THE INTERPRETATION OF QUANTUM MECHANICS: MANY WORLDS OR MANY WORDS?

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As cutting-edge experiments display ever more extreme forms of non-classical behavior, the prevailing view on the interpretation of quantum mechanics appears to be gradually changing. A (highly unscientific) poll taken at the 1997 UMBC quantum mechanics workshop gave the once alldominant Copenhagen interpretation less than half of the votes. The Many Worlds interpretation (MWI) scored second, comfortably ahead of the Consistent Histories and Bohm interpretations. It is argued that since all the above-mentioned approaches to nonrelativistic quantum mechanics give identical cookbook prescriptions for how to calculate things in practice, practical-minded experimentalists, who have traditionally adopted the "shut-up-and-calculate interpretation", typically show little interest in whether cozy classical concepts are in fact real in some untestable metaphysical sense or merely the way we subjectively perceive a mathematically simpler world where the Schrödinger equation describes everything — and that they are therefore becoming less bothered by a profusion of worlds than by a profusion of words.

Common objections to the MWI are discussed. It is argued that when environment-induced decoherence is taken into account, the experimental predictions of the MWI are identical to those of the Copenhagen interpretation except for an experiment involving a Byzantine form of "quantum suicide". This makes the choice between them purely a matter of taste, roughly equivalent to whether one believes mathematical language or human language to be more fundamental.

I. INTRODUCTION

At the quantum mechanics workshop to which these proceedings are dedicated, held in August 1997 at UMBC, the participants were polled as to their preferred interpretation of quantum mechanics. The results are shown in Table 1.

Interpretation	Votes
Copenhagen	13
Many Worlds	8
Bohm	4
Consistent Histories	4
Modified dynamics (GRW/DRM)	1
None of the above/undecided	18

Although the poll was highly informal and unscientific (several people voted more than once, many abstained, etc), it nonetheless indicated a rather striking shift in opinion compared to the old days when the Copenhagen interpretation reigned supreme. Perhaps most striking of all is that the Many Worlds interpretation (MWI), proposed by Everett in 1957 [1–3] but virtually unnoticed for about a decade [4,5], has survived 25 years of fierce criticism and occasional ridicule to become the number one challenger to the leading orthodoxy, ahead of the Bohm [6], Consistent Histories [7] and GRW [8] interpretations. Why has this happened? The purpose of the present paper is to briefly summarize the appeal of the MWI in the light of recent experimental and theoretical progress, and why much of the traditional criticism of it is being brushed aside.

II. THE MWI: WHAT IT IS AND WHAT IT ISN'T

Much of the old criticism of the MWI was based on confusion as to what it meant. Here we grant Everett the final say in how the MWI is defined, since he did after all invent it [1], and take it to consist of the following postulate alone:

• EVERETT POSTULATE:

All isolated systems evolve according to the Schrödinger equation $\frac{d}{dt}|\psi\rangle = -\frac{i}{\hbar}H|\psi\rangle$.

Although this postulate sounds rather innocent, it has far-reaching implications:

- 1. **Corollary 1:** the entire Universe evolves according to the Schrödinger equation, since it is by definition an isolated system.
- 2. Corollary 2: there can be no definite outcome of quantum measurements (wavefunction collapse), since this would violate the Everett postulate.

Because of corollary 1, "universally valid quantum mechanics" is often used as a synonym for the MWI. What is to be considered "classical" is therefore *not* specified axiomatically (put in by hand) in the MWI — rather, it can be derived from the Hamiltonian dynamics as described in Section III B, by computing decoherence rates.

How does corollary 2 follow? Consider a measurement of a spin 1/2 system (a silver atom, say) where the states "up" and "down" along the z axis are denoted $|\uparrow\rangle$ and $|\downarrow\rangle$. Assuming that the observer will get happy if she measures spin up, we let $|\ddot{-}\rangle$, $|\ddot{-}\rangle$ and $|\ddot{-}\rangle$ denote the states of the observer before the measurement, after perceiving spin up and after perceiving spin down, respectively. If the measurement is to be described by a unitary Schrödinger time evolution operator $U = e^{-iH\tau/\hbar}$ applied to the total system, then U must clearly satisfy

$$U|\uparrow\rangle\otimes|\ddot{}\rangle = |\uparrow\rangle\otimes|\ddot{}\rangle \quad \text{and} \quad U|\downarrow\rangle\otimes|\ddot{}\rangle = |\downarrow\rangle\otimes|\dot{}\rangle.$$
(1)

