially separated, previously interacting systems.

In the end, therefore, the most important thing to notice about Einstein's

[21]This addition is needed for Einstein's incompleteness argument to work: quantum mechanics is held to be 'incomplete' because in the situation exhibited in the *Gedankenexperiment* it assigns different ψ-functions to what counts, on the basis of the 'separation principle', as one and the same real state. Yet one might object that Einstein's 'completeness' condition is, nevertheless, *too* strong. 'Completeness' surely requires of the ψ-function that it describe *all* of the properties of a system in a given real state, so that different real states must be described by different ψ-functions (which is all that the weaker EPR completeness condition demands). But why require, conversely, that different ψ-functions *always* be correlated with different real states? Might there not be situations in which the differences between two ψ-functions (phase differences, for example) are inessential from the point of view of the system whose real state they aim to describe? Einstein's completeness condition would, indeed, be too strong if it required that literally *every* difference between ψ-functions mirror a difference in the real state of the system in question; but such was not Einstein's intention. The kind of difference with which Einstein was concerned is clear from his argument: ψ_B and $\psi_{\underline{B}}$ differ in the predictions they yield for the results of certain objective, local measurements on system B. This is not an inessential difference, and Einstein was right in holding that it is excluded by the demand for completeness, for the only way to account for such a difference (assuming that both ψ-functions provide 'correct' descriptions) is by assuming that at least one of the two ψ-functions, or perhaps each of them, gives an incomplete description. (For example, if ψ_B attributed a definite position to B, but not a definite momentum, it would be incomplete in its description of B's momentum; but, of course, Einstein's argument does not require any such reference to specific parameters or 'elements of reality'.) Two different ψ-functions can give correct descriptions of one and the same real state only if one of them, at least, tells less than the whole truth about that real state.

[22]Einstein to Schrödinger, 19 June 1935: 'Bemerkung: Ob die ψ_B und $\psi_{\underline{B}}$ als Eigenfunktionen von Observabeln B, $\underline{B}$ aufgefasst werden können ist mir *wurst*.' The emphasis is Einstein's, which suggests that he himself saw this as an important contrast with the EPR argument.

incompleteness argument is that for the crucial contradiction to be derived one must assume that spatially separated systems possess their own separate real states. This is what Einstein asserts when he says, 'After the collision, the real state of (AB) consists precisely of the real state of A and the real state of B, which two states have nothing to do with one another.' Of course, one must also assume locality, in order to exclude the possibility of instantaneous changes in B's real state as a result of manipulations performed on A. But separability is an even more basic assumption, for if the two systems partake, somehow, of the same joint reality, then an interference with one of them is automatically an interference with its Siamese twin. And thus the assignment of different ψ-functions to system B as a result of different measurements on A would not contradict the claim that the ψ-function is a complete description in the sense of being correlated one-to-one with the real state of B, for different real states of B could have been engendered by the measurements carried out on its companion.[23] However, such changes in B's real state are ruled out if we assume that B and A possess *separate* real states.[24]

[23]It is here that my reconstruction of Einstein's incompleteness argument most differs from the reconstruction offered by Fine in his 'Einstein's Critique of Quantum Theory' (note 4, above) and in his more recent essay, 'What is Einstein's Statistical Interpretation, or Is It Einstein for Whom Bell's Theorem Tolls?', *Topoi* (fall 1984). Fine characterizes the 'separation principle' as referring to separate real *properties* of the interacting systems, whereas I take it to refer to the underlying separate real *states* of the systems. The difference might seem inconsequential, especially if one regards the real state as simply the set of a system's real properties (Fine reports that this is his understanding of the relation between states and properties — personal communication). But I believe that Fine's approach contributes to a blurring of the distinction between separability and locality, a distinction which Fine does not draw in his analysis of Einstein's argument. (Of course, Einstein himself had not yet clearly distinguished separability and locality in his 1935 correspondence with Schrödinger, but I claim that the distinction is implicit even there, and Einstein does make it explicit later on.) The problem is that we too easily fall into thinking of properties, more so than states, as inherently local, perhaps because the *manifestations* of properties are paradigmatically local — when we measure, say, the spin of an electron, the measurement is performed at a specific place, so we tend to think that the property thereby revealed also resides, at least for the moment, at that place. If we think of properties in this way, then the 'independence' of these properties appears to consist solely in their immunity to influence from events separated from them by a space-like interval. 'Independence' thus seems to be a matter of locality alone. What we overlook is the possibility that locally manifested properties might belong to a physical system that is itself not decomposable into separate, local chunks, a system whose 'parts' do not possess separate real states. Thus the role of the separability assumption can be obscured if we focus our attention on the independence of properties, rather than on the more basic independence of systems and their states.

[24]It might be argued that there is yet another hidden premise even in Einstein's own incompleteness argument, namely, the premise that counterfactual conditionals have truth values; for it seems that Einstein is at least implicitly asserting that ψ_B, for example, is the ψ-function which we 'would' assign to B if we 'were' to measure the appropriate observable on A (and likewise for $\psi_{\underline{B}}$). I have no settled opinion on whether it is reasonable to impute such an assumption to Einstein, nor about whether his realism should, in consequence, be taken to include a commitment to the existence of real universals of the dispositional variety. Quite apart from the historical question of Einstein's actual *beliefs*, does his *argument* require this additional assumption? I think not. All one need assume is that the experiment can be repeated twice, with the same kinds of particles (same states), and with a different observable being measured on system A on each occasion. The real state of system B is the same on both occasions, but different ψ-functions are assigned to it, so the ψ-function is not correlated one-to-one with the real state. One must now assume that it makes sense to talk of the 'same kind' of particle being present on different occasions, and this talk of 'kinds' entails

It is in his recognizing the need for an explicit separability assumption that Einstein's originality and insight are made manifest. On my reading, the need for this assumption is the 'main point' which Einstein said was 'buried by the erudition' in the EPR argument, where it is simply asserted that:

> . . . since at the time of measurement the two systems no longer interact, no real change can take place in the second system in consequence of anything that may be done to the first system. This is, of course, merely a statement of what is meant by the absence of an interaction between the two systems.[25]

One could claim that the separability principle is implicit even here in the EPR argument, for it is assumed that there are two systems, each with its own real state, instead of just one. But by failing to make an explicit assumption of separability, the EPR argument leaves the door open to the very kind of reply which Bohr in fact made:

> Of course there is in a case like that just considered no question of a mechanical disturbance of the system under investigation during the last critical stage of the measuring procedure. But even at this stage there is essentially the question of *an influence on the very conditions which define the possible types of predictions regarding the future behavior of the system*. Since these conditions constitute an inherent element of the description of any phenomenon to which the term 'physical reality' can be properly attached, we see that the argumentation of the mentioned authors does not justify their conclusion that quantum-mechanical description is essentially incomplete.[26]

Bohr concedes that there is no physical disturbance of the second system, but he claims that there is, nevertheless, an influence on certain conditions (*i.e.*, the total experimental arrangement) which are *part of the reality* we aim to describe. In effect, Bohr denies that the system with which we are concerned (system B in Einstein's example) has its own *separate* reality. Einstein's strategy of making separability an explicit assumption forces the issue with Bohr in a way which the EPR argument does not, for it compels anyone who seeks Bohr's way out explicitly to deny separability, which is something Einstein thought no one would be willing to do.

a commitment to real universals of a set-theoretic variety, but these universals are a far more modest addition to one's ontology than the non-extensional universals lurking behind counterfactual conditionals.

