5

THE QUANTUM THEORY
OF RADIATION

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Introduction

In the attempts to give a theoretical interpretation of the mechanism of interaction between radiation and matter, two apparently contradictory aspects of this mechanism have been disclosed. On the one hand, the phenomena of interference, on which the action of all optical instruments essentially depends, claim an aspect of continuity of the same character as that involved in the wave theory of light, especially developed on the basis of the laws of classical electrodynamics. On the other hand, the exchange of energy and momentum between matter and radiation, on which the observation of optical phenomena ultimately depends, claims essentially discontinuous features. These have even led to the introduction of the theory of light-quanta, which in its most extreme form denies the wave constitution of light. At the present state of science it does not seem possible to avoid the formal character of the quantum theory which is shown by the fact that the interpretation of atomic phenomena does not involve a description of the mechanism of the discontinuous processes, which in the quantum theory of spectra are designated as transitions between stationary states of the atom. On the correspondence principle it seems nevertheless possible, as it will be attempted to show in this paper, to arrive at a consistent description of optical phenomena by connecting the discontinuous effects occurring in atoms with the continuous radiation field in a somewhat different manner from what is usually done. The essentially new assumption introduced in § 2 that the atom, even before a process of transition between two stationary states takes

place, is capable of communication with distant atoms through a virtual radiation field, is due to Slater.* Originally his endeavour was in this way to obtain a harmony between the physical pictures of the electrodynamical theory of light and the theory of light-quanta by coupling transitions of emission and absorption of communicating atoms together in pairs. It was pointed out by Kramers, however, that instead of suggesting an intimate coupling between these processes, the idea just mentioned leads rather to the assumption of a greater independence between transition processes in distant atoms than hitherto perceived. The present paper is the result of a mutual discussion between the authors concerning the possible importance of these assumptions for the elaboration of the quantum theory, and may in various respects be considered as a supplement to the first part of a recent treatise by Bohr, dealing with the principles of the quantum theory, in which several of the problems dealt with here are treated more fully.†

1. Principles of the quantum theory

The electromagnetic theory of light not only gives a wonderfully adequate picture of the propagation of radiation through free space, but has also to a wide extent shown itself adapted for the interpretation of the phenomena connected with the interaction of radiation and matter. Thus a general description of the phenomena of emission, absorption, refraction, scattering, and dispersion of light may be obtained on the assumption that the atoms contain electrified particles which can perform harmonic oscillations round positions of stable equilibrium, and which will exchange energy and momentum with the radiation fields according to the classical laws of electrodynamics. On the other hand, it is well known that these phenomena exhibit a number of features which are contradictory to the consequences of the classical electrodynamical theory. The first phenomenon where such contradictions were firmly established was the law of tempera-

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* J. C. Slater, Nature, March 1st, 1924, p. 307.
† N. Bohr, Zs. f. Phys. 13 (1923) 117. An English translation has recently appeared in the Publications of the Cambridge Philosophical Society under the title: 'On the application of the Quantum Theory to atomic structure. I: The fundamental postulates of the quantum theory.' This treatise, which also contains more detailed references to the literature, will in the following be quoted as 'P.Q.T.'

ture radiation. Starting from the classical conception of emission and absorption of radiation by a particle performing harmonic oscillations, Planck found that, in order to obtain agreement with the experiments on temperature radiation, it was necessary to introduce the auxiliary assumption that in a statistical distribution only certain states of the oscillating particles have to be taken into account. For these distinguished states the energy was found to be equal to a multiple of the quantum $h\omega$, where ω is the natural frequency of the oscillator and h is a universal constant. Independent of radiation phenomena, this result obtained, as Einstein pointed out, a direct support from experiments on the specific heat of solids. At the same time, this author put forward his well-known theory of 'light-quanta', according to which radiation should not be propagated through space as continuous trains of waves in the classical theory of light, but as entities, each of which contains the energy $h\nu$, concentrated in a minute volume, where h is Planck's constant and ν the quantity which in the classical picture is described as the number of waves passing in unit time. Although the great heuristic value of this hypothesis is shown by the confirmation of Einstein's predictions concerning the photoelectric phenomenon, still the theory of light-quanta can obviously not be considered as a satisfactory solution of the problem of light propagation. This is clear even from the fact that the radiation 'frequency' ν appearing in the theory is defined by experiments on interference phenomena which apparently demand for their interpretation a wave constitution of light.