Therefore if the atom is originally in a superposition $\alpha |\uparrow\rangle + \beta |\downarrow\rangle$, then the Everett postulate implies that the state resulting after the observer has interacted with the atom is

$$U(\alpha|\uparrow\rangle + \beta|\downarrow\rangle) \otimes |\vdots\rangle = \alpha|\uparrow\rangle \otimes |\vdots\rangle + \beta|\downarrow\rangle \otimes |\div\rangle.$$
(2)

In other words, the outcome is not $|\uparrow\rangle \otimes | \Rightarrow\rangle$ or $|\downarrow\rangle \otimes | \Rightarrow\rangle$ with some probabilities, merely these two states in superposition. Very few physicists have actually read Everett's book (printed in [2]), which has lead to a common misconception that it contains a second postulate along the following lines:

• What Everett does **NOT** postulate: At certain magic instances, the the world undergoes some sort of metaphysical "split" into two branches that subsequently never interact.

This is not only a misrepresentation of the MWI, but also inconsistent with the Everett postulate, since the subsequent time evolution could in principle make the two terms in equation (2) interfere. According to the MWI, there is, was and always will be only one wavefunction, and only decoherence calculations, not postulates, can tell us when it is a good approximation to treat two terms as non-interacting.

III. COMMON CRITICISM OF THE MWI

A. "It doesn't explain why we perceive randomness"

Everett's brilliant insight was that the MWI *does* explain why we perceive randomness even though the Schrödinger equation itself is competely causal. To avoid linguistic confusion, it is crucial that we distinguish between [9]

- the *outside view* of the world (the way a mathematical thinks of it, *i.e.*, as an evolving wavefunction), and
- the *inside view*, the way it is perceived from the subjective frog perspective of an observer in it.

 $| \vdots \rangle$ and $| = \rangle$ have by definition perceived two opposite measurement outcomes from their inside views, but share the same memory of being in the state $| \vdots \rangle$ moments earlier. Thus $| = \rangle$ describes an observer who remembers performing a spin measurement and observing the definite outcome $|\downarrow\rangle$. Suppose she measures the z-spin of n independent atoms that all have spin up in the x-direction initially, *i.e.*, $\alpha = \beta = 1/\sqrt{2}$. The final state corresponding to equation (2) will then contain 2^n terms of equal weight, a typical term corresponding to a seemingly random sequence of ups and downs, of the form

$$2^{-n/2}|\downarrow\downarrow\uparrow\uparrow\uparrow\uparrow\downarrow\downarrow\uparrow\cdots\rangle\otimes|\ddot{\neg}\ddot{\neg}\ddot{\neg}\ddot{\neg}\ddot{\neg}\ddot{\neg}\ddot{\cdots}\dot{\neg}\rangle.$$
(3)

Thus the perceived inside view of what happened according to an observer described by a *typical* element of the final superposition is a seemingly random sequence of ups and downs, behaving as if generated though a random process with probabilities $p = \alpha^2 = \beta^2 = 0.5$ for each outcome. This can be made more formal if we replace "∴" by "0", replace "∵" by "1", and place a decimal point in front of it all. Then the above observer state $|\ddot{\neg}\ddot{\neg}\ddot{\neg}\ddot{\neg}\ddot{\neg}\ddot{\neg}\ddot{\neg}\ddot{\neg}\ddot{\neg}\dot{\neg}\dot{\neg}\dot{\neg}\rangle = |.0010111001...\rangle$, and we see that in the limit $n \to \infty$, each observer state corresponds to a real number on the unit interval (written in binary). According to Borel's theorem on normal numbers [10,11], almost all (all except for a set of Borel measure zero) real numbers between zero and one have a fraction 0.5 of their decimals being "1", so in the same sense, almost all terms in our wavefunction describe observers that have perceived the conventional quantum probability rules to hold. It is in this sense that the MWI predicts apparent randomness from the inside viewpoint while maintaining strict causality from the outside viewpoint.* For a clear and pedagogical generalization to the general case with unequal probabilities, see [1,2].