[25] *Op. cit.*, note 1, p. 779.

[26] Bohr, 'Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?', p. 700. Curiously, there is no comment on Bohr's paper in the Einstein – Schrödinger correspondence from the summer of 1935. However, in a letter of 13 July 1935 Schrödinger complains about the obscurity of Bohr's brief earlier reply, published that same day, 13 July, as a letter to the editor of *Nature* (Bohr, 'Quantum Mechanics and Physical Reality', *Nature, Lond.* 136 (1935), 65). And in a letter of 23 March 1936 Schrödinger reports on a conversation he had recently had in London with Bohr in which Bohr said he regarded it as 'high treason' for men like von Laue, Schrödinger and Einstein to try to ensnare quantum mechanics with the EPR paradox.

That Einstein himself understood the role of the 'separation principle' in just this way — namely, as forcing the issue with Bohr — is clear from his reply to Henry Margenau's contribution to the collection, *Albert Einstein: Philosopher – Scientist*. Margenau sought to save quantum mechanics from the EPR critique by rejecting the projection postulate, which he termed an 'inadvertancy'.[27] But Einstein believed that Margenau had missed the point; he explains:

> Of the 'orthodox' quantum theoreticians whose position I know, Niels Bohr's seems to me to come nearest to doing justice to the problem. Translated into my own way of putting it, he argues as follows:
>
> If the partial systems A and B form a total system which is described by its ψ-function $\psi(AB)$, there is no reason why any mutually independent existence (state of reality) should be ascribed to the partial systems A and B viewed separately, *not even if the partial systems are spatially separated from each other at the particular time under consideration*. The assertion that, in this latter case, the real situation of B could not be (directly) influenced by any measurement taken on A is, therefore, within the framework of quantum theory, unfounded and (as the paradox shows) unacceptable.
>
> By this way of looking at the matter it becomes evident that the paradox forces us to relinquish one of the following two assertions:
>
> (1) the description by means of the ψ-function is *complete*.
>
> (2) the real states of spatially separated objects are independent of each other.
>
> On the other hand, it is possible to adhere to (2), if one regards the ψ-function as the description of a (statistical) ensemble of systems (and therefore relinquishes (1)). However, this view blasts the framework of the 'orthodox quantum theory'.[28]

By Einstein's own account, the central issue in his dispute with Bohr concerns the separability principle. Bohr defends the completeness thesis by repudiating the separability principle; Einstein defends separability by denying completeness. As Einstein himself says, you cannot 'get at the talmudist' unless you assume the 'separation principle'.

Einstein first published his own incompleteness argument in March of 1936, in the *Journal of the Franklin Institute*, where it forms a small part of his long and interesting essay, 'Physik und Realität'. I quote the argument in full:

[27] H. Margenau, 'Einstein's Conception of Reality', in *Albert Einstein: Philosopher – Scientist*, p. 265.

[28] Einstein, 'Remarks Concerning the Essays Brought Together in this Co-operative Volume', in *Albert Einstein: Philosopher – Scientist*, pp. 681 – 82. Ironically, many students of Einstein take this passage to be a reaffirmation of the EPR point of view. Typical is the comment of Max Jammer: 'In his "Reply to Criticisms,". . . Einstein explicitly reaffirmed, notwithstanding the objections advanced by Bohr and others, the view expressed in the 1935 paper.' See Jammer's *The Philosophy of Quantum Mechanics: The Interpretations of Quantum Mechanics in Historical Perspective* (New York: John Wiley, 1974), p. 187.

> Consider a mechanical system consisting of two partial systems A and B which interact with each other only during a limited time. Let the ψ function before their interaction be given. Then the Schrödinger equation will furnish the ψ function after the interaction has taken place. Let us now determine the physical state of the partial system A through a measurement which is as complete as possible. Then quantum mechanics allows us to determine the ψ function of the partial system B from the measurements made, and from the ψ function of the total system. This determination, however, gives a result which depends upon *which* of the state variables of A have been measured (for instance, coordinates or momenta). Since there can be only *one* physical state of B after the interaction, which state cannot reasonably be considered to depend upon the kinds of measurements I carry out on the system A separated from B, it is thus shown that the ψ function is *not* unambiguously correlated with the physical state. This correlation of several ψ functions to the same physical state of system B shows again that the ψ function cannot be interpreted as a (complete) description of a physical state (of an individual system). Here also the correlation of the ψ function to an ensemble of systems eliminates every difficulty.[29]

The brevity of this argument alone should have suggested to careful readers how it differs from the EPR argument. It should also have been noticed that, as in the letter to Schrödinger, Einstein here makes no reference to incompatible observables or the Heisenberg indeterminacy principle. Without doubt, the differences would have been more noticeable had Einstein emphasized in this essay the role of his 'separation principle', whose consequences are asserted, but not highlighted, when he says, '. . . there can be only *one* physical state of B after the interaction, which state cannot reasonably be considered to depend upon the kinds of measurements I carry out on the system A separated from B.' But even though the 'separation principle' is not explicitly distinguished, Einstein has here presented for the second time an argument fundamentally different from that in the EPR paper.[30]

It was ten or eleven years later, in 1946 or 1947, when Einstein next drafted a written statement of his views on incompleteness for the public eye. The occasion was his preparation of an intellectual autobiography for the Library of Living Philosophers volume, *Albert Einstein: Philosopher – Scientist*. The discussion of incompleteness which is found here represents an important stage in the development of Einstein's thinking, because it contains the first clear statement of the distinction between separability and locality.

[29]*Op. cit.*, note 10, p. 341. (My translation — D.H.)

[30]Einstein is himself partly responsible for confusion about the differences between his incompleteness argument and the EPR argument. The paragraph just quoted from 'Physik und Realität' is preceded by the following remark: 'Such an interpretation [the statistical ensemble interpretation] also overcomes a paradox recently presented by myself, together with two co-workers, a paradox which refers to the following situation:'. *Op. cit.*, note 10, p. 341. (My translation — D.H.) On a quick reading, this comment suggests that Einstein's argument and the EPR argument are the same. But Einstein has chosen his words here with care. All that he really says is that his argument makes use of the *Gedankenexperiment* developed in the EPR paper. One can easily understand that Einstein would not want to hurt Podolsky by airing in public the misgivings about the EPR paper which he was willing to share in his private correspondence with Schrödinger.

