ARTICLE IV.

On the Conservation of Force; a Physical Memoir.

By Dr. H. HELMHOLTZ.

[Read before the Physical Society of Berlin on the 23rd of July, 1847.
Berlin. G. Reimer.]

CONTENTS.

INTRODUCTION.

THE principal contents of the present memoir show it to be addressed to physicists chiefly, and I have therefore thought it judicious to lay down its fundamental principles purely in the form of a physical premise, and independent of metaphysical considerations,—to develope the consequences of these principles, and to submit them to a comparison with what experience has established in the various branches of physics. The deduction of the propositions contained in the memoir may be based on either of two maxims; either on the maxim that it is not possible by any combination whatever of natural bodies to derive an unlimited amount of mechanical force, or on the assumption that all actions in nature can be ultimately referred to attractive or repulsive forces, the intensity of which depends solely upon the distances between the points by which the forces are exerted. That both these propositions are identical is shown at the commencement of the memoir itself. Meanwhile the important bearing which they have upon the final aim of the physical sciences may with propriety be made the subject of a special introduction.

The problem of the sciences just alluded to is, in the first place, to seek the laws by which the particular processes of nature may be referred to, and deduced from, general rules. These rules,—for example, the law of the reflexion and refraction of light, the law of Mariotte and Gay-Lussac regarding the volumes of gases,—are evidently nothing more than general ideas by which the various phænomena which belong to them are connected together. The finding out of these is the office of the experimental portion of our science. The theoretic portion seeks, on the contrary, to evolve the unknown causes of the processes from the visible actions which they present; it seeks to comprehend these processes according to the laws of causality. We are justified, and indeed impelled in this proceeding, by the conviction that every change in nature *must* have a sufficient cause. The proximate causes to which we refer phænomena may, in themselves, be either variable or invariable; in the former case the above conviction impels us to seek for causes to account for the change, and thus we proceed until we at length arrive at final causes which are unchangeable, and which therefore must, in all cases where the exterior conditions are the same, produce the same invariable effects. The final aim of the theoretic natural sciences is therefore to discover the ultimate and unchangeable causes of natural phænomena. Whether all the processes of nature be actually referrible to such,—whether nature is capable of being completely comprehended, or whether changes occur which are not subject to the laws of necessary causation, but spring from spontaneity or freedom, this is not the place to decide; it is at all events clear that the science whose object it is to comprehend nature must proceed from the assumption that it is comprehensible, and in accordance with this assumption investigate and conclude until, perhaps, she is at length admonished by irrefragable facts that there are limits beyond which she cannot proceed.

Science regards the phænomena of the exterior world according to two processes of abstraction: in the first place it looks upon them as simple existences, without regard to their action upon our organs of sense or upon each other; in this aspect they are named *matter*. The existence of matter in itself is to us something tranquil and devoid of action: in it we distinguish merely the relations of space and of quantity (mass), which is assumed to be eternally unchangeable. To matter, thus regarded, we must not ascribe qualitative differences, for when we speak of different kinds of matter we refer to differences of action, that is, to differences in the forces of matter. Matter in itself can therefore partake of one change only,—a change which has reference to space, that is, motion. Natural objects are not, however, thus passive; in fact we come to a knowledge of their existence solely from their actions upon our organs of sense, and infer from these actions a something which acts. When, therefore, we wish to make actual application of our idea of matter, we can only do it by means of a second abstraction, and ascribe to it properties which in the first case were excluded from our idea, namely the capability of producing effects, or, in other words, of exerting force. It is evident that in the application of the ideas of matter and force to nature the two former should never be separated: a mass of pure matter would, as far as we and nature are concerned, be a nullity, inasmuch as no action could be wrought by it either upon our organs of sense or upon the remaining portion of nature. A pure force would be something which must have a basis, and yet which has no basis, for the basis we name matter. It would be just as erroneous to define matter as something which has an actual existence, and force as an idea which has no corresponding reality. Both, on the contrary, are abstractions from the actual, formed in precisely similar ways. Matter is only discernible by its forces, and not by itself.

We have seen above that the problem before us is to refer back the phænomena of nature to unchangeable final causes. This requirement may now be expressed by saying that for final causes unchangeable forces must be found. Bodies with unchangeable forces have been named in science (chemistry) elements. Let us suppose the universe decomposed into elements possessing unchangeable qualities, the only alteration possible to such a system is an alteration of position, that is, motion; hence, the forces can be only moving forces dependent in their action upon conditions of space.

To speak more particularly: the phænomena of nature are to be referred back to motions of material particles possessing unchangeable moving forces, which are dependent upon conditions of space alone.

Motion is the alteration of the conditions of space. Motion, as a matter of experience, can only appear as a change in the relative position of at least two material bodies. Force, which originates motion, can only be conceived of as referring to the relation of at least two material bodies towards each other; it is therefore to be defined as the endeavour of two masses to alter their relative position. But the force which two masses exert upon each other must be resolved into those exerted by all their particles upon each other; hence in mechanics we go back to forces exerted by material points. The relation of one point to another, as regards space, has reference solely to their distance apart: a moving force, therefore, exerted by each upon the other, can only act so as to cause an alteration of their distance, that is, it must be either attractive or repulsive.

Finally, therefore, we discover the problem of physical natural science to be, to refer natural phænomena back to unchangeable attractive and repulsive forces, whose intensity depends solely upon distance. The solvability of this problem is the condition of the complete comprehensibility of nature. In mechanical calculations this limitation of the idea of moving force has not yet been assumed: a great number, however, of general principles referring to the motion of compound systems of bodies are only valid for the case that these bodies operate upon each other by unchangeable attractive or repulsive forces; for example, the principle of virtual velocities; the conservation of the motion of the centre of gravity; the conservation of the principal plane of rotation; of the moment of rotation of free systems, and the conservation of *vis viva*. In terrestrial matters application is made chiefly of the first and last of these principles, inasmuch as the others refer to systems which are supposed to be completely free; we shall however show that the first is only a special case of the last, which therefore must be regarded as the most general and important consequence of the deduction which we have made.

Theoretical natural science therefore, if she does not rest contented with half views of things, must bring her notions into harmony with the expressed requirements as to the nature of simple forces, and with the consequences which flow from them. Her vocation will be ended as soon as the reduction of natural phænomena to simple forces is complete, and the proof given that this is the only reduction of which the phænomena are capable.

I. *The principle of the Conservation of* vis viva.

We will set out with the assumption that it is impossible, by any combination whatever of natural bodies, to produce force continually from nothing. By this proposition Carnot and Clapeyron have deduced theoretically a series of laws, part of which are proved by experiment and part not yet submitted to this test, regarding the latent and specific heats of various natural bodies. The object of the present memoir is to carry the same principle, in the same manner, through all branches of physics; partly for the purpose of showing its applicability in all those cases where the laws of the phænomena have been sufficiently investigated, partly, supported by the manifold analogies of the known cases, to draw further conclusions regarding laws which are as yet but imperfectly known, and thus to indicate the course which the experimenter must pursue.

The principle mentioned can be represented in the following manner:—Let us imagine a system of natural bodies occupying certain relative positions towards each other, operated upon by forces mutually exerted among themselves, and caused to move until another definite position is attained; we can regard the velocities thus acquired as a certain mechanical work and translate them into such. If now we wish the same forces to act a second time, so as to produce again the same quantity of work, we must, in some way, by means of other forces placed at our disposal, bring the bodies back to their original position, and in effecting this a certain quantity of the latter forces will be consumed. In this case our principle requires that the quantity of work gained by the passage of the system from the first position to the second, and the quantity lost by the passage of the system from the second position back again to the first, are always equal, it matters not in what way or at what velocity the change has been effected. For were the quantity of work greater in one way than another, we might use the former for the production of work and the latter to carry the bodies back to their primitive positions, and in this way procure an indefinite amount of mechanical force. We should thus have built a *perpetuum* *mobile* which could not only impart motion to itself, but also to exterior bodies.

If we inquire after the mathematical expression of this principle, we shall find it in the known law of the conservation of *vis viva*. The quantity of work which is produced and consumed may, as is known, be expressed by a weight m, which is raised to a certain height h; it is then mgh, where g represents the force of gravity. To rise perpendicularly to the height h, the body m requires the velocity $v=\sqrt{2gh}$, and attains the same by falling through the same height. Hence we have $\frac{1}{2}mv^2=mgh$; and hence we can set the half of the product mv^2, which is known in mechanics under the name of the *vis viva* of the body m, in the place of the quantity of work. For the sake of better agreement with the customary manner of measuring the intensity of forces, I propose calling the quantity $\frac{1}{2}mv^2$ the quantity of *vis viva*, by which it is rendered identical with the quantity of work. For the applications of the doctrine of *vis viva* which have been hitherto made this alteration is of no importance, but we shall derive much advantage from it in the following. The principle of the conservation of *vis viva*, as is known, declares that when any number whatever of material points are set in motion, solely by such forces as they exert upon each other, or as are directed against fixed centres, the total sum of the *vires vivæ*, at all times when the points occupy the same relative position, is the same, whatever may have been their paths or their velocities during the intervening times. Let us suppose the *vires vivæ* applied to raise the parts of the system or their equivalent masses to a certain height, it follows from what has just been shown, that the quantities of work, which are represented in a similar manner, must also be equal under the conditions mentioned. This principle however is not applicable to all possible kinds of forces; in mechanics it is generally derived from the principle of virtual velocities, and the latter can only be proved in the case of material points endowed with attractive or repulsive forces. We will now show that the principle of the conservation of *vis viva* is alone valid where the forces in action may be resolved into those of material points which act in the direction of the lines which unite them, and the intensity of which depends only upon the distance. In mechanics such forces are generally named central forces. Hence, conversely, it follows that in all actions of natural bodies upon each other, where the above principle is capable of general application, even to the ultimate particles of these bodies, such central forces must be regarded as the simplest fundamental ones.

Let us consider the case of a material point with the mass m, which moves under the influence of several forces which are united together in a fixed system A; by mechanics we are enabled to determine the velocity and position of this point at any given time. We should therefore regard the time t as primitive variable, and render dependent upon it,—the ordinates x, y, z of m in a system of coordinates, definite as regards A, the tangential velocity q, the components of the latter parallel to the axes, $u=\frac{dx}{dt}$, $v=\frac{dy}{dt}$, $w=\frac{dz}{dt}$, and finally the components of the acting forces

$$X=m\frac{du}{dt},\quad Y=m\frac{dv}{dt},\quad Z=m\frac{dw}{dt}.$$

Now according to our principle $\frac{1}{2}\, m\, q^2$, and hence also q^2, must be always the same when m occupies the same position relative to A; it is not therefore to be regarded merely as a function of the primitive variable t, but also as a function of the coordinates x, y, z only; so that

$$d(q^2)=\frac{d(q^2)}{dx}dx+\frac{d(q^2)}{dy}dy+\frac{d(q^2)}{dz}dz. \quad . \quad . \quad . \quad (1)$$

As $q^2=u^2+v^2+w^2$, we have $d(q^2)=2\,udu+2vdv+2\,wdw$. Instead of u let us substitute its value $\frac{dx}{dt}$, and instead of du its value $\frac{Xdt}{m}$, the corresponding values of v and w being also used, we have

$$d(q^2)=\frac{2X}{m}dx+\frac{2Y}{m}dy+\frac{2Z}{m}dz. \quad . \quad . \quad . \quad . \quad (2)$$

As the equations (1) and (2) must hold good together for all values whatever of dx, dy, dz, it follows that

$$\frac{d(q^2)}{dx}=\frac{2X}{m},\quad \frac{d(q^2)}{dy}=\frac{2Y}{m}\ \text{and}\ \frac{d(q^2)}{dz}=\frac{2Z}{m}.$$

But if q^2 is a function of x, y, and z merely, it follows that X, Y, and Z, that is, the direction and magnitude of the acting forces, are purely functions of the position of m in respect to A.

