hibiting the simplest conceivable form of regularity. Among these, next to the straight line and the circle, the most important were the ellipse and the hyperbola. We see the last two embodied—at least very nearly so—in the orbits of the heavenly bodies.

It seems that the human mind has first to construct forms independently before we can find them in things. Kepler's marvelous achievement is a particularly fine example of the truth that knowledge cannot spring from experience alone but only from the comparison of the inventions of the intellect with observed fact.

MAXWELL'S INFLUENCE ON THE EVOLUTION OF THE IDEA OF PHYSICAL REALITY

On the one hundredth anniversary of Maxwell's birth. Published, 1931, in James Clerk Maxwell: A Commemoration Volume, *Cambridge University Press.*

The belief in an external world independent of the perceiving subject is the basis of all natural science. Since, however, sense perception only gives information of this external world or of "physical reality" indirectly, we can only grasp the latter by speculative means. It follows from this that our notions of physical reality can never be final. We must always be ready to change these notions—that is to say, the axiomatic basis of physics—in order to do justice to perceived facts in the most perfect way logically. Actually a glance at the development of physics shows that it has undergone far-reaching changes in the course of time.

The greatest change in the axiomatic basis of physics—in other words, of our conception of the structure of reality—since Newton laid the foundation of theoretical physics was brought about by Faraday's and Maxwell's work on electromagnetic phenomena. We will try in what follows to make this clearer, keeping both earlier and later developments in sight.

According to Newton's system, physical reality is characterized by the concepts of space, time, material point, and force (reciprocal action of material points). Physical events, in Newton's view, are to be regarded as the motions, governed by fixed laws, of material points in space. The material point is our only mode of representing reality when dealing with changes taking place in it, the solitary representative of the real, in so far as the real is capable of change. Perceptible bodies are obviously responsible for the concept of the material point; people conceived it as an analogue of mobile bodies, stripping these of the characteristics of extension, form, orientation in space, and all "inward" qualities, leaving only inertia and translation and adding the concept of force. The material bodies, which had led psychologically to our formation of the concept of the "material point," had now themselves to be regarded as systems of material points. It should be noted that this theoretical scheme is in essence an atomistic and mechanistic one. All happenings were to be interpreted purely mechanically—that is to say, simply as motions of material points according to Newton's law of motion.

The most unsatisfactory side of this system (apart from the difficulties involved in the concept of "absolute space" which have been raised once more quite recently) lay in its description of light, which Newton also conceived, in accordance with his system, as composed of material points. Even at that time the question, What in that case becomes of the material points of which light is composed, when the light is absorbed?, was already a burning one. Moreover, it is unsatisfactory in any case to introduce into the discussion material points of quite a different sort, which had to be postulated for the purpose of representing ponderable matter and light respectively. Later on, electrical corpuscles were added to these, making a third kind, again with completely different characteristics. It was, further, a fundamental weakness that the forces of reciprocal action, by which events are determined, had to be assumed hypothetically in a perfectly arbitrary way. Yet this conception of the real accomplished much: how came it that people felt themselves impelled to forsake it?

In order to put his system into mathematical form at all, Newton had to devise the concept of differential quotients and propound the laws of motion in the form of total differential equations—perhaps the greatest advance in thought that a single individual was ever privileged to make. Partial differential equations were not necessary for this purpose, nor did Newton make any systematic use of them; but they were necessary for the formulation of the mechanics of deformable bodies; this is connected with the fact that in these problems the question of *how* bodies are supposed to be constructed out of material points was of no importance to begin with.

Thus the partial differential equation entered theoretical physics as a handmaid, but has gradually become mistress. This began in the nineteenth century when the wave-theory of light established itself under the pressure of observed fact. Light in empty space was explained as a matter of vibrations of the ether, and it seemed idle at that stage, of course, to look upon the latter as a conglomeration of material points. Here for the first time the partial differential equation appeared as the natural expression of the primary realities of physics. In a particular department of theoreticai physics the continuous field thus appeared side by side with the material point as the representative of physical reality. This dualism remains even today, disturbing as it must be to every orderly mind.

If the idea of physical reality had ceased to be purely atomic, it still remained for the time being purely *mechanistic;* people still tried to explain all events as the motion of inert masses; indeed no other way of looking at things seemed conceivable. Then came the great change, which will be associated for all time with the names of Faraday, Maxwell, and Hertz. The lion's share in this revolution fell to Maxwell. He showed that the whole of what was then known about light and electromagnetic phenomena was expressed in his well-known double system of differential equations, in which the electric and the magnetic fields appear as the dependent variables. Maxwell did, indeed, try to explain, or justify, these equations by the intellectual construction of a mechanical model.