Einstein begins his argument again by distinguishing two positions which physicists might take on the completeness question. Physicist A defends what in 1935 Einstein called the 'Born' interpretation, according to which the ψ-function provides an incomplete description of the always-definite properties of an individual system. Physicist B defends the view Einstein earlier labelled the 'Schrödinger' interpretation, according to which the ψ-function gives a complete description of the objectively indefinite (pre-measurement) properties of an individual system. Einstein next sketches the *Gedankenexperiment* involving two previously interacting systems, this time labelled S_1 and S_2. Then he presents the heart of the incompleteness argument:

> Now it appears to me that one may speak of the real state of the partial system S_2. To begin with, before performing the measurement on S_1, we know even less of this real state than we know of a system described by the ψ-function. But on *one* assumption we should, in my opinion, unconditionally hold fast: The real situation (state) of system S_2 is independent of what is done with system S_1, which is spatially separated from the former. According to the type of measurement I perform on S_1, I get, however, a very different ψ_2 for the second partial system. (ψ_2, ψ_2^1, . . .) But now the real state of S_2 must be independent of what happens to S_1. Thus, different ψ-functions can be found (depending on the choice of the measurement on S_1) for the same real state of S_2. (One can only avoid this conclusion either by assuming that the measurement on S_1 changes (telepathically) the real state of S_2, or by generally denying independent real states to things which are spatially separated from one another. Both alternatives appear to me entirely unacceptable.)
>
> If now the physicists A and B accept this reasoning as sound, then B will have to give up his position that the ψ-function is a complete description of a real situation. For in this case it would be impossible that two different types of ψ-functions could be correlated with the same situation (of S_2).[31]

This is, for the most part, a straightforward restatement of the argument we have seen twice before. But one piece of it warrants careful scrutiny.

Look at the parenthetical statement at the end of the first paragraph, where Einstein specifies the only two ways to avoid the conclusion that leads to the claim of incompleteness:

> (One can only avoid this conclusion either by assuming that the measurement on S_1 changes (telepathically) the real state of S_2, or by generally denying independent real states to things which are spatially separated from one another. Both alternatives appear to me entirely unacceptable.)

In effect, these are the only two ways one can deny the 'separation principle', and they correspond, respectively, to the denials of what I call locality and separability: either the separate real state of S_2 *is* changed by what happens in

[31] *Op. cit.*, note 11, p. 84 and p. 86. (My translation — D.H.)

a region separated from S_2 by a space-like interval, or spatially separated systems do not possess separate states.

It is almost as if Einstein stumbled, in this negative way, upon the *clear* distinction between separability and locality. But the insight thus won was not to be lost, and when in 1948 Einstein published his most thorough discussion of the incompleteness argument, the distinction between separability and locality is presented at the outset in direct, lucid terms. The context was a special double issue of the Swiss journal, *Dialectica*, devoted to essays on Bohr's 'complementarity' interpretation of quantum mechanics. Einstein's paper, 'Quanten-Mechanik und Wirklichkeit', is of major historical significance, but it is far less well known than many of his other writings, no doubt because it has never been made widely available in an English translation and is not included in any of the standard collections of his works.[32]

As had become his custom, Einstein begins his essay with the distinction between the two positions one might take on the relation between the ψ-function and some real state of affairs of an individual system. The point of view now labelled 'Ia' holds that the system possesses determinate values of all of its observables, even if these values cannot be determined simultaneously by measurement, from which it follows that the ψ-function gives an incomplete description of the real situation. The other point of view, 'Ib', holds that determinate values of observables do not really exist until they are brought into being by measurements. On this point of view, the ψ-function provides, in principle, a complete description.

The promised distinction between separability and locality is then developed in the course of two pregnant paragraphs, which I quote in their entirety (they form the whole of section II in Einstein's essay):

> If one asks what is characteristic of the realm of physical ideas independently of the quantum-theory, then above all the following attracts our attention: the concepts of physics refer to a real external world, *i.e.*, ideas are posited of things that claim a 'real existence' independent of the perceiving subject (bodies, fields, *etc.*), and these ideas are, on the other hand, brought into as secure a relationship as possible with sense impressions. Moreover, it is characteristic of these physical things that they are conceived of as being arranged in a space-time continuum. Further, it appears to be essential for this arrangement of the things introduced in physics that, at a specific time, these things claim an existence independent of one another, insofar as these things 'lie in different parts of space'. Without such an assumption of the mutually independent existence (the 'being-thus') of spatially distant things, an assumption which originates in everyday thought, physical thought in the sense familiar to us would not be possible. Nor does one see how physical laws could be formulated and

[32] *Op. cit.*, note 12. An English version of this essay is contained in the English edition of the Born – Einstein correspondence: M. Born (ed.), *The Born – Einstein Letters*, I. Born (trans.) (London: Macmillan, 1971), pp. 168 – 73. But this volume is now out of print, and in any case the translation is badly flawed.

> tested without such a clean separation. Field theory has carried out this principle to the extreme, in that it localizes within infinitely small (four-dimensional) space-elements the elementary things existing independently of one another that it takes as basic, as well as the elementary laws it postulates for them.
>
> For the relative independence of spatially distant things (A and B), this idea is characteristic: an external influence on A has no *immediate* effect on B; this is known as the 'principle of local action', which is applied consistently only in field theory. The complete suspension of this basic principle would make impossible the idea of the existence of (quasi-) closed systems and, thereby, the establishment of empirically testable laws in the sense familiar to us.[33]

As I read these paragraphs, I find Einstein making two basic claims. First, if we are to maintain a realistic attitude toward the 'things' we posit in physics, it is *'essential'* that we assume 'the mutually independent existence . . . of spatially distant things'. Indeed, Einstein says, we could neither test nor even formulate physical laws 'without such a clean separation'. Second, it is *'characteristic'* of the independence of any two such spatially separated things (but *not* essential) that an external influence on one has no immediate effect on the other. But now these two claims are simply Einstein's formulations of what I call separability and locality.

The remainder of 'Quanten-Mechanik und Wirklichkeit' consists of what we now recognize as Einstein's standard argument for incompleteness. He outlines the EPR *Gedankenexperiment*, showing that different ψ-functions are assigned to S_2 depending upon the kind of measurement we perform on S_1. He observes that this circumstance is compatible with the completeness assumption (interpretation Ib) taken alone. But then he adds:

> Matters are different, however, if one seeks to hold on to principle II — the autonomous existence of the real states of affairs present in two separated parts of space, R_1 and R_2 — simultaneously with the principles of quantum mechanics. In our example the complete measurement on S_1 of course implies a physical interference which only affects the portion of space R_1. But such an interference cannot immediately influence the physically real in a distant portion of space R_2. From that it would follow that every statement regarding S_2 which we are able to make on the basis of a complete measurement on S_1 must also hold for the system S_2 if, after all, no measurement whatsoever ensued on S_1. That would mean that for S_2 all statements that can be derived from the postulation of ψ_2 or ψ_2', etc. must hold simultaneously. This is naturally impossible, if ψ_2, ψ_2', are supposed to signify mutually distinct real states of affairs of S_2, i.e. one comes into conflict with interpretation 'Ib' of the ψ-function.[34]

Once again, the argument is that the incompleteness of quantum mechanics follows only if one assumes both locality and separability; there are distinct

[33] *Op. cit.*, note 12, pp. 321–22. (My translation — D.H.)
[34] *Ibid.*, p. 323. (My translation — D.H.)

real states of affairs in the parts of space R_1 and R_2, and an event in R_1 can exert no immediate influence on the state of affairs in R_2 (the term, 'principle II', alludes to section II of the paper).