Let us now imagine, instead of the system A, a single material point a, it follows from what has been just proved, that the direction and magnitude of the force exerted by a upon m is only affected by the position which m occupies with regard to a. But the only circumstance, as regards position, that can affect the action between the two points is the distance ma; the law, therefore, in this case would require to be so modified, that the direction and magnitude of the force must be functions of the said distance, which we shall name r. Let us suppose the co-ordinates referred to any system of axes whatever whose origin lies in a, we have then

$$md(q^2) = 2\mathrm{X}dx + 2\mathrm{Y}dy + 2\mathrm{Z}dz = 0. \quad . \quad . \quad . \quad (3)$$

as often as

$$d(r^2) = 2xdx + 2ydy + 2zdz = 0$$

that is, as often as

$$dz = -\frac{xdx + ydy}{z};$$

setting this value in equation (3), we obtain

$$\left(\mathrm{X} - \frac{x}{z}\mathrm{Z}\right)dx + \left(\mathrm{Y} - \frac{y}{z}\mathrm{Z}\right)dy = 0$$

for any values whatever of dx and dy; hence also singly

$$\mathrm{X} = \frac{x}{z}\mathrm{Z} \text{ and } \mathrm{Y} = \frac{y}{z}\mathrm{Z},$$

that is to say, the resultant must be directed towards the origin of coordinates, or towards the point a.

Hence in systems to which the principle of the conservation of force can be applied, in all its generality, the elementary forces of the material points must be central forces.

II. *The principle of the Conservation of Force.*

We will now give the law for the cases where the central forces act, a still more general expression.

Let ϕ be the intensity of the force which acts in the direction of r, which is to be regarded as positive when it attracts, and as negative when it repels, then we have

$$\mathrm{X} = -\frac{x}{r}\phi; \ \mathrm{Y} = -\frac{y}{r}\phi; \ \mathrm{Z} = -\frac{z}{r}\phi; \quad . \quad . \quad . \quad (1)$$

and from equation (2) of the foregoing section, we have

$$md(q^2) = -2\frac{\phi}{r}(xdx + ydy + zdz)\text{ ; hence}$$

$$\frac{1}{2}md(q^2) = -\phi dr\text{ ;}$$

or when Q and R, q and r represent corresponding tangential velocities and distances,

$$\frac{1}{2}mQ^2 - \frac{1}{2}mq^2 = -\int_r^R \phi dr. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Let us regard this equation more closely; we find at the left-hand side the difference of the *vires vivæ* possessed by m at two different distances. To understand the import of the quantity $\int_r^R \phi dr$, let us suppose the intensities of ϕ which belong to different points of the connecting line ma erected as ordinates at these points, then the above quantity would denote the superficial content of the space enclosed between the two ordinates r and R. As this surface may be regarded as the sum of the infinite number of ordinates which lie between r and R, it therefore represents the sum of the intensities of the forces which act at all distances between R and r. Calling the forces which tend to move the point m, before the motion has actually taken place, *tensions*, in opposition to that which in mechanics is named *vis viva*, then the quantity $\int_r^R \phi dr$ would be *the sum of the tensions* between the distances R and r, and the above law would be thus expressed:—The increase of *vis viva* of a material point during its motion under the influence of a central force is equal to the sum of the tensions which correspond to the alteration of its distance.

Let us suppose the case of two points operated upon by an attractive force, at the distance R; by the action of the force they will be drawn to less distances r, their velocity, and consequently *vis viva*, will be increased; but if they should be driven to greater distances r, their *vis viva* must diminish and must finally be quite consumed. We can therefore distinguish, in the case of attractive forces, the sum of the tensions for the distances between $r=0$ and $r=\text{R}$, $\int_0^R \phi dr$, as those which yet remain, but those between $r=\mathrm{R}$ and $r=\infty$ as those already consumed; the former can immediately act, the latter can only be called into action by an equivalent loss of *vis viva*. It is the reverse with repulsive forces. If the points are situate at the distance R, as the distance becomes greater *vis viva* will be gained, and the still existing tensions are those between $r=\mathrm{R}$ and $r=\infty$, those lost are between $r=0$ and $r=\mathrm{R}$.

To carry our law through in quite a general manner, let us suppose any number whatever of material points with the masses m_1, m_2, m_3, &c. denoted generally by m_a; let the components of the forces which act upon these parallel to the axes be X_a, Y_a, Z_a, the components of the velocities along the same axes u_a, v_a, w_a, the tangential velocity q_a; let the distance between m_a and m_b be r_{ab}, the central force between both being ϕ_{ab}. For the single point m_n we have, analogous to equation (1),

$$X_n=\Sigma\left[(x_a-x_n)\frac{\phi_{an}}{r_{an}}\right]=m_n\frac{du_n}{dt},$$

$$Y_n=\Sigma\left[(y_a-y_n)\frac{\phi_{an}}{r_{an}}\right]=m_n\frac{dv_n}{dt},$$

$$Z_n=\Sigma\left[(z_a-z_n)\frac{\phi_{an}}{r_{an}}\right]=m_n\frac{dw_n}{dt},$$

where the sign of summation Σ includes all members which are obtained by putting in the place of the index a the separate indices 1, 2, 3, &c., with the exception of n.

Multiplying the first equation by $dx_n=u_n dt$, the second by $dy_n=v_n dt$, the third by $dz_n=w_n dt$, and supposing the three equations thus obtained to be formed for every single point of m_b, as it is already done for m_n; adding all together, we obtain

$$\Sigma\left[(x_a-x_b)dx_b\frac{\phi^{ab}}{r_{ab}}\right]=\Sigma\left[\frac{1}{2}m_a d(u^2{}_a)\right]$$

$$\Sigma\left[(y_a-y_b)dy_b\frac{\phi^{ab}}{r_{ab}}\right]=\Sigma\left[\frac{1}{2}m_a d(v^2{}_a)\right]$$

$$\Sigma\left[(z_a-z_b)dz_b\frac{\phi^{ab}}{r_{ab}}\right]=\Sigma\left[\frac{1}{2}m_a d(cw^2{}_a)\right].$$

The members of the left-hand series will be obtained by placing instead of a all the single indices 1, 2, 3, &c., and in each case for b also all the values of b, which are greater or smaller than a already possesses. The sums divide themselves therefore into two portions, in one of which a is always greater than b, and in the other always smaller, and it is clear that for every member of the one portion

$$(x_p - x_q)dx \frac{\phi_{pq}}{r_{pq}},$$

a member

$$(x_q - x_p)dx_p \frac{\phi_{pq}}{r_{pq}}$$

must appear in the other portion: adding both together, we obtain

$$-(x_p - x_q)(dx_p - dx_q)\frac{\phi_{pq}}{r_{pq}}:$$

drawing the sums thus together, adding all three and setting

$$\frac{1}{2}d\Big[(x_a - x_b)^2 + (y_a - y_b)^2 + (z_a - z_b)^2\Big] = r_{ab}dr_{ab},$$

we obtain

$$-\Sigma\Big[\phi_{ab}dr_{ab}\Big] = \Sigma\Big[\frac{1}{2}m_a d(q^2{}_a)\Big], \quad . \quad . \quad . \quad . \quad . \quad (3)$$

or

$$-\Sigma\Big[\int_{r_{ab}}^{R_{ab}} \phi_{ab}dr_{ab}\Big] = \Sigma\Big[\frac{1}{2}m_a Q^2{}_a\Big] - \Sigma\Big[\frac{1}{2}m_a q^2{}_a\Big], \quad . \quad . \quad (4)$$

where R and Q, as well as r and q, denote contemporaneous values.

We have here at the left-hand side again the sum of the tensions consumed, on the right the *vis viva* of the entire system, and we can now express the law as follows:—In all cases of the motion of free material points under the influence of their attractive and repulsive forces, whose intensity depends solely upon distance, the loss in tension is always equal to the gain in *vis viva*, and the gain in the former equal to the loss in the latter. Hence *the sum of the existing tensions and vires vivæ is always constant.* In this most general form we can distinguish our law as *the principle of the conservation of force.*

In the deduction of the law as given above, nothing is changed if a number of the points, which we will denote generally by the letter d, are supposed to be fixed, so that q_d is constantly $=0$; the form of the law will then be

$$\Sigma[\phi_{ab}dr_{ab}]+\Sigma[\phi_{ad}dr_{ad}]=-\Sigma\left[\frac{1}{2}m_b d(q^2{}_b)\right] \quad . \quad . \quad . \qquad (5)$$

It remains to be shown in what relation the principle of the conservation of force stands to the most general law of statics, the so-called principle of virtual velocities. This follows immediately from equations (3) and (5). If equilibrium is to set in when a certain arrangement of the points ma takes place, that is, if in case these points come to rest, hence $q_a=0$, they remain at rest, hence $dq_a=0$, it follows from equation (3),

$$\Sigma[\phi_{ab}dr_{ab}]=0\,; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (6)$$

or in case that forces act upon them from points m_b without the system, by equation (5),

$$\Sigma[\phi_{ab}dr_{ab}]+\Sigma[\phi_{ad}dr_{ad}]=0. \quad . \quad . \quad . \quad . \qquad (7)$$

In these equations under dr are understood alterations of distance consequent on the small displacements of the point m_a, which are permitted by the conditions of the system. We have seen, in the former deductions, that an increase of *vis viva*, hence a transition from rest to motion, can only be effected by an expenditure of tension; in correspondence with this, the last equations declare that in cases where in no single one of the possible directions of motion tension in the first moment is consumed, the system once at rest must remain so.