But he made use of several such constructions at the same time and took none of them really seriously, so that the equations alone appeared as the essential thing and the field strengths as the ultimate entities, not to be reduced to anything else. By

the turn of the century the conception of the electromagnetic field as an ultimate entity had been generally accepted and serious thinkers had abandoned the belief in the justification, or the possibility, of a mechanical explanation of Maxwell's equations. Before long they were, on the contrary, actually trying to explain material points and their inertia on field theory lines with the help of Maxwell's theory, an attempt which did not, however, meet with complete success.

Neglecting the important *individual* results which Maxwell's life-work produced in important departments of physics, and concentrating on the changes wrought by him in our conception of the nature of physical reality, we may say this: before Maxwell people conceived of physical reality—in so far as it is supposed to represent events in nature—as material points, whose changes consist exclusively of motions, which are subject to total differential equations. After Maxwell they conceived physical reality as represented by continuous fields, not mechanically explicable, which are subject to partial differential equations. This change in the conception of reality is the most profound and fruitful one that has come to physics since Newton; but it has at the same time to be admitted that the program has by no means been completely carried out yet. The successful systems of physics which have been evolved since rather represent compromises between these two schemes, which for that very reason bear a provisional, logically incomplete character, although they may have achieved great advances in certain particulars.

The first of these that calls for mention is Lorentz's theory of electrons, in which the field and the electrical corpuscles appear side by side as elements of equal value for the comprehension of reality. Next come the special and general theories of relativity, which, though based entirely on ideas connected with the field-theory, have so far been unable to avoid the independent introduction of material points and total differential equations.

The last and most successful creation of theoretical physics, namely quantum-mechanics, differs fundamentally from both the schemes which we will for the sake of brevity call the Newtonian and the Maxwellian. For the quantities which figure in its laws make no claim to describe physical reality itself, but only the *probabilities* of the occurrence of a physical reality that we have in view. Dirac, to whom, in my opinion, we owe the most perfect exposition, logically, of this theory, rightly points out that it would probably be difficult, for example, to give a theoretical description of a photon such as would give enough information to enable one to decide whether it will pass a polarizer placed (obliquely) in its way or not.

I am still inclined to the view that physicists will not in the long run content themselves with that sort of indirect description of the real, even if the theory can eventually be adapted to the postulate of general relativity in a satisfactory manner. We shall then, I feel sure, have to return to the attempt to carry out the program which may be described properly as the Maxwellian—namely, the description of physical reality in terms of fields which satisfy partial differential equations without singularities.

ON THE METHOD OF THEORETICAL PHYSICS

The Herbert Spencer lecture, delivered at Oxford, June 10, 1933. Published in Mein Weltbild, *Amsterdam: Querido Verlag, 1934.*

If you want to find out anything from the theoretical physicists about the methods they use, I advise you to stick closely to one principle: don't listen to their words, fix your attention on their deeds. To him who is a discoverer in this field, the products of his imagination appear so necessary and natural that he regards them, and would like to have them regarded by others, not as creations of thought but as given realities.

These words sound like an invitation to you to walk out of this lecture. You will say to yourselves, the fellow's a working physicist himself and ought therefore to leave all questions of the structure of theoretical science to the epistemologists.

Against such criticism I can defend myself from the personal point of view by assuring you that it is not at my own instance but at the kind invitation of others that I have mounted this rostrum, which serves to commemorate a man who fought hard all his life for the unity of knowledge. Objectively, however, my enterprise can be justified on the ground that it may, after

all, be of interest to know how one who has spent a lifetime in striving with all his might to clear up and rectify its fundamentals looks upon his own branch of science. The way in which he regards its past and present may depend too much on what he hopes for the future and aims at in the present; but that is the inevitable fate of anybody who has occupied himself intensively with a world of ideas. The same thing happens to him as to the historian, who in the same way, even though perhaps unconsciously, groups actual events round ideals which he has formed for himself on the subject of human society.

Let us now cast an eye over the development of the theoretical system, paying special attention to the relations between the content of the theory and the totality of empirical fact. We are concerned with the eternal antithesis between the two inseparable components of our knowledge, the empirical and the rational, in our department.

We reverence ancient Greece as the cradle of western science Here for the first time the world witnessed the miracle of a logical system which proceeded from step to step with such precision that every single one of its propositions was absolutely indubitable—I refer to Euclid's geometry. This admirable triumph of reasoning gave the human intellect the necessary confidence in itself for its subsequent achievements. If Euclid failed to kindle your youthful enthusiasm, then you were not born to be a scientific thinker.

But before mankind could be ripe for a science which takes in the whole of reality, a second fundamental truth was needed, which only became common property among philosophers with the advent of Kepler and Galileo. Pure logical thinking cannot yield us any knowledge of the empirical world; all knowledge of reality starts from experience and ends in it. Propositions arrived at by purely logical means are completely empty as regards reality. Because Galileo saw this, and particularly because he drummed it into the scientific world, he is the father of modern physics—indeed, of modern science altogether.

If, then, experience is the alpha and the omega of all our knowledge of reality, what is the function of pure reason in science?