'Quanten-Mechanik und Wirklichkeit' is Einstein's last statement of this incompleteness argument. Over the years he several times discussed a still different incompleteness argument, one similar in character to the Schrödinger cat paradox, in that it focuses on an alleged incompleteness in the quantum mechanical description of macroscopic observables.[35] But this latter argument is not as profound as the one based on the separability and locality principles, and so will not be analyzed here.

I have examined Einstein's incompleteness argument at length and in detail partly because it is important to set aright in the historical record. But there are other dividends as well. For one thing, we are now better prepared to understand the way assumptions of physics work in grounding Einstein's realism. For another thing, we are also better prepared to understand the physical implications of the experiments recently conducted to test local hidden-variable theories, experiments which probe the kinds of correlations between previously interacting systems that were at the heart of the EPR *Gedankenexperiment*. I want briefly to consider both issues.

3. The Physical Roots of Einstein's Realism

That Einstein was a realist, nearly everyone will agree. The question is: 'What kind of realist?' What Einstein's incompleteness argument shows is that his realism was no mere philosophical prejudice. On the contrary, it is securely anchored in quite definite assumptions not only about the nature of theories, but also about the physical world itself. And foremost among these physical assumptions is what I call separability or what Einstein calls the 'mutually

[35]An especially clear statement of one version of this 'macroscopic' incompleteness argument is contained in Einstein's late essay, 'Elementare Überlegungen zur Interpretation der Grundlagen der Quanten-Mechanik', in *Scientific Papers Presented to Max Born* (New York: Hafner, 1953), pp. 33 – 40. A somewhat different version is developed much earlier, in a letter from Einstein to Schrödinger of 8 August 1935, where Einstein considers a 'quantum mechanical' description of a chemically unstable pile of gunpowder. However precise the initial description of the state of the gunpowder might be, the quantum state evolves after a sufficient length of time into a superposition of exploded and unexploded states. But the gunpowder itself is by that time either definitely exploded or definitely unexploded. Thus, Einstein concludes, the quantum mechanical description is incomplete. (The correspondence gives evidence that Einstein's gunpowder argument was the inspiration for Schrödinger's more refined 'cat paradox'; on 19 August 1935, Schrödinger wrote to Einstein: 'In einem längeren Aufsatz, den ich eben geschrieben, bringe ich ein Beispiel, dass Deinem explodierenden Pulverfass sehr ähnlich ist.' The 'cat paradox' was first presented in Schrödinger's 'Die gegenwärtige Situation in der Quantenmechanik', *Die Naturwissenschaften* 23 (1935), 807 – 12, 824 – 28, 844 – 49.) Though not as profound as the argument based on the 'separation principle', the 'macroscopic' incompleteness argument was evidently regarded by Einstein as the more persuasive of the two because of our strong intuitions about the definiteness of macroscopic properties.

independent existence of spatially distant things'.

Look again at the first of the paragraphs quoted from his 1948 essay, 'Quanten-Mechanik und Wirklichkeit'. His commitment to realism is unambiguous:

> . . . the concepts of physics refer to a real external world, *i.e.* ideas are posited of things that claim a 'real existence' independent of the perceiving subject (bodies, fields, *etc.*) . . .

Equally unambiguous, however, is his commitment to separability as a necessary physical condition for this realism:

> Further, it appears to be essential for this arrangement of the things introduced in physics that, at a specific time, these things claim an existence independent of one another, insofar as these things 'lie in different parts of space'.

What is less clear is why Einstein believed separability to be a necessary condition for realism. There is one important clue in the quoted passage, where Einstein asserts that separability is a necessary condition for testability:

> Without such an assumption of the mutually independent existence . . . of spatially distant things, an assumption which originates in everyday thought, physical thought in the sense familiar to us would not be possible. Nor does one see how physical laws could be formulated and tested without such a clean separation.

There is another clue in the next paragraph, where Einstein says that locality is an additional necessary condition for testability:

> The complete suspension of this basic principle would make impossible the idea of the existence of (quasi-)closed systems and, thereby, the establishment of empirically testable laws in the sense familiar to us.

So the argument is that *both* separability and locality are necessary conditions for testability, the latter in particular because it grounds the existence of closed systems. But still one asks, 'Why?'

Little light is shed on this question by any of Einstein's published writings; one remark not intended for publication, however, might point us toward an answer. This remark is found in Max Born's commentary on his correspondence with Einstein. In his Waynflete Lectures, *Natural Philosophy of Cause and Chance*, Born tried to illustrate Einstein's attitude toward quantum mechanics with quotations from two of Einstein's letters, and as a courtesy he solicited Einstein's reaction to the manuscript, only to find that Einstein felt himself misunderstood.[36] In March of 1948, around the time when he was at work on the essay for *Dialectica*, Einstein returned Born's manuscript with a number of marginal comments and one long summary comment about his commitment to realism:

> I just want to explain what I mean when I say that we should try to hold on to physical reality. We are, to be sure, all of us aware of the situation regarding what will turn out to be the basic foundational concepts in physics: the point-mass or the particle is surely not among them; the field, in the Faraday – Maxwell sense, might be, but not with certainty. But that which we conceive as existing ('actual') should somehow be localized in time and space. That is, the real in one part of space, A, should (in theory) somehow 'exist' independently of that which is thought of as real in another part of space, B. If a physical system stretches over the parts of space A *and* B, then what is present in B should somehow have an existence independent of what is present in A. What is actually present in B should thus not depend upon the type of measurement carried out in the part of space, A; it should also be independent of whether or not, after all, a measurement is made in A.
>
> If one adheres to this program, then one can hardly view the quantum-theoretical description as a *complete* representation of the physically real. If one attempts, nevertheless, so to view it, then one must assume that the physically real in B undergoes a sudden change because of a measurement in A. My physical instincts bristle at that suggestion.
>
> However, if one renounces the assumption that what is present in different parts of space has an independent, real existence, then I do not at all see what physics is supposed to describe. For what is thought to be a 'system' is, after all, just conventional, and I do not see how one is supposed to divide up the world objectively so that one can make statements about the parts.[37]

The first of these three paragraphs just restates the ideas of separability and locality. The second points out that the assumption of the completeness of quantum mechanics can only be reconciled with the separability principle if one denies locality. The third is the most interesting, for what Einstein suggests here is that the separability principle is necessary because it provides the only imaginable objective principle for the individuation of physical systems. Needless to say, this is an important and provocative claim, because quantum mechanics, interpreted as a theory about individual systems, denies the separability principle.[38] Moreover, I will argue in the next section that the Bell experiments

[36]See Born's *Natural Philosophy of Cause and Chance* (Oxford: Oxford University Press, 1949), pp. 122 – 23.