It is known that all the laws of statics may be deduced from the above equations. The most important consequence as regards the nature of the acting forces is this: instead of the arbitrary small displacements of the points m, let us suppose such introduced as might take place were the system in itself firmly united, so that in equation (7) every $dr_{ab}=0$, it follows singly,

$$\Sigma[\phi_{ab}dr_{ab}]=0, \text{ and}$$

$$\Sigma[\phi_{ab}dr_{ab}]=0.$$

Then the exterior, as well as the interior forces, must satisfy among themselves the conditions of equilibrium. Hence, if any system whatever of natural bodies be brought by the action of exterior forces into a certain position of equilibrium, the equilibrium will not be destroyed—1, if we imagine the single points of the system in their present position to be rigidly united to each other; and 2, if we then remove the forces which the points exert upon each other. From this however it follows further: If the forces which two material points exert upon each other be brought into equilibrium by the action of exterior forces, the equilibrium must continue, when, instead of the mutual forces of the points, a rigid connexion between them is substituted. Forces, however, which are applied to two points of a rigid right line can only be in equilibrium when they lie in this line and are equal and opposite. It follows therefore for the forces of the points themselves, which are equal and opposed to the exterior ones, that they must act in the direction of the line of connexion, and hence must be either attractive or repulsive.

The preceding propositions may be collected together as follows:—

1. Whenever natural bodies act upon each other by attractive or repulsive forces, which are independent of time and velocity, the sum of their *vires vivæ* and tensions must be constant; the maximum quantity of work which can be obtained is therefore a limited quantity.

2. If, on the contrary, natural bodies are possessed of forces which depend upon time and velocity, or which act in other directions than the lines which unite each two separate material points, for example, rotatory forces, then combinations of such bodies would be possible in which force might be either lost or gained *ad infinitum*.

3. In the case of the equilibrium of a system of bodies under the operation of central forces, the exterior and the interior forces must, each system for itself, be in equilibrium, if we suppose that the bodies of the system cannot be displaced, the whole system only being moveable in regard to bodies which lie without it. A rigid system of such bodies can therefore never be set in motion by the action of its interior forces, but only by the operation of exterior forces. If, however, other than central forces had an existence, rigid combinations of natural bodies might be formed which could move of themselves without needing any relation whatever to other bodies.

III. *The application of the principle in Mechanical Theorems.*

We will now turn to the special application of the law of the constancy of force. In the first place, we will briefly notice those cases in which the principle of the conservation of *vis viva* has been heretofore recognized and made use of.

1. *All motions which proceed under the influence of the general force of gravitation*; hence those of the heavenly and the ponderable terrestrial bodies. In the former case the law pronounces itself in the increase of velocity which takes place when the paths of the planets approach the central body, in the unchangeableness of the greater axes of their orbits, their time of rotation and orbital revolution. In the latter case, by the known law that the terminal velocity depends only upon the perpendicular height fallen through, and that this velocity, when it is not destroyed by friction or by unelastic eoncussion, is exactly sufficient to carry the body to the same height as that from which it has fallen; that the height of ascent of a certain weight is used as the unit of measure in our machines has been already mentioned.

2. *The transmission of motion through the incompressibly solid and fluid bodies,* where neither friction nor concussion takes place. Our general principle finds for this case expression in the known fact, that a motion transmitted and altered by mechanical powers, diminishes in force as it increases in velocity. Let us suppose that by means of any machine whatever, to which mechanical force is uniformly applied, the weight m is raised with the velocity c, by another mechanical arrangement the weight nm may be raised, but only with the velocity $\frac{c}{n}$, so that in both cases the quantity of tension developed by the machine in the unit of time is expressed by mgc, where g represents the intensity of gravity.

3. *The motions of perfectly elastic, solid, and fluid bodies.* As condition of complete elasticity, we must to the ordinary definition, that the body which has been changed in form or volume completely regains its primitive condition, add, that no friction takes place between the particles in the interior. In the laws of these motions our principle was first recognised and most frequently made use of. Among the most common cases of its application to solids may be mentioned the collision of elastic bodies, the

laws of which may be readily deduced from our principle; the conservation of the centre of gravity, and the manifold elastic vibrations which continue without fresh excitement, until, through the friction of the interior parts and the yielding up of motion to exterior bodies, they are destroyed. In fluid bodies, liquid (evidently also elastic, but endowed with a high modulus of elasticity and a position of equilibrium of the particles) as well as gaseous, (with low modulus of elasticity and without position of equilibrium) motions are in general propagated by undulations. To these belong the waves on the surfaces of liquids, the motion of sound, and probably also those of light and radiant heat.

The *vis viva* of a single particle Δm in a medium which is traversed by a train of waves, is evidently to be determined from the velocity which it possesses at its position of equilibrium. The general equation of waves determines, as is known, the velocity u, when a^2 is the intensity, λ the length of the wave, α the velocity of propagation, x the abscissa, and t the time, as follows:—

$$u = a \,.\, \cos\left[\frac{2\pi}{\lambda}(x - \alpha t)\right].$$

For the position of equilibrium u is $= a$, hence the *vis viva* of the particle Δm during the undulatory motion $\frac{1}{2}\Delta m a^2$ is proportional to the intensity. If the waves expand spherically from a centre, masses continually increasing in bulk are set in motion, and hence the intensity must diminish, if the *vis viva* is to remain the same. Now as the masses embraced by the waves increase as the square of the distance, the law follows as a consequence, that the intensities diminish in the reciprocal ratio.

The laws of reflexion, refraction and polarization of light at the limit of two media of different wave-velocity, are known to have been deduced by Fresnel from the assumption that the motion of the limiting particles in both media is the same, and from the conservation of *vis viva*. By the interference of two trains of waves we have no destruction of *vis viva*, but merely another distribution. Two trains of waves of the intensities a^2 and b^2 which do not interfere, give to all points on which they strike the intensity $a^2 + b^2$; if they interfere, the maxima possess the intensity $(a + b)^2$, that is, $2ab$ more, and the minima $(a - b)^2$, just as much less than $a^2 + b^2$.

The *vis viva* of elastic waves is only destroyed by such processes as we denominate absorption. The absorption of sonorous waves we find to be chiefly effected by concussion against yielding unelastic bodies, for example, curtains and coverlets; they may therefore be regarded as a communication of motion to the bodies in question, in which the motion is destroyed by friction. Whether motion can be destroyed by the friction of the air-particles against each other is a question which cannot yet perhaps be decided. The absorption of rays of heat is accompanied by a proportional development of heat; how far the latter corresponds to a certain equivalent of force, we will consider in the next section. The conservation of force would take place if the quantity of heat radiated from one body appeared again in the body into which it was radiated, provided that none was lost by conduction, and no portion of the rays escaped elsewhere. The theorem is certainly assumed in investigations upon radiant heat, but I am aware of no experiments which furnish the proof of it. As regards the absorption of light by imperfectly transparent or totally opake bodies, we are acquainted with three peculiarities. In the first place, phosphorescent bodies absorb the light in such a manner that they yield it up again afterwards. Secondly, most luminous rays, perhaps all of them, appear to excite heat. The obstacles to the belief in the identity of the luminous, calorific, and chemical rays of the spectrum have been lately disappearing more and more; the heat-equivalent of the chemical and luminous rays appears to be very inconsiderable in comparison to their intense actions upon the eye. If, however, the similarity of these differently acting rays does not permit of being established, then the end of the motion of light must undoubtedly be declared to be unknown. Thirdly, in many cases the light absorbed developes chemical action. We must here distinguish two species of action; first, where the mere incitement to chemical activity is communicated, as in the case of those bodies which induce catalytic action, for example, the action upon a mixture of chlorine and hydrogen; and secondly, those in which it is opposed to chemical action, as in the decomposition of salts of silver and the action upon the green portions of plants. In most of these processes however, the effect of light is so little known, that we are able to form no judgement regarding the magnitude of the forces developed. The latter appear to be considerable in quantity and intensity only in the actions on the green portions of plants.

IV. *The Force-equivalent of Heat.*

The mechanical processes in which an absolute loss of force has been heretofore assumed, are—

1. *The collision of unelastic bodies.*—The loss is mostly connected with the change of form and the compression of the body struck; hence with the increase of the tensions, we also find, by the repetition of such shocks, a considerable development of heat; for example, by hammering a piece of metal; finally, a portion of the motion will be communicated as sound to the contiguous solid and gaseous bodies.

2. *Friction,* both at the surface of two bodies which move over each other, and also that arising in the interior from the displacement of the particles. In the case of friction certain small changes in the molecular constitution of the bodies take place, especially when they commence to rub against each other; afterwards the surfaces generally accommodate themselves to each other, so that this change becomes a vanishing quantity in the further course of the motion. In many cases such changes do not at all appear, for example, when fluids rub against solid bodies or against each other. Besides those already mentioned, thermic and electric changes always take place.

It is customary in mechanics to represent friction as a force which acts against the existing motion, and the intensity of which is a function of the velocity. This mode of representing the subject is only made use of for the sake of calculation, and is evidently an extremely incomplete expression of the complicated process of action and reaction of the molecular forces. From this customary manner of regarding the subject, it was inferred that by friction *vis viva* was absolutely lost; and the same was assumed in the case of unelastic collision. It is not however here taken into account that, disregarding the increase of the tensions caused by the compression of the body rubbed or struck, the heat developed is also the representant of a force by which we can develope mechanical actions; the electricity developed, whose attractions and repulsions are direct mechanical actions, and the heat it excites, an indirect one, has also been neglected. It remains therefore to be asked whether the sum of these forces always corresponds to the mechanical force which has been lost. In those cases where the molecular changes and the development of electricity are to a great extent avoided, the question would be, whether for a certain loss of mechanical force a definite quantity of heat is always developed, and how far can a quantity of heat correspond to a mechanical force. For the solution of the first question but few experiments have yet been made. Joule * has measured the heat developed by the friction of water in narrow tubes, and that developed in vessels in which the water was set in motion by a paddle-wheel; in the first case he found that the heat which raises 1 kilogramme of water 1°, was sufficient to raise 452 kilogrammes through the height of 1 metre; in the second case he found the weight to be 521 kilogrammes. His method of measurement however meets the difficulty of the investigation so imperfectly, that the above results can lay little claim to accuracy †. Probably the above numbers are too high, inasmuch as in his proceeding a quantity of heat might have readily escaped unobserved, while the necessary loss of mechanical force in other portions of the machine is not taken into account.

Let us now turn to the further question, how far heat can correspond to an equivalent of force. The material theory of heat must necessarily assume the quantity of caloric to be constant; it can therefore develope mechanical forces only by its effort to expand itself. In this theory the force-equivalent of heat can only consist in the work produced by the heat in its passage from a warmer to a colder body; in this sense the problem has been treated by Carnot and Clapeyron, and all the consequences of the assumption, at least with gases and vapours, have been found corroborated.

To explain the heat developed by friction, the material theory must either assume that it is communicated by conduction as supposed by Henry ‡, or that it is developed by the compression of the surfaces and of the particles rubbed away, as supposed by

* Philosophical Magazine, S. 3. vol. xxvii. p. 205.

† It must be remembered that the writer was acquainted with the earlier experiments only of Mr. Joule.—Ed.

‡ Mem. of the Society of Manchester, t. v. p. 2. London, 1802.