B. "It doesn't explain why we don't perceive weird superpositions"

That's right! The Everett postulate doesn't! Since the state corresponding to a superposition of a pencil lying in two macroscopically different positions on a table-top is a perfectly permissible quantum state in the MWI, why do we never perceive such states? Indeed, if we were to balance a pencil exactly on its tip, it would by symmetry fall down in a superposition of all directions (a calculation shows that this takes about 30 seconds),

^{*}It is interesting to note that Borel's 1909 theorem made a strong impression on many mathematicians of the time, some of whom had viewed the entire probability concept with a certain suspicion, since they were now confronted with a theorem in the heart of classical mathematics which could be reinterpreted in terms of probabilities [11]. Borel would undoubtedly have been interested to know that his work showed the emergence of a probability-like concept "out of the blue" not only in in mathematics, but in physics as well.

thereby creating such a macrosuperposition state. The inability to answer this question was originally a serious weakness of the MWI, which can equivalently be phrased as follows: why is the position representation so special? Why do we perceive macroscopic objects in approximate eigenstates of the position operator \mathbf{r} and the momentum operator **p** but never in approximate eigenstates of other Hermitian operators such as $\mathbf{r} + \mathbf{p}$? The answer to this important question was provided by the realization that environment-induced decoherence rapidly destroys macrosuperpositions as far as the inside view is concerned, but this was explicitly pointed out only in the 70's [12] and 80's [13], more than a decade after Everett's original work. This elegant mechanism is now well-understood and rather uncontroversial [14], and the interested reader is referred to [15] and a recent book on decoherence [16] for details. Essentially, the position basis gets singled out by the dynamics because the field equations of physics are local in this basis, not in any other basis.

Historically, the collapse postulate was introduced to suppress the off-diagonal density matrix elements elements corresponding to strange macrosuperpositions (cf. [17]). However, many physicists have shared Gottfried's view that "the reduction [collapse] postulate is an ugly scar on what would be beautiful theory if it could be removed" [18], since it is not accompanied by any equation specifying when collapse occurs (when the Everett postulate is violated). The subsequent discovery of decoherence provided precisely such an explicit mechanism for suppression of off-diagonal elements, which is essentially indistinguishable from the effect of a postulated Copenhagen wavefunction collapse from an observational (inside) point of view (e.g. [19]). Since this eliminates arguably the main motivation for the collapse postulate, it may be a principal reason for the increasing popularity of the MWI.

C. "It's too weird for me"

The reader must choose between two tenable but diametrically opposite paradigms regarding physical reality and the status of mathematics:

- **PARADIGM 1:** The outside view (the mathematical structure) is physically real, and the inside view and all the human language we use to describe it is merely a useful approximation for describing our subjective perceptions.
- **PARADIGM 2:** The subjectively perceived inside view is physically real, and the outside view and all its mathematical language is merely a useful approximation.

What is more basic — the inside view or the outside view? What is more basic — human language or mathematical language? Note that in case 1, which might be termed the *Platonic paradigm*, all of physics is ultimately a mathematics problem, since an infinitely intelligent mathematician given the equations of the Universe could in principle *compute* the inside view, *i.e.*, compute what self-aware observers the Universe would contain, what they would perceive, and what language they would invent to describe their perceptions to one another. Thus in the Platonic paradigm, the axioms of an ultimate "Theory of Everything" would be purely mathematical axioms, since axioms or postulates in English regarding interpretation would be derivable and thus redundant. In paradigm 2, on the other hand, there can never be a "Theory of Everything", since one is ultimately just explaining certain verbal statements by other verbal statethis is known as the infinite regress problem ments -(e.g., [20]).

The reader who prefers the Platonic paradigm should find the MWI natural, whereas the reader leaning towards paradigm 2 probably prefers the Copenhagen interpretation. A person objecting that the MWI is "too weird" is essentially saying that the inside and outside views are extremely different, the latter being "weird", and therefore prefers paradigm 2. In the Platonic paradigm, there is of course no reason whatsoever to expect the inside view to resemble the outside view, so one expects the correct theory to seem weird. One reason why theorists are becoming increasingly positive to the MWI is probably that past theoretical breakthroughs have shown that the outside view really is very different from the inside view. For instance, a prevalent modern view of quantum field theory is (e.q., [21,22]) that the standard model is merely an effective theory, a lowenergy limit of a yet to be discovered theory that is even more removed from our cozy classical concepts (perhaps involving superstrings in 26 dimensions, say). General Relativity has already introduced quite a gap between the outside view (fields obeying covariant partial differential equations on a 4-dimensional manifold) and the inside view (where we always perceive spacetime as locally Minowski, and our perceptions depend not only on where we are but also on how fast we are moving).