[37]*Op. cit.*, note 13. (My translation — D.H.)

[38]In the quantum mechanical interaction formalism, which was developed by Schrödinger in response both to EPR and his correspondence with Einstein (see the references in note 15, above), the joint state of two previously interacting systems is represented as a non-factorizable superposition of products of separate states. For example, the joint state of a correlated pair of spin-½ particles of the kind studied in the modern versions of the EPR *Gedankenexperiment* could be written thus: $\psi(AB) = 2^{-\frac{1}{2}}(\psi_z^+(A)\psi_z^-(B) - \psi_z^-(A)\psi_z^+(B))$, where $\psi_z^+(A)$ and $\psi_z^-(A)$ represent the $+\frac{1}{2}$ and $-\frac{1}{2}$ z-spin eigenstates of particle A (and similarly for particle B). What is important for our purposes

can be held to refute separability. Does that mean we are left with no objective basis for physics?

Einstein does not say why he believes separability to be the only objective principle of individuation for physical systems. He just says that he can see no alternative. Without forcing an interpretation on him, can we guess what his reasons might have been? Separability works as an objective principle of individuation by providing at least a sufficient condition for distinguishing any two physical systems, namely, spatial separation. Thus, implicitly, it counts even every infinitesimal region of space-time as a separate physical system. Were we not to individuate systems in this fashion, the only alternative would be to provide an objective criterion whereby some, but of course not all, otherwise distinct regions of space-time would be, as it were, knit together, blended or united to form larger, non-decomposable, but still individual physical systems. Such, presumably, is the kind of alternative Einstein could not imagine. From his point of view, it must have appeared as though any such criterion would be arbitrary, on the assumption that there are no intrinsic connections between different regions of space-time.

But again, quantum mechanics denies the separability principle, assigning non-decomposable, joint states even to widely separated previously interacting systems, at least until one of the systems subsequently interacts with another system. So was Einstein wrong about the impossibility of alternative criteria of individuation? Or is the quantum theory's assignment of non-decomposable joint states to previously interacting systems based upon a non-objective criterion of 'connection' between systems? One might indeed follow Einstein and argue that quantum mechanics establishes these links in a non-objective way. After all, from one point of view every system is in continuous interaction with every other system if only through gravitational interactions. So it would seem arbitrary to pick out only certain interactions as establishing the special quantum links between systems. But I think quantum mechanics does individuate systems objectively, which is to say that there *is* an alternative which Einstein did not envisage. What that alternative is I will explain in the next section.

Earlier I claimed that physical realism is not simply *assumed* by Einstein, but is *grounded* in the deeper assumption of separability. And now I have argued that Einstein regarded separability as at least a *necessary* condition for realism. One further link between realism and separability needs still to be exhibited in order to show just how firm the grounding was in Einstein's view.

Like so many realists before him, Einstein speaks of the real world which physics aims to describe as the real *'external'* world, and he does so in such a way as to suggest that the *independence* of the real — its not being dependent

is that such a state description cannot be re-expressed as a simple product of separate state functions for A and B. That is, for all $\psi(A)$ and $\psi(B)$, $\psi(AB) \neq \psi(A)\psi(B)$. The non-separability implicit in this manner of description is broken only when one of the systems interacts with yet another system.

in any significant way upon ourselves as observers — is grounded in this 'externality'. For most other realists this talk of 'externality' is at best a suggestive metaphor. But for Einstein it is no metaphor. 'Externality' is a relation of spatial separation, and the separability principle, the principle of 'the mutually independent existence of spatially distant things', asserts that any two systems separated by so much as an infinitesimal spatial interval always possess separate states. Once we realize that observer and observed are themselves just previously interacting physical systems, we see that their independence is grounded by the separability principle, along with the independence of all other physical systems. Certain properties of the observed object can be changed by the observation interaction, but the locality principle guarantees that such changes cannot take place if the separation between observer and observed is space-like, as in the EPR *Gedankenexperiment*, and in any case the separability principle guarantees that the observed object always possesses its own cluster of properties, even if these might be changed by the observation. The independence of the real consists not in any immunity to change, but in autonomy even through observation induced changes.

It should now be clear why Einstein found it so hard to entertain the possibility of denying the separability principle, and thus why he insisted that quantum mechanics is not the final word about the microworld; for quantum mechanics, at least when interpreted as a theory about individual systems, denies the separability principle and, with it, the very foundation of Einstein's realism.[39]

4. Separability, Locality and the Physical Implications of the Bell Experiments

For my purposes, the most valuable dividend yielded by this investigation of Einstein's views on incompleteness is that it suggests a new way of understanding Bell's theorem and the Bell experiments, themselves the offspring of the original EPR *Gedankenexperiment*. Specifically, I claim that these experiments should be interpreted as refuting the separability principle. I want briefly to explain why and to indicate some of the larger physical and philosophical implications of such an interpretation.

Would it be unfair to say that the recent striking progress in the experimental tests of local hidden-variable theories by Aspect and his co-workers has not

[39]On an ensemble interpretation, the non-separability of the description of previously interacting systems would be preserved, but since the ψ-function would be interpreted as referring to an ensemble of pairs of systems, it would not follow that the real states of individual pairs of systems in the ensemble are similarly non-separable. The statistics could be 'entangled' in the peculiar quantum mechanical way without the individual systems being likewise 'entangled'. Schrödinger first introduced the concept of 'entanglement' ['Verschränkung'] in his 'cat paradox' paper (see note 35, above).

been matched by progress of a theoretical sort?[40] Consensus exists on only one point, which is that the experimentally determined correlations between previously interacting systems consistently violate the Bell inequality. Bell claimed that the predictions of any local hidden-variable theory must satisfy the inequality, which would imply that local hidden-variable theories have therefore been refuted. But even this has been denied.[41]

Still, even if it were agreed that local hidden-variable theories have been refuted, confusion would remain, for it is not clear what is implied by the repudiation of local hidden-variable theories. Should we opt for non-local hidden-variable theories? Most physicists decline this alternative since it requires the sacrifice of special relativity. Are we then to accept some peculiar kind of 'quantum non-locality'? If this alternative is more palatable, it is only because it is so poorly understood that we are not sure what sacrifices it might entail. Is quantum mechanics incompatible with special relativity? We think not, but then what is this 'non-locality' which experiment seems to be forcing upon us?

I am convinced that most of this confusion is to be traced back to the way

[40]See the references in note 3. By employing time-varying analyzers, Aspect's group has succeeded in ruling out the possibility of subluminal physical effects being transmitted between the two wings of the apparatus.