Berthollet*. The first of these assumptions lacks all experimental proof; if it were true, then in the neighbourhood of the rubbed portions a cold proportionate to the intense heat often developed must be observed. The second assumption, without dwelling upon the altogether improbable magnitude of action, which according to it must be ascribed to the almost imperceptible compression of the hydrostatic balance, breaks down completely when it is applied to the friction of fluids, or to the experiments where wedges of iron have been rendered red-hot and soft by hammering and pieces of ice melted by friction†; for here the softened iron and the water of the melted ice could not remain in a compressed condition. Besides this, the development of heat by the motion of electricity proves that the quantity of heat can be actually increased. Passing by frictional and voltaic electricity—because it might here be suspected that, by some hidden relation of caloric to electricity, the former was transferred from the place where it was originated and deposited in the heated wire—two other ways of producing electric tensions by purely mechanic agencies in which heat does not at all appear, are still open to us, namely, by induction and by the motion of magnets. Suppose we possess a completely insulated body positively electric and which cannot part with its electricity; an insulated conductor brought near to it will show free +E, we can discharge this upon the interior coating of a battery and remove the conductor, which will then show −E; this latter can of course be discharged upon the exterior surface of the first or upon a second battery. By repeating this process, it is evident that we can charge a battery of any magnitude whatever as often as we please, and by means of its discharge can develope heat, which nowhere disappears. We shall, on the contrary, have consumed a certain amount of force, for at each removal of the negatively-charged conductor from the inducing body the attraction between both is to be overcome. This process is essentially carried out when the electrophorus is used to charge a Leyden jar. The same takes place in magneto-electric machines; as long as magnet and keeper are moved opposite to each other, electric currents are excited which develope heat in the connecting wire; and in-

* *Statique Chimique*, t. i. p. 247.

† Humphry Davy, Essay on Heat, Light, and the Combinations of Light.

asmuch as they constantly act in a sense contrary to the motion of the keeper, they destroy a certain amount of mechanical force. Here evidently heat *ad infinitum* may be developed by the bodies constituting the machine, while it nowhere disappears. That the magneto-electric current developes heat instead of cold, in the portion of the spiral directly under the influence of the magnet, Joule has endeavoured to prove experimentally*. From these facts, it follows that the quantity of heat can be absolutely increased by mechanical forces, that therefore calorific phænomena cannot be deduced from the hypothesis of a species of matter, the mere presence of which produces the phænomena, but that they are to be referred to changes, to motions, either of a peculiar species of matter, or of the ponderable or imponderable bodies already known, for example of electricity or the luminiferous æther. That which has been heretofore named the quantity of heat, would, according to this, be the expression, first, of the quantity of *vis viva* of the calorific motion, and, secondly, of the quantity of those tensions between the atoms, which, by changing the arrangement of the latter, such a motion can develope. The first portion would correspond to that which has been heretofore called free heat, the second with that which has been named latent heat. If it be permitted to make an attempt at rendering the idea of this motion still clearer, the view derived from the hypothesis of Ampère seems best suited to the present state of science. Let us imagine the bodies formed of atoms which themselves are composed of subordinate particles (chemical elements, electricity, &c.), in such an atom three species of motion may be distinguished,—1, displacement of the centre of gravity; 2, rotation round the centre of gravity; 3, displacement of the particles of the atom among themselves. The two first would be compensated by the forces of the neighbouring atoms, and hence transmitted to these in the form of undulations, a species of propagation which corresponds to the radiation of heat, but not to its conduction. Motions of the single particles of the atoms among themselves, would be compensated by the forces existing within the atom, and would communicate motion but slowly to the surrounding atoms, as a vibrating string sets a second in motion and thereby loses an equal quantity of motion

* Phil. Mag. S. 3. 1844.

itself; this description of motion seems to be similar to the conduction of heat. It is also clear, that such motions in the atoms may cause changes in the molecular forces, and consequently give rise to expansion or an alteration of the state of aggregation. Of what nature the motion is, we have no means whatever of ascertaining; the possibility of conceiving the phænomena of heat as being due to motion is, however, sufficient for our present object. The conservation of force in the case of these motions will hold good in all cases where hitherto the conservation of caloric has been assumed; for example, in all phænomena of radiation and conduction of heat from one body to another, and in the case of the appearance and disappearance of heat during changes of aggregation.

Of the different modes in which heat manifests itself, we have considered the cases where one body radiates into another, and where it is produced by mechanical force; further on we will examine the heat generated by electricity. It remains to consider the development of heat in chemical processes. It has been heretofore referred to the setting free of caloric which was previously latent in the combining bodies. According to this, we must ascribe to every simple body, and every chemical combination which is capable of entering into still further combinations of a higher order, a definite quantity of latent heat which is necessary to its chemical constitution. From this we derive the law, which has been also partially verified by experience, that when several bodies unite together to a chemical compound, the same quantity of heat is developed, no matter in what order the combination may have been effected*. According to our way of viewing the subject, the quantity of heat developed by chemical processes would be the quantity of *vis viva* produced by the chemical attractions, and the above law would be the expression for the principle of the conservation of force in this case.

As little as the conditions and laws of the generation of heat have been investigated, although such a generation undoubtedly occurs, this has been done with reference to the disappearance of heat. Hitherto we are only acquainted with cases in which chemical combinations have been decomposed, or less dense

* Hess, in Pogg. *Ann.* vol. l. 392; lvi. 598.

states of aggregation brought about, and thus heat rendered latent. Whether by the development of mechanical force heat disappears, which would be a necessary postulate of the conservation of force, nobody has troubled himself to inquire. I can only in respect to this cite an experiment by Joule*, which seems to have been carefully made. He found that air while streaming from a reservoir with a capacity of 136·5 cubic inches, in which it was subjected to a pressure of 22 atmospheres, cooled the surrounding water 4°·085 Fahr. when the air issued into the atmosphere, and therefore had to overcome the resistance of the latter. When, on the contrary, the air rushed into a vessel of equal size which had been exhausted of air, thus finding no resistance and exerting no mechanical force, no change of temperature took place.

We have now to examine the manner in which the views of Clapeyron† and Holtzmann‡ bear upon our own. Clapeyron starts from the notion that only by its passage from warm bodies to cold ones can heat be applied as a means of developing mechanical force, and that the maximum of the latter is gained when the transmission of the heat is effected between bodies of equal temperatures, the alterations of temperature being effected by the compression and dilatation of the heated bodies. This maximum must be the same for all natural bodies which by heating and cooling can produce mechanical force; for were it different, that body in which a certain quantity of heat was capable of producing the greatest action might be applied in the production of mechanical work, and a portion of the latter might be made use of to bring the heat back from the colder to the warmer source, and thus mechanical force to infinity might be gained, it being at the same time tacitly assumed that the quantity of heat cannot be changed by this process. The following is the analytical expression given by Clapeyron to this law:—

$$\frac{dq}{dv}\,\frac{dt}{dp}-\frac{dq}{dp}\cdot\frac{dt}{dv}=C,$$

* Philosophical Magazine, S. 3. vol. xxvi. p. 369.

† Scientific Memoirs, vol. i. Part 3 of 1st Series.

‡ *Ueber die Wärme und Elasticität der Gase und Dämpfe.* Manheim, 1845. Translated in full in vol. iv. Part 14., of Scientific Memoirs.

where q denotes the quantity of heat contained by one body, t its temperature, both expressed as functions of the volume v, and the pressure p. $\frac{1}{C}$ is the mechanical work which the unit of heat (which can raise 1 kilogramme of water 1° Centigrade) produces when it passes into a temperature 1° lower. This is stated to be identical for all natural bodies, but to vary with the temperature. For gases the formula is

$$C = v\frac{dq}{dv} - p\frac{dq}{dp}.$$

Clapeyron's inferences from the general validity of this formula, at least in the case of gases, have many analogies on their side which agree with experience. His deduction of the law can only be admitted when the quantity of heat is regarded as unchangeable. Further, his more special formulas for gases, which alone are supported by experiment, follow also from the formula of Holtzmann, as we shall immediately show. With regard to the general formula, he has only sought to show that the law which follows from it is at least not contradicted by experiment. This law is, that when the pressure on different bodies, taken at the same temperature, is a little increased, quantities of heat will be developed which are proportional to the expansibility of the bodies by heat. I will here merely draw attention to what must be regarded as at least a very improbable consequence of this law. Compression of water at the point of maximum density would develope no heat, and between this and the freezing-point it would develope cold.

Holtzmann sets out from the consideration that a certain quantity of heat which enters a gas can cause in it either an increase of temperature or an expansion. The quantity of work thus produced by the heat he assumed to be the mechanical equivalent of the heat; he calculated from the experiments of Dulong upon sound, that the heat which raises the temperature of 1 kilogramme of water 1° Centigrade, would raise a weight of 374 kilogrammes 1 metre. This method of calculation, regarded from our point of view, is only admissible when the entire *vis viva* of the heat communicated is actually returned as mechanical force, hence the sum of the *vis viva* and tensions, that is, the quantity of free and of latent heat, is just the same in the more expanded gases as in the denser ones of the same temperature. According to this, a gas which expands without producing work must exhibit no change of temperature, which indeed appears to follow from the above-mentioned experiment of Joule; and thus the increase and diminution of temperature by compression and dilatation would, under ordinary circumstances, be due to the excitation of heat by mechanical force, and *vice versâ*. In support of the correctness of Holtzmann's law, a great number of consequences from it which agree with experiment might be adduced; more particularly the deduction of the formula for the elasticity of the vapour of water at different temperatures.

Joule determines from his own experiments the force-equivalent, which Holtzmann, from the experiments of others, reckons at 374, to be 481, 464, 479; while, by friction, the force-equivalent for the unit of heat he found to be 452 and 521.

The formula of Holtzmann for gases coincides with that of Clapeyron, only in the former the undetermined function C of the temperature is found, and by this means the complete determination of the integral is rendered possible. The former formula is

$$\frac{pv}{a}=v\frac{dq}{dv}-p\frac{dq}{dp},$$

where a is the force-equivalent of the unit of heat; the formula of Clapeyron is

$$\mathrm{C}=v\frac{dq}{dv}-p\frac{dq}{dp}.$$

Both are therefore coincident, when $\mathrm{C}=\frac{pv}{a}$; or, as $p=\frac{k}{v}\left(1+\alpha t\right)$ where a is the coefficient of expansion, and k is a constant, when

$$\frac{1}{\mathrm{C}}=\frac{a}{k(1+\alpha t)}.$$

The values found by Clapeyron for $\frac{1}{\mathrm{C}}$ agree pretty well with this formula, as is shown by the following table:—

Temperature.	Calculated by Clapeyron.			According to the formula.
	a	*b.*	*c.*	
0°	1·410	....	1·586	1·544
35·5	....	1·365	1·292	1·366
78·8	....	1·208	1·142	1·198
100	....	1·115	1·102	1·129
156·8	....	1·076	1·072	0·904

The number *a* is calculated from the velocity of sound in air, the series *b* from the latent heat of the vapours of æther, alcohol, water, and oil of turpentine, *c* from the expansive force of the vapour of water at different temperatures. Clapeyron's formula for gas is, according to this, identical with Holtzmann's; its applicability to solid and liquid bodies remains as yet doubtful.