One reason why experimentalists are becoming increasingly positive to the MWI is probably that they have recently produced so many "weird" (but perfectly repeatable) experimental results (Bell inequality violations with kilometer baselines [23], molecule interferometry [24], vorticity quantization in a macroscopically large amount of liquid Helium [25], etc.), and therefore simply accept that the world is a weirder place than we thought it was and get on with their calculations.

D. "Many words" objections

The questions addressed in Sections III A and III B are in the author's opinion quite profound, and were answered thanks to the ingenuity of Everett and the dis-

coverers of decoherence, respectively. However, there are also a number of questions/objections that in the author's opinion belong in the category "many words", being issues of semantics rather than physics. When discussing the MWI, it is of course within the context of the Platonic paradigm described above, paradigm 1, where equations are ultimately more fundamental than words. Since human language is merely something that certain observers have invented to describe their subjective perceptions, many words describe concepts that by necessity are just useful approximations (cf. [26]). We know that the classical concept of gas pressure is merely an approximation that breaks down if we consider atomic scales, and in the Platonic paradigm, we should not be surprised if we find that other traditional concepts (e.g.,that of physical probability, and indeed the entire notion of a classical world) also turn out to be merely convenient approximations.

As an example of a "many words" objection, let us consider the rather subtle claim that the MWI does not justify the use of the word "probability" [27]. When our observer is described by the state $| : \rangle$ before measuring her atom, there is no aspect of the measurement outcome of which she has epistemological uncertainty (lack of knowledge): she simply knows that with 100% certainty, she will end up in a superposition of $| :: \rangle$ and $| ;; \rangle$. After the measurement, there is still no epistemological uncertainty, since both $| :: \rangle$ and $| :: \rangle$ know what they have measured. For those who feel that the word probability should only be used when there is true lack of knowledge, probabilities can readily be introduced by performing the experiment while the observer is sleeping, and placing her bed in one of two identical rooms depending on the outcome [28]. On awakening, the observer described by either of the two states in the superposition can thus say that she is in the first room with 50% probability in the sense that she has lack of knowledge as to where she is. If there were 2^n identical rooms and n measurements dictated the room number in binary, then the observers in the final superposition could compute probabilities for observing specific numbers of zeroes and ones in their room number. Moreover, these could have been computed in advance of the experiment, used as gambling odds, etc., before the orthodox linguist would allow us to call them probabilities, which is why they are a useful concept regardless of what we call them.

Let us also consider a paper entitled "Against Many-Worlds Interpretations" by Kent [29,30]. Although most of its claims were subsequently shown to result from misconceptions [31] (as to the definition of the MWI, as to the mathematical distinction between "measure" [of a subset] and "norm" [of a vector], etc.), it also contained a number of objections in the "many words" category. In Section II.A, the author states that "one needs to define [...] the preferred basis [...] by an axiom." According to what preconceived notion is this necessary, since decoherence can determine the preferred basis dynamically? In the foreword to a 1997 version of this paper [30], it is not only suggested that MWI adherents "represent a relatively small minority" and "tend to be working in other areas of physics" (both in apparent contradiction to the above-mentioned poll), but also that they "tend to have non-standard views on the nature of scientific theories". In our terminology, this "objection" presumable reflects the obvious fact that MWI adherents subscribe to paradigm 1 rather than 2. Moreover, Galileo once held "non-standard" views on the epicycle theory of planetary motion.

A large number of other objections have been raised against the MWI, tacitly based on some variant of paradigm 2. The opinion of this author is that *if paradigm 1 is adopted*, then there are *no* outstanding problems with the MWI when decoherence is taken into account (as discussed in *e.g.* [16,19,32]).