[41]Fine is the principal hold-out. He proposes what he calls 'prism' models for the Bell-type correlation experiments, the key innovation in these models being that one allows for the possibility that one or both of the two previously interacting systems are not detected by their respective analyzers. If the non-detection occurs in just the right cases, the inherently weak correlations implied by local hidden-variable theories can be masked in such a way as to make them appear stronger than they 'really' are. See Fine's 'Correlations and Physical Locality', in *PSA 1980*, Volume 2, P. D. Asquith and R. N. Giere (eds.) (East Lansing, Michigan: Philosophy of Science Association, 1981), pp. 535 – 62, and his 'Antinomies of Entanglement: The Puzzling Case of the Tangled Statistics', *Journal of Philosophy* 79 (1982), 733 – 47. Fine suggests that his prism models are more 'realistic' than other models, inasmuch as actual particle detection apparatus falls short of perfect efficiency. But the prism models require far more than just random non-detection in order for the correlations to come out in line with the correlations predicted by quantum mechanics; failures of detection must occur in quite specific cases, and it can be objected that Fine has provided no physically plausible mechanism whereby to guarantee just the right failures. An experiment has been designed which could be used to test the prism models, but it has not been carried out. See T. K. Lo and A. Shimony, 'Proposed Molecular Test of Local Hidden-Variables Theories', *Physical Review A* 23 (1981), 3003 – 12. For a critical discussion of Fine's prism models, see A. Shimony, 'Critique of the Papers of Fine and Suppes', in *PSA 1980*, Vol. 2, pp. 572 – 80. In another paper, 'What is Einstein's Statistical Interpretation?' (note 23, above), Fine argues that the concept of a prism model provides the best interpretation of Einstein's ideas on the interpretation of quantum mechanics. On at least a casual reading, Einstein seems to have endorsed explicitly an ensemble interpretation of the ψ-function, rather than anything like a prism model (see, for example, the last sentence in the passage quoted from his 'Physik und Realität', note 29, above). But Fine argues that Einstein's specific remarks about ensembles are not compatible with an ensemble interpretation, because, among other things, a traditional ensemble interpretation would imply a measurement induced transition to a subensemble only after a specific measurement *result* is obtained, whereas according to Fine, Einstein allows for such transitions as soon as the *kind* of measurement is determined, and before a specific *result* is obtained. In my opinion, this is just a misreading of Einstein. Consider, for example, this comment of Einstein's: 'Then quantum mechanics allows us to determine the ψ-function of the partial system $\underline{B}$ from the measurements made, and from the ψ function of the total system' ('Physik und Realität', p. 341). Clearly, by the phrase 'the measurements made', Einstein means 'the measurement results'. The prism interpretation saves Einstein from refutation by the results of the Bell experiments, but I do not think Einstein's words will bear such an interpretation.

in which the Bell inequality was originally derived. There is something peculiar about the derivation, something that is common to all of the more recent and otherwise more elegant derivations. Our newly won understanding of Einstein's incompleteness argument will help us to make sense of the matter.

The problem is that when one writes down the hypothetical *hidden* state of the joint system (the pair of previously interacting systems), one writes down a *single* state for the joint system, not a product of separate states. This is peculiar because a shocking feature of the quantum mechanical description of previously interacting systems, indeed, one of the quantum theory's greatest affronts to our classical intuitions, is precisely its requiring the employment of such a single, non-factorized joint state. One would think that if there were any interesting difference between quantum mechanics and hidden-variable theories, it would lie here. Certainly Einstein saw non-separability to be a surprising, non-classical feature of the quantum theory under its 'orthodox' (Bohrian) interpretation. So why not ask under what circumstances a hidden-variable theory could accommodate a *separable* description of previously interacting systems?

It is not that the possibility of a separable hidden-variable theory has been ignored in the investigations inspired by Bell's theorem. In fact, Bell mentions the possibility at the start of his original paper, but he dismisses the need for explicit consideration of separable hidden-variable theories:

> Some might prefer a formulation in which the hidden variables fall into two sets, with A dependent on one and B on the other; this possibility is contained in the above, since λ stands for any number of variables and the dependences thereon of A and B are unrestricted.[42]

Bell is correct; the possibility of a separable description is 'contained' in the use of just a single, non-factorized hidden state. The question is, how is it so 'contained'? Bell's approach makes it appear as though some kind of locality condition were the only prerequisite for the derivation of the inequality, and thus makes it appear as though the negative results of the subsequent experiments refute both separable and non-separable hidden-variable theories alike, as long as they are local. What Bell did not consider is the possibility that the separability of the hidden state is implicit in the derivation of the inequality, in the sense that any hidden-variable theory whose predictions conform to the Bell inequality would be a separable theory. Though we have not realized it, we might therefore all along have been testing not simply local hidden-variable theories, but separable, local hidden-variable theories.

I suspect that most of our trouble in understanding so-called 'quantum non-locality' is a result of this more basic confusion. We focus our attention on the apparent demonstration of non-local effects mysteriously communicated

[42] J. S. Bell, 'On the Einstein, Podolsky and Rosen Paradox' (see note 3), p. 196.

between two systems separated by a space-like interval, without pausing to ask the deeper question of whether there really are *two* systems, or just *one*. We can and should clear up this confusion.

In another paper I show that we can derive the Bell inequality from two independent assumptions — the separability principle and the locality principle. Separability is there formulated as an assumption about the existence of separate probability measures for the two systems, measures defined over pairs of measurement outcomes and measurement contexts, together with an assumption about the manner in which a joint measure is to be constructed out of separate measures. Locality is formulated as an assumption about the invariance of such a separate probability measure under conditionalization on the state of the distant apparatus. I also show that any hidden-variable theory whose predictions satisfy the Bell inequality is separable, at least with regard to those aspects of the hidden state which are at issue in the Bell experiments.[43]

We should not be surprised that the Bell inequality can be derived in this fashion. Reflect for a moment on my reconstruction of Einstein's incompleteness argument. I present Einstein as claiming that the incompleteness of quantum mechanics is a consequence of two assumptions — separability and locality. Now I claim that the Bell inequality is a consequence of the same two assumptions. But what would have been shown if the Bell experiments had turned out the opposite of the way they did? That is, what if the correlations between previously interacting systems had turned out to *satisfy* the Bell