V. *The Force-equivalent of Electric Processes.*

Statical Electricity.—Machine electricity can act in two ways as the cause of the generation of force; in the first place, the electricity itself moves with the body that bears it, in obedience to its attractive or repulsive forces; secondly, by its motion *in* the body that bears it, heat is generated. The first mechanical actions have, as is known, been deduced from hypothesis of two fluids which attract or repel with a force inversely proportional to the square of the distance; so far as experiment has been compared with theory, both have been found to agree. According to our primitive deduction, the conservation of force must take place with such forces. We will therefore enter only so far into the more special laws of the mechanical actions of electricity as is necessary for deducing the law of the electrical development of heat.

Let $e_{\prime}$ and $e_{\prime\prime}$ be two electric elements referred to a unit which at the distance 1 repels an equal quantity of similar electricity with the force 1. When the opposite electricities are distinguished by opposite signs, and the distance between $e_{\prime}$ and $e_{\prime\prime}$ is called r, the intensity of the central force is

$$\phi = -\frac{e_{\prime} e_{\prime\prime}}{r^2}.$$

The gain in *vis viva* caused by passing from the distance R to r is

$$-\int_{R}^{r}\phi dr=\frac{e_{I}e_{II}}{R}-\frac{e_{I}e_{II}}{r}.$$

When they pass from the distance ∞ to r, the expression is $-\frac{e_{I}e_{II}}{r}$. Let us denote this last quantity, which is the sum of the tensions consumed by the motion from ∞ to r and of the *vis viva* produced, in conformity with Gauss in his magnetical researches as the *potential* of the two electric elements for the distance r; the increase of *vis viva* due to any motion whatever is then equal to the excess of the potential at the end of the route over its value at the beginning.

Calling the sum of the potentials of an electrical element, with reference to the whole of the elements of an electrified body, the potential of the element towards the body, and the sum of the potentials of all the elements of an electrified body towards all the elements of another, the potential of the two bodies, we obtain again the gain in *vis viva* in the difference of the potentials, provided that the distribution of the electricity in the bodies is not changed, hence that the bodies are idio-electric. If the distribution undergo a change, the magnitude of the electric tensions in the body itself becomes altered, and the *vis viva* gained must then be different.

By all methods of electrifying, equal quantities of positive and negative electricity are generated; in the neutralization of the electricities between two bodies, one of which, A, contains as much positive electricity as the other, B, does of negative, half of the positive electricity goes from A to B, and half the negative from B to A. Let the potentials of the bodies upon themselves be W_a and W_b, their potential towards each other V, we then find the entire *vis viva* which has been gained, when we subtract the potentials of the moving electric masses on themselves and towards each of the other masses, before transfer, from the same potentials after the transfer has been effected. It is here to he remarked that the potential of two masses changes its sign when the sign of one of the masses is changed. Hence the following potentials come under consideration :—

1. of the moved $+\frac{1}{2}$E from A

upon itself	$\frac{1}{4}(W_b - W_a)$
towards the moved $-\frac{1}{2}$E . .	$\frac{1}{4}(V - V)$
towards the motionless $\frac{1}{2}$E . .	$\frac{1}{4}(-V - W_a)$
towards the motionless $-\frac{1}{2}$E .	$\frac{1}{4}(-W_b - V)$.

2. of the moved $-\frac{1}{2}$E from B

upon itself	$\frac{1}{4}(W_a - W_b)$
towards the moved $+\frac{1}{2}$E . .	$\frac{1}{4}(V - V)$
towards the motionless $-\frac{1}{2}$E .	$\frac{1}{4}(-V - W_b)$
towards the motionless $+\frac{1}{2}$E .	$\frac{1}{4}(-W_a - V)$

$$\text{Sum } -\left(V + \frac{W_a + W_b}{2}\right)$$

This quantity gives us therefore the maximum of the *vis viva* to be generated and the quantity of tension gained by electrifying.

To obtain, instead of these potentials, more familiar ideas, let us consider as follows. Suppose surfaces to be constructed for which the potential of an electric element, which lies in them, in regard to several other electrified bodies which are present, possesses equal values, and let us call these surfaces of equilibrium, then must the motion of an electric particle from any point whatever of one of them to any point of another of them, always increase the *vis viva* by the same quantity; a motion on the surface itself, on the contrary, will not alter the velocity of the particle. Hence the resultant of the whole of the attractive forces of the electricity for every single point of space must be perpendicular to the surface of equilibrium which passes through it, and every surface which is at right angles to this resultant must be a surface of equilibrium.

The equilibrium of electricity in a conductor cannot take place until the resultants of the whole of the attractive forces of its own electricity, and such other electrified bodies as may happen to be present, are perpendicular to its surface; because, were it otherwise, the electric particles must be moved along the surface. Consequently the surface of an electrified conductor is itself a surface of equilibrium, and the *vis viva* gained by an infinitely small electric particle in its passage from one conductor to another is constant. Let C_a denote the *vis viva* gained by the unit of positive electricity in its passage from the surface of the conductor A to an infinite distance, so that C_a is positive for positive charges; A_a the potential of the same quantity of electricity in regard to A when it occupies a certain point upon the surface of A; A_b the same in regard to B; W_a the potential of A upon itself; W_b the same of B; V that of A upon B, and Q_a the quantity of electricity in A, Q_b in B; the *vis viva* gained by the particle e in its passage from an infinite distance to the surface A is

$$-eC_a = e(A_a + A_b).$$

If instead of e we set successively all the electric particles of the surface A, and for A_a and A_b the corresponding potentials, and add all, we obtain

$$-Q_aC_a = V + W_a.$$

In like manner, for the conductor B,

$$-Q_bC_b = V + W_b.$$

The constant C must not only possess the same value for one and the same conductor, but also for separate conductors, if the latter, when connected together in a manner by which the distribution of their electricities is not sensibly changed, exchange no electricity with each other; that is, it must possess the same value for all conductors possessing the same free tension. As unit of measure for the free tension of an electrified body, we can make use of a quantity of electricity which, distributed over a sphere of the radius 1, placed beyond the distance where induction can take place, is in electric equilibrium with the said body. If the electricity be distributed uniformly over the sphere, the exterior action, as is known, will be the same as if the electricity was concentrated at its centre. Denoting the mass of electricity by E, the radius of the sphere by $R=1$, for this sphere we have the constant

$$C=\frac{E}{R}=E;$$

that is, the constant C is equal to the free tension.

In accordance with this, the tensions of two conductors which contain equal quantities Q of positive and negative electricity are

$$-\left(V+\frac{W_a+W_b}{2}\right)=Q\left(\frac{C_a-C_b}{2}\right).$$

As C_b is negative, the algebraic difference C_a-C_b is equal to the absolute sum of both. If the capacity of discharge of B be very great, and consequently C_b nearly $=0$, the quantity of the electric tensions is $\frac{QC_a}{2}=-\frac{V+W_a}{2}$; if the distance between the two conductors be also very great, the above becomes $-\frac{1}{2}W_a$.

We have found that the *vis viva* generated by the motion of two electric masses is equal to the decrease of the sum $\frac{Q_aC_a+Q_bC_b}{2}$. This *vis viva* is gained as mechanical force, if the veloeity of the electricity in the bodies be a vanishing quantity when compared with the velocity of propagation of the electric motion; we must obtain it as heat when this is not the case. The heat Θ developed by the discharge of equal quantities Q of the opposed electricities is therefore

$$\Theta=\frac{1}{2a}Q(C_a-C_b),$$

where a is the mechanical equivalent of the unit of heat, or when $C_b=0$, as is the case with batteries whose external coating is not insulated, and whose capacity of discharge is S, so that $CS=Q$,

$$\Theta=\frac{1}{2a}QC=\frac{1}{2a}\frac{Q^2}{S}.$$

Riess* has proved by experiment that, with different charges and different numbers of similarly constructed jars, the quantity of heat developed in every single portion of the same connecting

* Pogg. *Ann.* vol. xliii. p. 47.

wire is proportional to the quantity $\frac{Q^2}{S}$. He however denotes by S the surface of the coating of the jars. With similarly constructed jars, however, this must be proportional to the capacity of discharge. Vorsselmann de Heer* and Knochenhauer† have also deduced from their respective experiments, that the development of heat with equal charges of the same battery remains the same, however the connecting wire may be altered. The latter has proved the same law to exist in branches of the connecting wire. With regard to the quantity $\frac{1}{2a}$, we as yet possess no experimental data.

It is easy to explain this law if we assume that the discharge of a battery is not a simple motion of the electricity in one direction, but a backward and forward motion between the coatings, in oscillations which become continually smaller until the entire *vis viva* is destroyed by the sum of the resistances. The notion that the current of discharge consists of alternately opposed currents is favoured by the alternately opposed magnetic actions of the same; and secondly, by the phænomena observed by Wollaston while attempting to decompose water by electric shocks, that both descriptions of gases are exhibited at both electrodes. This assumption also explains, why in these experiments the electrodes must possess the smallest surface possible.

Galvanism.—With regard to galvanic phænomena, we have to distinguish two classes of conductors:—1. Those which conduct in the manner of metals, and follow the law of the tension series. 2. Those which do not follow this law. To the latter belong all compound liquids, which undergo during conduction a decomposition proportional to the quantity of electricity conducted.

We can classify the experimental facts in accordance with the above, 1, into such as take place between conductors of the first class only—the charging with different electricities of different metals which are in contact; and 2, those between conductors of both classes, the electric tensions in open, and the electric currents in closed circuits. By any combination whatever of conductors of the first class electric currents can

* Pogg. *Ann.* vol. xlviii. p. 292, and the remark of Riess, *ibid.* p. 320.