IV. IS THE MWI TESTABLE?

A. The "shut-up-and-calculate" recipe

When comparing the contenders in Table 1, it is important to distinguish between their experimental predictions and their philosophical interpretation. When confronted with experimental questions, adherents of the first four will all agree on the following cookbook prescription for how to compute the right answer, which we will term the "shut-up-and-calculate"[†] recipe:

Use the Schrödinger equation in all your calculations. To compute the probability for what you personally will perceive in the end, simply convert to probabilities in the traditional way at the instant when you become mentally aware of the outcome. In practice, you can convert to probabilities much earlier, as soon as the superposition becomes "macroscopic", and you can determine when this occurs by a standard decoherence calculation.

The fifth contender (a dynamical reduction mechanism such as that proposed by Ghirardi, Rimini & Weber) is the only one in the table to prescribe a different calculational recipe, since it modifies the Heisenberg equation of motion $\dot{\rho} = -i[H, \rho]/\hbar$ by adding an extra term [8].

B. Quantum suicide

The fact that the four most popular contenters in Table 1 have given identical predictions for all experiments performed so far probably explains why practical-minded

[†]The author is indebted to Anupam Garg for this phrase.

physicists show so little interest in interpretational questions. Is there then *any* experiment that could distinguish between say the MWI and the Copenhagen interpretation using currently available technology? (Cf. [33,34].) The author can only think of one: a form of quantum suicide in a spirit similar to so-called quantum roulette. It requires quite a dedicated experimentalist, since it is amounts to an iterated and faster version of Schrödinger's cat experiment [35] — with you as the cat.

The apparatus is a "quantum gun" which each time its trigger is pulled measures the z-spin of a particle in the state $(|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2}$. It is connected to a machine gun that fires a single bullet if the result is "down" and merely makes an audible click if the result is "up". The details of the trigger mechanism are irrelevant (an experiment with photons and a half-silvered mirror would probably be cheaper to implement) as long as the timescale between the quantum bit generation and the actual firing is much shorter than that characteristic of human perception, say 10^{-2} seconds. The experimenter first places a sand bag in front of the gun and tells her assistant to pull the trigger ten times. All contenders in Table 1 agree that the "shutup-and-calculate" prescription applies here, and predict that she will hear a seemingly random sequence of shots and duds such as "bang-click-bang-bang-bang-click-clickbang-click-click." She now instructs her assistant to pull the trigger ten more times and places her head in front of the gun barrel. This time the shut-up-and-calculate recipe is inapplicable, since probabilities have no meaning for an observer in the dead state $|\stackrel{\times\times}{\frown}\rangle$, and the contenders will differ in their predictions. In interpretations where there is an explicit non-unitary collapse, she will be either dead or alive after the first trigger event, so she should expect to perceive perhaps a click or two (if she is moderately lucky), then "game over", nothing at all. In the MWI, on the other hand, the state after the first trigger event is

$$U \frac{1}{\sqrt{2}} \left(|\uparrow\rangle + |\downarrow\rangle \right) \otimes |\ddot{\downarrow}\rangle = \frac{1}{\sqrt{2}} \left(|\uparrow\rangle \otimes |\ddot{\downarrow}\rangle + |\downarrow\rangle \otimes |\overset{\times\times}{\frown}\rangle \right).$$
(4)

Since there is exactly one observer having perceptions both before and after the trigger event, and since it occurred too fast to notice, the MWI prediction is that $| " \rangle$ will hear "click" with 100% certainty. When her assistant has completed his unenviable assignment, she will have heard ten clicks, and concluded that collapse interpretations of quantum mechanics are ruled out at a confidence level of $1 - 0.5^n \approx 99.9\%$. If she wants to rule them out at "ten sigma", she need merely increase *n* by continuing the experiment a while longer. Occasionally, to verify that the apparatus is working, she can move her head away from the gun and suddenly hear it going off intermittently. Note, however, that almost all terms in the final superposition will have her assistant perceiving that he has killed his boss. Many physicists would undoubtedly rejoice if an omniscient genie appeared at their death bed, and as a reward for life-long curiosity granted them the answer to a physics question of their choice. But would they be as happy if the genie forbade them from telling anybody else? Perhaps the greatest irony of quantum mechanics is that if the MWI is correct, then the situation is quite analogous if once you feel ready to die, you repeatedly attempt quantum suicide: you *will* experimentally convince yourself that the MWI is correct, but you can never convince anyone else!