[43] Howard, 'Non-Separability or Non-Locality? On the Physical Implications of the Bell Experiments' (in preparation). In my argument I draw heavily upon an important theorem recently proved by Jon Jarrett in his 'On the Physical Significance of the Locality Conditions in the Bell Arguments', *Nous* (forthcoming). Jarrett shows that the factorizability condition (of joint measurement probabilities, not states) from which the Bell inequality is standardly derived, a condition which Jarrett labels 'strong locality', can be decomposed into two independent conditions which he terms 'completeness' and 'weak locality'. Jarrett proves that the conjunction of 'completeness' and 'weak locality' implies 'strong locality', which, conversely, implies *both* 'completeness' and 'weak locality'. 'Weak locality' is shown to correspond to the relativistic locality constraint. In my paper I show how Jarrett's 'completeness' condition can be reformulated as a condition on the factorizability of the hidden state, which is to say that I replace his condition by my 'separability' condition. 'Strong locality' is then seen to be a consequence of 'separability' and 'weak locality', and to imply both of them in turn. Since, as Fine has shown, 'strong locality' is not only a sufficient but also a necessary condition for the satisfaction of the Bell inequality, it follows that all hidden-variable theories whose predictions conform to the Bell inequality are both separable and local. See Fine's 'Hidden Variables, Joint Probability, and the Bell Inequalities', *Physical Review Letters* **48** (1982), 291 – 95. For another point of view on the interpretation of Jarrett's two conditions, see A. Shimony, 'Controllable and Uncontrollable Non-Locality', *Proceedings of International Symposium on Foundations of Quantum Mechanics* (Tokyo: Physical Society of Japan, 1984), pp. 225 – 30. A distinction similar to that between separability and locality is drawn by P. Heywood and M. L. G. Redhead, in their 'Nonlocality and the Kochen – Specker Paradox', *Foundations of Physics* 13 (1983), 481 – 99; they use the terminology 'ontological locality' and 'environmental locality'. They also prove a theorem which is remarkably similar in structure and conclusion to Einstein's incompleteness argument. Where Einstein exhibits a contradiction between separability, locality, and the completeness condition, they demonstrate a contradiction between ontological locality, environmental locality and a pair of constraints on the assignment of values to physical magnitudes, constraints which they argue must be accepted on any realistic interpretation of quantum mechanics.

inequality, thus confirming the predictions of local hidden-variable theories? We would have concluded that quantum mechanics is incomplete, that it needs supplementation by additional variables more fully to describe reality. And so Einstein would have been vindicated on precisely the grounds he expected. He believed that physical reality is separable and that all effects are local effects; and he believed quantum mechanics incomplete because it does not accommodate all the reality there must be in a separable, local universe.

If my derivation of the Bell inequality is sound, then the interpretation of the results of the Bell experiments is simple. We must give up either separability or locality. And those two alternatives correspond, respectively, to accepting either non-separable quantum mechanics or non-local hidden-variable theories. But if these are our only alternatives, then most of us would likely prefer the former alternative, on the grounds that special relativistic locality constraints are too much a part of our physics to be sacrificed to the cause of saving separability, all the more so because we have ready at hand a highly successful non-separable quantum mechanics, but no well-developed non-local hidden-variable theory. In fact, I believe that Einstein himself would have followed us in this choice had he been forced to choose between these two alternatives, because the locality principle has the flavor of those high-level constraints, like the conservation of mass-energy and the second law of thermodynamics, which Einstein said should guide the development of our theories, whereas the separability principle has more of the flavor of those 'constructive' principles, like the atomic hypothesis, which Einstein thought were too often an impediment to scientific progress.[44]

But to say that we accept the non-separable quantum theory is not to say that we yet understand all that such acceptance entails. Einstein's conception of separability points to one major but still largely unappreciated consequence of quantum non-separability. Let me quote again just one sentence from Einstein's *Dialectica* paper (the 'principle' referred to here is the principle of 'the mutually independent existence of spatially distant things' — the separability principle):

> Field theory has carried out this principle to the extreme, in that it localizes within infinitely small (four-dimensional) space-elements the elementary things existing independently of one another that it takes as basic, as well as the elementary laws it postulates for them.[45]

The point is that field theories are, by their very nature, separable theories. Thus, the acceptance of the quantum theory's non-separability entails the repudiation

[44]See Einstein's 'My Theory', *The Times* (London), 28 November 1919, p. 13. This essay is reprinted, but with the wrong bibliographical information, as 'Time, Space and Gravitation' in Einstein's *Out of My Later Years* (New York: Philosophical Library, 1950), pp. 54–58.

[45]*Op. cit.*, note 12, p. 321.

of this most basic feature of field theories, including *general relativity*, inasmuch as it is standardly formulated as a field theory. To be more specific, as soon as we assume that the metric tensor is well defined *at every point in the space-time manifold*, we implicate ourselves in the now questionable assumption of separability.[46]

It is not the point of this paper to attempt a reconciliation of quantum mechanics and general relativity, but I would like to suggest a direction in which a reconciliation might be sought, and my doing so will bring us back to an issue left unresolved at the end of the previous section, namely, can quantum mechanics provide an objective criterion for the individuation of physical systems?

At a minimum, acceptance of quantum non-separability requires that we replace the concept of a metric tensor well-defined at every point of the space-time manifold with a different kind of structure which is well defined *only* for certain non-separable *regions* of space-time. I have no clear idea what form such a structure might take. Perhaps some structure from which we could derive an average curvature for the region would work. Whatever structure we settle upon, one other problem must be solved, however, and that is the prior problem of determining what should count as a non-separable region of space-time. Here I have a simple proposal. We should make the existence of quantum correlations a criterion of non-separability. After all, if it were not for the existence of these peculiar correlations which violate the Bell inequality, the separability principle would not be threatened. In other words, what I suggest is that instead of taking the quantum correlations as a puzzle needing explanation, we should make these correlations themselves the explanation, or at least the criterion for the existence of the kind of non-separability of space-time regions which I claim quantum mechanics entails.

This proposal should assuage the fears of those who share Einstein's worry that there is no objective criterion for the individuation of physical systems other than spatial separation. The experimentally determinable quantum correlations are as objective as spatial separation. For practical reasons they may not actually be determinable in all cases, but the quantum theory tells us where to look for them.

[46]Was Einstein right in arguing that field theories are by their very nature separable theories? It might be objected that he was wrong, since under certain circumstances the value of the metric at a point is not 'locally' determined, being, for example, a function of the boundary conditions on some neighborhood of the point. But this objection is based on a misunderstanding. The question is not how the value of the metric is determined, but whether it is presumed to be always well-defined at every point of the manifold. In this sense, general relativity, like any other field theory, is radically separable in the sense indicated. It would also not do to object that general relativity employs other structures in addition to the metric tensor, such as the stress–energy tensor. In a field theory, some structure must be fundamental, and as I understand Einstein, he took the metric tensor to be the fundamental field structure. Be that as it may, the question is not which structure is fundamental. The question is whether the fundamental structure, whatever it is taken to be, is assumed always to be well-defined at every point of the manifold.