† Pogg. *Ann.* vol. lxii. p. 364; lxiv. p. 64.

never be generated, but electric tensions only. These tensions, however, are not equivalent to a certain quantity of force like those heretofore considered, which implied a disturbance of the electric equilibrium; galvanic tensions are, on the contrary, consequent upon the establishing of the electric equilibrium; no motion of the electricity can be generated by them further than a change of distribution of the electricity consequent upon a change of position. Let us suppose all the metals of the earth brought into connexion with each other, and the corresponding distribution of electricity established; by no other combination of the same metals can any one of them suffer an alteration of its tension until contact is established with a conductor of the second class. The idea of the force of contact, the force which is active at the place where two different metals touch each other, and which developes and sustains the different electric tensions of the latter, has not hitherto been rendered more determinate than it is here, because the attempt to embrace the phænomena resulting from the contact of conductors of the first and second classes was made at a time when the constant and distinguishing feature of the phænomena, namely the chemical process, was not yet properly recognized. From this indefinite mode of regarding the subject, it would certainly appear that the force of contact is such that by means of it infinite quantities of free electricity, and hence mechanical force, might be generated, if a conductor of the second class could be found which was not electrolysed during the conduction. It was precisely this circumstance which excited such a resistance to the contact theory, notwithstanding the simplicity and precision of the explanations which it furnished*. The principle which we have thus far advocated contradicts the idea of such a force directly, if it do not recognise also the necessity of the chemical processes. If this however be admitted, if it be assumed that the conductors of the second class do not follow the law of the tension series, just because they conduct by electrolysis, then the idea of the force of contact is capable of great simplification, and may at once be referred to attractive and repulsive forces. All phænomena exhibited by conductors of the first

* Faraday's Experimental Researches in Electricity, 17th Series; Phil. Trans. 1840, p. 1, No. 2071; and Pogg. *Ann.* vol. liii. p. 568.

class may evidently be deduced from the assumption that different chemical substances possess different attractive forces for both electricities, and that these attractions are only exerted at insensible distances; while the mutual attractions of the electricities themselves are exerted at measurable distances. The force of contact would, according to this, be the difference of the attractive forces which the metallic particles at the place of contact exert upon the electricities at this place; and electric equilibrium would occur when an electric particle which passes from the one to the other neither gains nor loses *vis viva* further. Let $c_{\prime}$ and $c_{\prime\prime}$ be the free tensions of the two metals, $a_{\prime}e$ and $a_{\prime\prime}e$ the *vires vivæ* gained by the electric particle e by its passage to the one or the other uncharged metal, then the force gained by its passage from one to the other charged metal is

$$e(a_{\prime}-a_{\prime\prime})-e(c_{\prime}-c_{\prime\prime}).$$

In the case of equilibrium this must be $=0$, and hence

$$a_{\prime}-a_{\prime\prime}=c_{\prime}-c_{\prime\prime},$$

that is, the difference of tension in different portions of the same metal must be constant, and in different metals must follow the law of the tension series.

In galvanic currents, with respect to the conservation of force, we have specially to consider the following actions: the generation of heat, chemical processes, and polarization. The electrodynamic actions shall be considered under the head of magnetism. The development of heat is common to all currents; in regard to the two other actions, we can divide the currents into those which excite chemical decomposition only, those which excite polarization only, and those which excite both.

We will, in the first place, investigate the conditions of the conservation of force in those circuits in which the polarization is completely annulled, inasmuch as these are the only ones for which we have arrived at determinate quantitative laws. The intensity of a current I in a battery of n elements is given by the law of Ohm as

$$\mathrm{I}=\frac{n\mathrm{A}}{\mathrm{R}},$$

where the constant A is the electromotive force of a single element, and R is the resistance of the circuit; A and R are in this circuit independent of the intensity. During a certain interval the action of such a circuit will be changed in no respect save in its chemical conditions and quantity of heat; the law of the conservation of force would therefore require that the heat to be gained by the chemical processes which have taken place must be equal to the quantity actually gained. In a simple portion of a metallic conductor, with the resistance r, the heat developed in a certain time t is, according to Lenz*,

$$\vartheta = \mathrm{I}^2 rt,$$

where as unit for r the length of wire is taken in which the unit of current developes the unit of heat in the unit of time. For branching wires, where the resistance in the single branches is denoted by r_a, the total resistance r is given by the equation

$$\frac{1}{r} = \Sigma\left[\frac{1}{r_a}\right],$$

the intensity I in the branch r_n by

$$\mathrm{I}_n = \frac{\mathrm{I}r}{r_n};$$

hence the heat ϑ^n in the same branch,

$$\vartheta_n = \mathrm{I}^2 r^2 \cdot \frac{1}{r_n} t,$$

and the total heat developed in all the branches

$$\vartheta = \Sigma[\vartheta_a] = \mathrm{I}^2 r^2 \Sigma\left[\frac{1}{r_a}\right] t = \mathrm{I}^2 r \,.\, t.$$

Hence the total quantity of heat developed in any circuit whatever, where the conduction is effected through any number of branches, if the law of Lenz be true for fluid conductors, as found by Joule,

$$\Theta = \mathrm{I}^2 \mathrm{R}t = n\mathrm{AI}t.$$

We have two kinds of constant batteries, those constructed according to the system of Daniell and those on Grove's principle. In the first the chemical process consists in the dissolution of the positive metal in an acid, while the negative one is precipitated from its saline solution. Let us assume as the

* Pogg. *Ann.* vol. lix. p. 203 and 407. From the Bulletin of the Acad. of Scien. St. Petersburg.

unit of intensity a current which in the unit of time decomposes an equivalent of water (say 1 gramme); then in the time t, the equivalent of the positive metal dissolved will be nIt, and the same quantity of the negative will be precipitated. Calling the heat which an equivalent of the positive metal developes by its oxidation and the solution of the oxide in the contiguous acid a_z, and the same for the negative a_c, then the quantity of heat to be developed chemically would be

$$=nIt(a_z-a_c).$$

The chemical would therefore be equal to the electrical if

$$A=a_z-a_c,$$

that is, if the electromotive force of two of the so-combined metals be proportional to the difference of the quantities of heat developed by their combustion and by their combination with acids.

In the elements constructed upon Grove's principle, the polarization is annulled by permitting the hydrogen separated to reduce a fluid rich in oxygen which surrounds the negative metal. To these belong the batteries of Grove and of Bunsen: amalgamated zinc, dilute sulphuric acid, fuming nitric acid, platinum or coal; further, those batteries in which chromic acid is made use of, and which have been subjected to exact measurement: amalgamated zinc, dilute sulphuric acid, solution of bichromate of potash with sulphuric acid, copper or platinum. The chemical processes are the same in the two batteries in which nitric acid is used, and likewise the same in the two in which chromic acid is used; from this it would follow, according to the deduction just made, that the electromotive forces must be also equal, which, indeed, by the measurement of Poggendorff, is proved to be exactly the case. The battery prepared from coal and chromic acid is very inconstant, and possesses a considerably higher electromotive force, at least at the beginning; this battery is therefore not to be included here, but belongs to those in which polarization is exhibited. In these constant circuits the electromotive force is independent of the negative metal; we can reduce them to the type of Daniell's battery if we regard as the negative element the particles of oxide of chromium and nitrous acid which are immediately in contact with the platinum; so that thus regarded an element of Grove or of Bunsen would become a circuit formed between zinc and nitrous acid, and those constructed with chromic acid circuits between zinc and oxide of chromium.

The batteries with polarization may be divided into two classes, such as excite polarization, but which cause no chemical decomposition, and such as give rise to both. To the former, which produce an inconstant and quickly disappearing current, belong the simple circuit of Faraday *, with combinations formed of solutions of caustic potash, sulphuret of potassium, and nitrous acid; further, those of the more strongly negative metals in common acids, when the positive are not able to decompose the acid; for example, copper with silver, gold, platinum, coal in sulphuric acid, and so forth; of the compound circuits, all those in which decomposition cells are introduced, the polarization of which overpowers the electromotive force of the other elements. Exact quantitative experiments on the intensity of these circuits have, up to the present time, on account of the great variability of the currents, not been carried out. In general the intensity of these currents seems to depend upon the nature of the immersed metal; their duration increases with the magnitude of the surfaces and with the diminution of the intensity of the current; they can be renewed, even after they have almost disappeared, by moving the metal in the fluid or in the air, by which the polarization of the plate against which hydrogen has been liberated is annulled. To such actions the residual currents, which exhibit themselves on fine galvanometric instruments, are probably due. The entire process is therefore an establishing of an electric equilibrium of the particles of the fluid and of the metals; the particles of fluid appear sometimes to undergo a modification of arrangement; and on the other side chemical changes occur, in many cases, at the metallic surfaces †. In compound circuits, where the polarization of plates originally alike is caused by the action of currents from other elements, we can obtain the lost force of the primitive current in the form of a secondary current, by removing the exciting elements, and causing the polarized

* Experimental Researches in Electricity, 16th series; Phil. Trans. 184. p. 1; and Pogg. *Ann.* vol. lii. p. 163 and 547.

† Ohm, in Pogg. *Ann.* vol. lxiii. p. 389.

metals to form a circuit among themselves. The absence of special facts prevents us from making closer application of the principle of the conservation of force in this case.

The most complicated case is presented by those circuits in which chemical decomposition and polarization take place side by side; to these belong the circuits in which gas is developed. The current in these cases is strongest at the commencement, and sinks more or less quickly to a point at which it remains pretty constant. With single elements of this description, or with compound batteries composed of such elements, the current of polarization ceases with extreme slowness; it is easier, on the contrary, to obtain constant currents by combining constant elements with inconstant ones, particularly if the plates of the latter be comparatively small. Hitherto however but few measurements have been made with such circuits; from the few which I have been able to find by Lenz* and Poggendorff†, it follows that the intensity of such currents, when different resistances of wire are introduced, cannot be expressed by the simple formula of Ohm; for when the constants are calculated from the low intensities, the results for higher intensities are found to be too great. It is therefore necessary to regard the numerator or denominator, or both, as functions of the intensity; the facts hitherto known do not enable us to decide which of these cases really takes place.

In applying the principle of the conservation of force to these currents we must divide them into two classes, into inconstant or polarization currents, with regard to which, what we have already expressed regarding the pure currents of polarization, and constant or decomposing currents, is applicable. The same mode of treatment is applicable to the latter and to the constant currents in which no gas is developed; the quantity of heat generated by the current must be equal to that due to chemical decomposition. For example, in a combination of zinc with a negative metal in dilute sulphuric acid, suppose the quantity of heat liberated by an atom of zinc during its solution and the expulsion of the hydrogen $a_z - a_h$, then the quantity of heat developed in the time dt would be

$$I(a_z - a_h)dt.$$

* Pogg. *Ann.* liv. 229.

† *Ann.* lxvii. 531.

If, now, the development of heat in all portions of the circuit were proportional to the square of the intensity, that is I^2Rdt, we should have, as before,

$$I=\frac{a_z-a_h}{R},$$

which is the simple formula of Ohm. As this however is not applicable in the present instance, it follows that there are certain transverse sections in the circuit in which the development of heat is subject to another law, and whose resistance therefore is not to be regarded as constant. If, for example, the heat liberated in any cross section whatever be directly proportional to the intensity, which among others must be the case with the heat due to a change of aggregation, hence $\vartheta=\mu I dt$, we have

$$I(a_z-a_h)=I^2r+I\mu$$

$$I=\frac{a_z-a_h-\mu}{r}.$$

The quantity μ would therefore appear in the numerator of the formula of Ohm. The resistance of such a transverse section would be $r=\frac{\vartheta}{I^2}=\frac{\mu}{I}$. If however the heat-development be not exactly proportional to the intensity, or, in other words, the quantity μ not constant, but increasing with the intensity, we then obtain the case which corresponds to the observations of Lenz and Poggendorff.