The author wishes to thank David Albert, Orly Alter, Geoffrey Chew, Angélica de Oliveira-Costa, Michael Gallis, Bill Poirier, Svend Erik Rugh, Marlan Scully, Robert Spekkens, Lev Vaidman, John Wheeler and Wojciech Zurek (some of whom disagree passionately with the opinions expressed in the present paper!) for thoughtprovoking and entertaining discussions. This work was supported by Hubble Fellowship #HF-01084.01-96A, awarded by the Space Telescope Science Institute, which is operated by AURA, Inc. under NASA contract NAS5-26555.

- [1] Everett, H. 1957, Rev. Mod. Phys, 29, 454
- [2] Everett, N. 1973, in *The Many-Worlds Interpretation of Quantum Mechanics*, ed. DeWitt, B. S. & Graham, N. (Princeton: Princeton Univ. Press)
- [3] Wheeler, J. A. 1957, Rev. Mod. Phys., 29, 463
- [4] Cooper, L. M. & van Vechten, D. 1969, Am. J. Phys, 37, 1212
- [5] DeWitt, B. S. 1971, Phys. Today, 23 (9), 30
- Bohm, D. & Hiley, B. J. 1993, The Undivided Universe: an Ontological Interpretation of Quantum Theory (London: Routledge)
- [7] Omnes, R. 1992, The Interpretation of Quantum Mechanics (Princeton: Princeton Univ. Press)
- [8] Ghirardi, G. C., Rimini, A. & Weber, T. 1986, Phys. Rev. D, 34, 470
- [9] Tegmark, M. 1997, preprint gr-qc/9704009
- [10] Borel, E. 1909, Rend. Circ. Mat. Paleremo, 26, 247
- [11] Chung, K. L. 1974, A Course in Probability Theory (New York: Academic)
- [12] Zeh, H. D. 1970, Found. Phys, 1, 69
- [13] Zurek, W. H. 1981, Phys. Rev. D, 24, 1516
 Zurek, W. H. 1982, Phys. Rev. D, 26, 1862
 Joos, E. & Zeh, H. D. 1985, Z. Phys. B, 59, 223
- [14] Zurek, W. H. 1991, Phys. Today, 44 (10), 36
- [15] Omnès, R. 1997, Phys. Rev. A, in press
- [16] Giulini, D., Joos, E., Kiefer, C., Kupsch, J., Stamatescu, I. O. & Zeh, H. D. 1996, Decoherence and the Appearance of a Classical World in Quantum Theory (Berlin: Springer)

- [17] von Neumann, J. 1932, Matematische Grundlagen der Quanten-Mechanik (Berlin: Springer)
- [18] Gottfried, K. 1989, contribution to Erice School "62 Years of Uncertainty", unpublished
- [19] Tegmark, M. 1993, Found. Phys. Lett, 6, 571
- [20] Nozick, R. 1981, *Philosophical Explanations* (Cambridge: Harvard Univ. Press)
- [21] Nielsen, H. B. 1983, Phil. Trans. Royal Soc. London, A310, 1983
- [22] Weinberg, S. 1995, The Quantum Theory of Fields (Cambridge: Cambridge Univ. Press)
- [23] Tapster, P. R., Rarity, J. G. & Owens, P. C. M. 1994, *Phys. Rev. Lett.*, **73**, 1923
- [24] Pritchard, D. et al. 1997, in these proceedings
- [25] Schwab, K.,Bruckner, N. & Packard, R. E. 1997, Nature, 386, 585
- [26] Page, D. N. A 1995, preprint gr-qc/9507025
- [27] Albert, D. 1997, private communication
- [28] Vaidman, L. 1996, quant-ph/9609006, Int. Stud. Phil. Sci., in press
- [29] Kent, A. 1990, Int. J. Mod. Phys A, 5, 1745
- [30] Kent, A. 1997, preprint gr-qc/9703089
- [31] Sakaguchi, T. 1997, preprint quant-ph/9704039
- [32] Zeh, H. D. 1993, Phys. Lett. A, 172, 189
- [33] Deutsch, D. 1986, in *Quantum Concepts of Space and Time*, ed. Penrose, R. & Isham, C. J. (Oxford: Caledonia)
 [34] Lockwood, M. 1989, *Mind*, *Brain & the Quantum* (New
- York: Blackwell)
- [35] Schrödinger, E. 1935, Naturwissenschaften, 23, 807

NOTE:

This paper and a number of related ones are available online at h t t p://www.sns.ias.edu/~max/everett.html