There are two potential misunderstandings which I want definitely to avoid. First, I am not proposing the abandonment of the continuum in favor of a discrete manifold. It makes no difference how large the chunks of space-time are, whether finite or infinitesimal. What matters is whether we regard spatial separation, *by itself*, as a sufficient condition for the individuation of physical systems. A separable theory based on a discrete manifold is just as much threatened by the Bell experiments, on my interpretation, as a separable theory based on the continuum. Second, I am not proposing the abandonment of either the equivalence principle or the principle of general covariance. What I am suggesting is that we cannot seek to satisfy these principles with field equations governing structures well-defined at every point in the manifold. Some other structure must be sought. The set of heuristic principles guiding this search will still contain the locality principle, the equivalence principle and the principle of general covariance. But it must now be enlarged by the addition of one new heuristic — the principle of quantum non-separability.

Earlier I ventured the opinion that if he were forced to choose between separability and locality, Einstein too would have chosen to give up separability in order to save locality. Would he have been prepared to accept the consequences of such a choice for general relativity? Einstein's commitment to the concept of the field as a framework for a unified theory is well known, but there is evidence which shows that this was not his most basic commitment, and that, indeed, he could consider abandoning the field concept. What he would not give up under any circumstances is the principle of general relativity. This emerges in a letter Einstein wrote on 2 May 1948 to Pauli, who had edited the special issue of *Dialectica* in which Einstein's 'Quanten-Mechanik und Wirklichkeit' appeared. After reading the essay, Pauli sent Einstein his comments, to which Einstein replied in a letter that concludes with this paragraph:

> Indeed, I have often said to you that I am not bent on having differential equations, but that I am intent on keeping the general relativity principle, whose heuristic power cannot be dispensed with. In spite of much searching, I have not succeeded in doing justice to the general relativity principle otherwise than through differential equations; perhaps one will discover such a possibility if he searches stubbornly enough for it.[47]

Remarks such as these suggest considerable flexibility, in principle, about the way one might develop a physics which 'does justice' to the general relativity principle. Einstein says that he has found no way other than through differential field equations; but he does not rule out alternatives.

[47]Einstein to Pauli, 2 May 1948: 'Ich habe Ihnen ja schon öfter gesagt, dass ich nicht auf Differentialgleichungen versessen bin, wohl aber auf das allgemeine Relativitäts-prinzip, dessen heuristische Kraft nicht entbehrt werden kann. Es ist mir eben trotz vielen Suchens nicht gelungen, dem allgemeinen Relativitäts-prinzip anders als durch Differentialgleichungen gerecht zu werden; vielleicht entdeckt einer eine derartige Möglichkeit, wenn er hartnäckig genug danach sucht.'

In fact, Einstein was willing to consider alternatives even more radical than what I have suggested here, including the abandonment of the continuum. In a letter to David Bohm of 28 October 1954, Einstein writes:

> In the last years several attempts have been made to complete quantum theory as you have also attempted. But it seems to me that we are still quite remote from a satisfactory solution of the problem. I myself have tried to approach this goal by generalizing the law of gravitation. But I must confess that I was not able to find a way to explain the atomistic character of nature. My opinion is that if an objective description through the field as elementary concept is not possible, then one has to find a possibility to avoid the continuum (together with space and time) altogether. But I have not the slightest idea what kind of elementary concepts could be used in such a theory.[48]

And in a famous remark in an appendix to the last edition of *The Meaning of Relativity*, a remark which must have been penned at about the same time as Einstein's letter to Bohm, in late 1954, Einstein says:

> One can give good reasons why reality cannot at all be represented by a continuous field. From the quantum phenomena it appears to follow with certainty that a finite system of finite energy can be completely described by a finite set of numbers (quantum numbers). This does not seem to be in accordance with a continuum theory, and must lead to an attempt to find a purely algebraic theory for the description of reality. But nobody knows how to obtain the basis of such a theory.[49]

It is not entirely clear what Einstein meant by this allusion to a 'purely algebraic theory', but what is clear is that Einstein was willing to entertain alternatives to field theories. Would his openness to alternatives have extended all the way to non-separable theories? We will never know.

[48]Einstein to Bohm, 28 October 1954. Bohm's efforts to 'complete' quantum mechanics, to which Einstein here refers, took the form of his proposed hidden variables interpretation. See D. Bohm, 'A Suggested Interpretation of the Quantum Theory in Terms of "Hidden" Variables. I & II', *Physical Review* **85** (1952), 166–79, 180–93.

[49]Einstein, 'Relativistic Theory of the Non-Symmetric Field', Appendix II in *The Meaning of Relativity*, 5th edn. (Princeton: Princeton University Press, 1956), pp. 165–66. Einstein's 'Note on the Fifth Edition' is dated 'December 1954'. This passage, which has puzzled many students of Einstein because it seems unconnected with any of Einstein's earlier reflections on field theories and quantum mechanics, should be compared to the following passage from Einstein's 1936 essay, 'Physik und Realität': '. . . it has been pointed out that the introduction of a spatio-temporal continuum may already be regarded as contrary to nature in view of the molecular structure of everything which happens in the small. Perhaps the success of Heisenberg's method points to a purely algebraic method for the description of nature, and to the elimination of continuous functions from physics. But then, in principle, the employment of the space-time continuum must also be given up. It is not unthinkable that human ingenuity will some day find methods which make it possible to follow this path. But in the meantime, this project appears similar to the attempt to breathe in a space devoid of air.' *Op. cit.*, note 10, p. 343. (My translation — D.H.) What this passage suggests is that Einstein's later remark about a 'purely algebraic theory' is meant as a reference to a theory like Heisenberg's matrix mechanics.

5. Conclusion

Enough of speculation. What I have tried to demonstrate in this paper is that Einstein's own argument for the incompleteness of quantum mechanics differs significantly from the Einstein, Podolsky and Rosen incompleteness argument, and differs in such a way that it helps us better to understand both the foundations of Einstein's realism and the physical implications of the experimental tests of local hidden-variable theories. If any other conclusion is to be drawn, it is that careful and sympathetic reading of the considered opinions of a thinker like Einstein is an imperative for the historian and philosopher of science, and for the physicist as well.

Let me close by quoting one final time from Einstein. This is the conclusion of his 1948 *Dialectica* paper, and his remarks now appear both ironic and edifying:

> As it appears to me, there can be no doubt that the physicists who hold the quantum-mechanical manner of description to be, in principle, definitive, will react to these considerations as follows: They will drop requirement II of the independent existence of the physical realities which are present in different portions of space; they can rightly appeal to the fact that the quantum-theory nowhere makes explicit use of this requirement.
>
> I grant this, but note: if I consider the physical phenomena with which I am acquainted, and especially those which are so successfully comprehended by means of quantum-mechanics, then, nevertheless, I nowhere find a fact which makes it appear to me probable that one has to give up requirement II. For that reason I am inclined to believe that the description afforded by quantum-mechanics is to be viewed . . . as an incomplete and indirect description of reality, that will again be replaced later by a complete and direct description.
>
> In any case, one should be on guard, in my opinion, against committing oneself dogmatically to the schema of current theory in the search for a unified basis for the whole of physics.[50]

[50] *Op. cit.*, note 12, p. 323. (My translation — D.H.)