The electromotive force of such a circuit, as soon as the current due to polarization has ceased, would, analogous to the constant circuits, be that between zinc and hydrogen: in the language of the contact theory, it would be that between zinc and the negative metal, lessened by the polarization of the latter in hydrogen. We must then regard this maximum of the polarization as independent of the intensity of the current, and differing for different metals exactly as the electromotive forces differ. The numerator of the formula of Ohm, calculated from measurements of intensity with different resistances, can, however, besides the electromotive force, contain a quantity which springs from the resistance at the points of transition, and which is perhaps different for different metals. That such a resistance exists follows out of the principle of the conservation of force, from the fact that the intensities of these circuits are not to be calculated from the law of Ohm, as the chemical processes remain the same. In support of the view that in circuits where the polarization has ceased, the numerator of the formula of Ohm is dependent on the nature of the negative metal, I have been unable to find any certain observations. In order to set the current of polarization quickly aside, it is necessary to increase as much as possible the density of the current on the polarized plate, partly through the introduction of cells with a constant intensity, and partly by diminishing the surface of the plate. In the experiments of Lenz and Saweljev*, which bear upon this point, the constancy of the current was, according to their own statement, not attained; the electromotive forces calculated from their observations contain therefore those of the currents of polarization also. They found for zinc and copper in sulphuric acid 0·51, for zinc and iron 0·76, for zinc and mercury 0·90.

I may remark, in conclusion, that to prove the equality of the heat developed chemically and electrically, an experiment has been made by Joule†. His method of experiment is however open to many objections‡. He assumes, for example, for the tangent compass, the law of tangents as correct up to the highest degrees; he did not work with constant currents, but calculated the intensity from the mean of the first and last deflections; he also assumes the constancy of the electromotive force, the resistance of the cells and the gas development. Hess has already drawn attention to the divergence of his quantitative determinations of heat from numbers found by others. In a notice in the *Comptes Rendus*, 1843, No. 16, E. Becquerel is said to have corroborated the same law empirically.

We have above seen ourselves necessitated to refer the idea of a contact-force to simple forces of attraction and repulsion, in order to bring it into coincidence with our principle. Let us now endeavour to refer the electric motions between metal and fluid to the same cause. Let us imagine the particles of the

* Bull. de la Classe Phys. Math. de l'Acad. d. Scienc. de St. Pétersburg, t. v. p. 1; and Pogg. *Ann.* lvii. 497.

† Phil. Mag. 1841, vol. xix. p. 275; and 1843, vol. xx. p. 204.

‡ See note to page 131.—Ed.

compound atom of a fluid endowed with different forces of attraction for the electricities, and thus differently electric. When these portions of the atoms are separated at the metallic electrodes, each atom, according to the law of electrolysis, yields up a quantity of $\pm E$, which is independent of the electromotive forces. We can therefore imagine that in the chemical combination itself the atoms are combined with equivalents of $\pm E$, which follow the same laws as the stochiometric equivalents of the ponderable substances in different combinations. If now two different metals are immersed in a fluid, without a chemical process taking place, the positive components will be attracted by the negative metal, and the negative components by the positive metal. The consequence will be an altered direction and distribution of the different electric fluid particles, the recurrence of which we recognise in a current of polarization. The moving force of this current would be the electric difference of the metals, and to this must therefore the intensity at the commencement be proportional; its duration must, the intensities being equal, be proportional to the number of the atoms which spread themselves over the plate, and consequently to the surface of the latter. In the currents which are accompanied by chemical decomposition a permanent equilibrium between the fluid particles and the metals is not attained, because the positively charged surface of the metal is continually removed, being itself converted into a portion of the fluid, and hence a perpetual renewal of the charge must take place behind it. Every atom of the positive metal which, united to an equivalent of positive electricity, enters into the solution, and for which an atom of the negative component is separated neutrally, causes an acceleration of the motion once commenced, whenever the attractive force of the first atom for the $+E$, denoted by a_z, is greater than that of the latter, a_c. The velocity would in this way increase to an unlimited extent, did not the loss of *vis viva* by the development of heat increase also at the same time. It will therefore merely increase until this loss $I^2R\,dt$ is equal to the consumption of tensions $I(a_z - a_c)$, or until

$$I = \frac{a_z - a_c}{R}.$$

I believe that in this division of galvanic currents into those which occasion polarization and those which give rise to chemical decomposition, as required by the conservation of force, is the only means whereby the difficulties of the theory of contact and of the chemical theory will be alike avoided.

Thermo-electric currents.—In these currents we must seek the origin of force in the actions discovered by Peltier at the place of contact, by which a current opposed to the given one is developed. Let us suppose the case of a constant hydro-electric current into the conducting wire of which a piece of another metal is soldered, the temperatures of the places of union being t' and t'', the electric current will then, during the element of time dt, generate in the entire conduction the heat I^2Rdt; besides this, at one of the points where the metals are soldered together, the quantity $q_{\prime}dt$ will be developed, and at the other the quantity $q_{\prime\prime}dt$ absorbed. Let the electromotive force of the entire circuit be A, hence $AIdt$ the heat to be generated chemically, it then follows from the law of the conservation of force,

$$AI = I^2R + q_{\prime} - q_{\prime\prime}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Let the electromotive force of the thermo-circuit be B_t, when one of the soldered junctions possesses the temperature t, and the other any constant temperature whatever, for example 0°; then, for the entire circuit, we have

$$I = \frac{A - B_{t_{\prime}} - B_{t_{\prime\prime}}}{R}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

When $t_{\prime} = t_{\prime\prime}$, we have

$$I = \frac{A}{R}.$$

This set in equation (1) gives

$$q_{\prime} = q_{\prime\prime},$$

that is, when the temperatures of the places of soldering are both the same and the intensity of the current constant, the heat developed and that absorbed must be equal, independently of the cross section. If we assumed that the process is the same in every point of the cross section, it would follow that the heat developed in equal spaces of different cross sections is proportional to the density of the current, and from this again, that the quantities generated by different currents in the whole of the transverse sections are directly proportional to the intensity of the current.

When the solderings are of different temperatures, it follows from equations (1) and (2), that

$$(B_{t_{\prime}}-B_{t_{\prime\prime}})I=q_{\prime}-q_{\prime\prime},$$

that is to say, with the same intensity of current both the force which generates and which absorbs the heat increases with the temperature, in the same proportion as the electromotive force.

I am thus far unacquainted with any quantitative experiments with which either of the inferences might be compared.

VI. *Force-equivalent of Magnetism and Electro-magnetism.*

Magnetism.—Through the attractive and repulsive forces which a magnet exerts upon other magnets, or upon soft iron, it is capable of generating a certain *vis viva.* As the phænomena of magnetic attraction may be completely deduced from the assumption of two fluids which attract or repel in the inverse ratio of the square of the distance, it follows, without going further than the deduction made at the commencement of this memoir, that in the motions of magnetic bodies the conservation of force must take place. For the sake of the following theory of induction, we must consider a little more closely the laws of these motions.

1. Let $m_{\prime}$ and $m_{\prime\prime}$ be two magnetic elements, referred to a unit which, at the distance 1, repels an equal quantity of a similar magnetism with the force 1. Let the opposed magnetisms be distinguished by opposite signs, and let r be the distance between $m_{\prime}$ and $m_{\prime\prime}$, the intensity of their central force is

$$\phi=-\frac{m_{\prime}m_{\prime\prime}}{r^2};$$

the gain of *vis viva* during the passage from an infinite distance to r is $-\frac{m_{\prime}m_{\prime\prime}}{r}$.

2. Let us call this quantity the potential of the two elements, and extending the term potential to magnetic bodies as in the case of electricity, we obtain the gain in *vis viva* during the motion of two bodies whose magnetism does not change, for instance of steel magnets, when we subtract from the potential at the end of the motion its value at the commencement of the motion. The gain of *vis viva* during the motion of magnetic

bodies liable to be changed by induction, will, on the contrary, as in the case of electricity, be measured by the alterations of the sum

$$V+\frac{1}{2}(W_a+W_b),$$

where V denotes the potential of the bodies towards each other, W_a and W_b those of the bodies upon themselves. If B be an unchangeable steel magnet, the approximation of a body with changeable magnetism generates a *vis viva* equal to the increase of the sum $V+\frac{1}{2}W_a$.

3. It is known that the exterior action of a magnet can always be represented by a certain distribution of its fluids over its surface; we can therefore for the potential of the magnet, substitute the potentials of such surfaces; we then find, as in the case of conducting electric surfaces, for a perfectly soft mass of iron A, which is magnetized by a magnet B, the gain C in *vis viva* for the unit of quantity of the positive magnetism, during the passage from the surface of the iron to an infinite distance, given by the equation

$$-QC=V+W_a.$$

Now as every magnet contains equal quantities of north and south magnetism, hence Q in each $=0$, it follows for such a piece of iron, or for a piece of steel of the same form, position, and distribution of magnetism, whose magnetism therefore is completely bound by the magnet B, that

$$V=-W_a.$$

4. But V is the *vis viva* generated by a steel magnet during its approximation until its magnetism is completely bound; according to this equation it must be the same no matter what may be the magnet to which it approaches, provided always that the approximation is continued till the magnetism is completely bound, for W_a remains always the same. The *vis viva* of a piece of iron, on the contrary, which has been approximated till the same distribution is effected, is, as shown above,

$$V+\frac{1}{2}W=-\frac{1}{2}W,$$

hence only half as great as that of the already magnetized mass. It is to be remembered that W is in itself negative, hence $-\frac{1}{2}W$ always positive.

If a piece of unmagnetized steel be caused to approach the influencing magnet, supposing it, when removed, to retain the magnetism imparted to it, $-\frac{1}{2}W$ will then be lost in mechanical force, and for this the magnet thus created is in a condition to produce $-\frac{1}{2}W$ more than the piece of unmagnetized steel.

Electro-magnetism.—Electro-dynamic phænomena have been referred by Ampère to attractive and repulsive forces exerted by the elements of a current, the intensity of which depends only upon the direction and velocity of the current. His deduction does not comprehend the phænomena of induction; the latter, together with the electro-dynamic, have been referred by W. Weber to the attractive and repulsive forces of the electric fluids themselves, the intensity of which depends on the velocity of approximation or of removal, and the increase of this velocity. Up to the present time no hypothesis has been established by which these phænomena could be referred to constant central forces. The laws of induced currents have been developed by Neumann*, by extending the law of Lenz for entire currents to the ultimate particles of the same, and these laws coincide in the case of closed currents with the developments of M. Weber. In like manner the laws of Ampère and Weber for the electro-dynamic actions of closed currents coincide with Grassmann's deduction of the same from rotatory forces†. Experience gives us no further intelligence; because thus far experiment has been resorted to only in the cases of closed or nearly closed currents. We will therefore confine the application of our principle to closed currents, and show that from it the same laws follow.

It has been already shown by Ampère that the electro-dynamic actions of a closed current can be always represented by a certain distribution of the magnetic fluids on a surface which is bounded by the current. Neumann has therefore extended the idea of the potential to closed currents, by setting for a current the potential of such a surface.

5. When a magnet moves under the influence of a current, the *vis viva* gained thereby must be furnished by the tensions consumed in the current. During the portion of time *dt*,

* Poggendorff's *Annalen*, lxvii. 31.

† *Ibid.* lxiv. 1.

according to the notation before made use of, these are, $\mathrm{AI}dt$ in units of heat, or $a\mathrm{AI}dt$ in mechanical units, where a is the mechanical equivalent of the unit of heat. The *vis viva* generated in the path of the current is $a\mathrm{I}^2\mathrm{R}dt$, that gained by the magnet is $\mathrm{I}\frac{d\mathrm{V}}{dt}dt$, where V represents its potential towards the conductor through which the unit of current passes. Hence

$$a\mathrm{AI}dt = a\mathrm{I}^2\mathrm{R}dt + \mathrm{I}\frac{d\mathrm{V}}{dt}.dt,$$

consequently

$$\mathrm{I} = \frac{\mathrm{A} - \frac{1}{a}\cdot\frac{d\mathrm{V}}{dt}}{\mathrm{R}}.$$

We can distinguish the quantity $\frac{1}{a}\frac{d\mathrm{V}}{dt}$ as a new electromotive force, that of the induced current. It always acts against that which moves the magnet in the direction which it follows, or which would increase its velocity. As this force is independent of the intensity of the current, it must remain the same, when before the motion of the magnet no current existed.

If the intensity be changeable, the whole induced current during a certain time is

$$\int \mathrm{I}dt = -\frac{1}{a\mathrm{R}}\int\frac{d\mathrm{V}}{dt}dt = \frac{1}{a}\frac{(\mathrm{V}_{\prime} - \mathrm{V}_{\prime\prime})}{\mathrm{R}},$$

where $\mathrm{V}_{\prime}$ denotes the potential at the beginning, and $\mathrm{V}_{\prime\prime}$ at the end of the motion. If the magnet comes from a very great distance, we have

$$\int \mathrm{I}dt = -\frac{\frac{1}{a}\mathrm{V}_{\prime\prime}}{\mathrm{R}},$$

independent of the route or the velocity of the magnet.

We can express the law thus:—The entire electromotive force of the induced current, generated by a change of position of a magnet relative to a closed conductor, is equal to the change which thereby takes place in the potential of the magnet towards the conductor, when the latter is traversed by the current $-\frac{1}{a}$. The unit of the electromotive force is here regarded as that by which the arbitrary unit of current is generated in the unit of resistance, the latter being that in which the above unit of current developes the unit of heat in the unit of time. The same law is deduced by Neumann, l. *c.* § 9, only instead of $\frac{1}{a}$ he has an undetermined constant ϵ.

6. When a magnet moves under the influence of a conductor, towards which its potential for the unit of current is ϕ, and of a piece of iron magnetized by this conductor, towards which its potential for the magnetism excited by the unit of current is χ, we have then, as before,

$$a\mathrm{AI}=a\mathrm{I}^2\mathrm{R}+\mathrm{I}\frac{d\phi}{dt}+\mathrm{I}\frac{d\chi}{dt};$$

hence

$$\mathrm{I}=\frac{\mathrm{A}-\frac{1}{a}\left(\frac{d\phi}{dt}+\frac{d\chi}{dt}\right)}{\mathrm{R}}.$$

The electromotive force of the induced current due to the presence of the piece of iron is therefore

$$-\frac{1}{a}\frac{d\chi}{dt}.$$

If in the electro-magnet the same distribution of the magnetism be effected by the current n, as by the approximated magnet, then, in accordance with what has been stated in No. 4, its potential $n\chi$ towards the magnet must be equal to its potential towards the conducting wire $n\mathrm{V}$, when V denotes the same for the unit of current: therefore $\chi=\mathrm{V}$. Hence when an induced current is excited by means of the magnetization of the piece of iron by the magnet, the electromotive force is

$$-\frac{1}{a}\frac{d\chi}{dt}=-\frac{1}{a}\frac{d\mathrm{V}}{dt},$$

and, as in No. 7 we have the whole current

$$\int\mathrm{I}dt=\frac{\frac{1}{a}(\mathrm{V}_{\prime}-\mathrm{V}_{\prime\prime})}{\mathrm{R}},$$

where $\mathrm{V}_{\prime}$ and $\mathrm{V}_{\prime\prime}$ are the potentials of the magnetized iron towards the conducting wire before and after magnetization. Neumann deduces this law from its analogy with the foregoing case.

7. When an electro-magnet has become magnetic under the influence of a current, through the induced current heat is lost;

if the iron be soft the same induced current will proceed in the opposite direction when the circuit is broken, and the heat will be gained again. If it be a piece of steel which retains its magnetism, the heat is permanently lost, and in place of it we gain mechanical magnetic force equal, as shown in No 4, to half the potential of the said magnet when the binding is complete. From analogy with the foregoing cases it does not appear to be improbable that, as Neumann has concluded, the electromotive force corresponds to its entire potential, and that a portion of the motion of the magnetic fluids on account of their velocity is lost as heat, which is gained in the magnet.

8. If two closed conductors of currents be moved towards each other, the intensity of the current may be changed in each. Let V be their potential towards each other for the unit of current, we must then, for the same reason as in the former case, have

$$A_{,}I_{,}+A_{,,}I_{,,}=I_{,}^{2}R_{,}+I_{,,}^{2}R_{,,}+\frac{1}{a}I_{,}I_{,,}\frac{dV}{dt}.$$

If the intensity of the current in the conductor $R_{,,}$ be much less than that in $R_{,}$, so that the electromotive force of induction which is excited by $R_{,,}$ in $R_{,}$ vanishes in comparison to the force $A_{,}$, and we can set $I=\frac{A_{,}}{R_{,}}$, we obtain from the equation

$$I_{,,}=\frac{A_{,,}-\frac{1}{a}I_{,}\frac{dV}{dt}}{R_{,,}}.$$

The electromotive force of induction is therefore the same that a magnet would generate which possesses the same electrodynamic force as the inducing current. This law has been proved experimentally by W. Weber*.

If, on the contrary, the intensity in $R_{,}$ be a vanishing quantity compared to that in $R_{,,}$, we find

$$I=\frac{A_{,}-\frac{1}{a}I_{,,}\frac{dV}{dt}}{R_{,}}.$$

The electromotive forces of the conductors upon each other are therefore equal, when the intensities of the currents are equal, whatever may be the form of the conductor.

Here again the total force of induction, which, during a certain

* Electro-dynamische Massbestimmungen, p. 71-75. See also Scientific Memoirs, Part XX. p. 489.

motion of the conductors towards each other, furnishes a current which itself is unchanged by the induction, is equal to the change of its potential towards the other conductor which is traversed by $-\frac{1}{a}$. In this form the law is deduced by Neumann from the analogy of the magnetic and electro-dynamic forces, l. *c.* § 10, and he extends it also to the case where the induction in motionless conductors is effected by the strengthening or weakening of the current. W. Weber shows the coincidence of his assumption, with regard to the electro-dynamic forces, with these theorems, l. *c.* p. 147–153. From the law of the conservation of forces we do not obtain any determination for this case; by the reaction of the induced upon the inducing current a weakening of the latter must occur, which corresponds to a loss of heat equivalent to that gained in the induced current. In the action of the current upon itself, the same relation must exist between the weakening of the current at the commencement, and the extra current. No further consequences can however be deduced here, inasmuch as the form of the augmentation of the currents is not known, and besides this the law of Ohm is not immediately applicable.

Of known natural processes those of organic existences are still to be considered. In plants the processes are chiefly chemical, and besides these a slight development of heat takes place, at least in some: but the principal fact is, that a vast quantity of chemical tensions is here stored up, the equivalent of which we again obtain as heat by the combustion of the plants. The only *vis viva* which we know to be absorbed in the accomplishment of this is that of the chemical solar rays; we are, however, totally at a loss for the means of comparing the force-equivalents which are thereby lost and gained. Animals present some points in this respect which we can lay hold of. These take in oxygen and the complicated oxidizable combinations which are generated by plants, and give back the same, for the most part burnt, as carbonic acid and water, but in part reduced to simpler combinations; hence they consume a certain quantity of chemical tensions, and generate in their place heat and mechanical force. As the latter compared with the quantity of heat represents but a small quantity of work, the question of the conservation of force is reduced to this, whether the combustion and metamorphosis of the substances which serve as nutriment generate a quantity of heat equal to that given out by animals. According to the experiments of Dulong and Despretz, this question can be approximately answered in the affirmative.

In conclusion I must refer to some remarks of Matteucci's which have been directed against the views advocated in this memoir, and which appear in the *Bib. Univ. de Genève*, No. 16, 1847, 15 May, p. 375. He proceeds from the proposition, that according to the above views a chemical process could not generate so much heat where it at the same time developes electricity, magnetism, or light, as when this is not the case. He takes pains to show by a series of measurements which he adduces, that zinc, during its solution in sulphuric acid, generates just as much heat where the solution is effected directly by chemical affinity as when it forms a circuit with platinum; and that an electric current produces just as much chemical and thermic action while it deflects a magnet as when no such deflection is produced. That Matteucci regards these facts as objections, is due to his total miscomprehension of the views which he undertakes to refute, which will be at once evident from a consideration of our statement of the subject. He then brings forward two calorimetric experiments on the heat which is developed by the combination of caustic baryta with concentrated or dilute sulphuric acid, and on that generated by the same electric current in a wire immersed in gases of different cooling capacities, whereby the above mass and the wire were sometimes glowing and sometimes not. He finds the quantity of heat in the former cases not less than in the latter. When we, however, reflect upon the incompleteness of our calorimetric arrangements, it will not appear extraordinary that differences of cooling through radiation, which are due to the fact that this radiation, according to its luminous or non-luminous nature, passes with less or more difficulty through the surrounding diathermanous bodies, escape observation. In the first experiment of Matteucci the union of the baryta with sulphuric acid was effected in a non-diathermanous leaden vessel, where the luminous rays were completely prevented from escaping outwards. The imperfections of Matteucci's methods in carrying out these measurements need not be further dwelt upon.

By what has been laid down in the foregoing pages, I believe I have proved that the law in question does not contradict any known fact in natural science, but in a great number of cases is, on the contrary, corroborated in a striking manner. I have endeavoured to state in the most complete manner possible, the inferences which flow from a combination of the law with other known laws of natural phænomena, and which still await their experimental proof. The object of this investigation was to lay before physicists as fully as possible the theoretic and practical importance of a law whose complete corroboration must be regarded as one of the principal problems of the natural philosophy of the future.

[J. T.]