In spite of the fundamental difficulties involved in the ideas of the quantum theory, it has nevertheless been possible to a certain extent to apply these conceptions, combined with information on the structure of the atom derived from other sources, to the interpretation of the results of investigations of the emission and absorption spectra of the elements. This interpretation is based on the fundamental postulate: that an atom possesses a number of distinguished states, the so-called 'stationary states', which are supposed to possess a remarkable stability, for which no interpretation can be derived from the concepts of classical electrodynamics. This stability comes to light in the circumstance that any change of the state of the atom must consist of a complete process of transition from one of these stationary states to another. The postulate obtains a connection with optical phenomena through the further assumption that when a transition between two

stationary states is accompanied by emission of radiation, this consists of a train of harmonic waves, whose frequency is given by the relation

$$hv = E_1 - E_2, \qquad (1)$$

where E_1 and E_2 are the values of the energy of the atom in the initial and in the final state of the process respectively. Inversely it is assumed that the reversed process of transition can take place by illumination with light with this same frequency. The applicability of these assumptions to the interpretation of the spectra of the elements is essentially due to the fact that it has been found possible in many cases to fix the energy in the stationary states of an isolated atom by means of simple rules referring to motions which with a high approximation obey the ordinary laws of electrodynamics (P.Q.T., Ch. I, § 2). The concepts of this theory, however, do not allow us to describe the details of the mechanism underlying the process of transition between the various stationary states.

At the present state of science it seems necessary, as regards the occurrence of transition processes, to content ourselves with considerations of probability. Such considerations have been introduced by Einstein,[*] who has shown how a remarkably simple deduction of Planck's law of temperature radiation can be obtained by assuming that an atom in a given stationary state may possess a certain probability of a 'spontaneous' transition in unit time to a stationary state of smaller energy content, and that in addition an atom, by illumination with external radiation of suitable frequency, may acquire a certain probability of performing an 'induced' transition to another stationary state with higher or smaller energy content. In connexion with the conditions of thermal equilibrium between radiation and matter, Einstein further arrived at the conclusion that the exchange of energy by the transition process is accompanied by an exchange of momentum of the amount hv/c, just as would be the case if the transition were accompanied by the starting or stopping of a small entity moving with the velocity of light c and containing the energy hv. He concluded that the direction of this momentum for the induced transitions is the same as the direction of propagation of the illuminating light-waves, but that for the spontaneous transitions the direction of the impulse is distributed according to probability laws. These results, which were considered as an argument for ascribing a

* A. Einstein, Phys. Zs. 18 (1917) 121.

certain physical reality to the theory of light-quanta, have recently found an important application in explaining the remarkable phenomena of the change of wave-length of radiation scattered by free electrons brought to light by A. H. Compton's * investigation on X-ray scattering. The application of probability considerations to the problem of temperature equilibrium between free electrons and radiation suggested by this discovery has recently been successfully treated by Pauli,† and the formal analogy of his results with the laws governing transition processes between stationary states of atoms has been emphasized by Einstein and Ehrenfest.**

In spite of the fundamental departure of the quantum theory of atomic processes from a picture based on the ordinary concepts of electrodynamics, the former must in a certain sense ultimately appear as a natural generalization of the latter. This is evident from the condition that in the limit, where we consider processes which depend on the statistical behaviour of a large number of atoms, and which involve stationary states where the difference between neighbouring stationary states is comparatively little, the classical theory leads to conclusions in agreement with the experiments. In the case of emission and absorption of spectral lines, the connexion between the two theories has led to the establishment of the 'correspondence principle', which postulates a general conjugation of each of the various possible transitions between stationary states with one of the harmonic oscillation components in which the electrical moment of the atom, considered as a function of the time, can be resolved (P.Q.T., Ch. II, § 2). This principle has afforded a basis for an estimation of probabilities of transition, and thereby for bringing the problem of intensities and polarization of spectral lines in close connexion with the motion of the electrons in the atom.

The correspondence principle has led to comparing the reaction of an atom on a field of radiation with the reaction on such a field which, according to the classical theory of electrodynamics, should be expected from a set of 'virtual' harmonic oscillators with frequencies equal to those determined by the equation (1) for the various possible transitions between stationary states (P.Q.T., Ch. III, § 3). Such a

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* A. H. Compton, Phys. Rev. 21 (1923) 483. See also P. Debye, Phys. Zs. 24 (1923) 161.

† W. Pauli, Zs. f. Phys. 18 (1923) 272.

** A. Einstein and P. Ehrenfest, Zs. f. Phys. 14 (1923) 301.

picture has been used by Ladenburg * in an attempt to connect the experimental results on dispersion quantitatively with considerations on the probability of transitions between stationary states. Also in the phenomenon of interaction between free electrons and radiation, the possibility of applying similar considerations is suggested by the analogy, emphasized by Compton, between the change of wave-length of the scattered rays and the classical Doppler effect of radiation from a moving source.

Although the correspondence principle makes it possible through the estimation of probabilities of transition to draw conclusions about the mean time which an atom remains in a given stationary state, great difficulties have been involved in the problem of the time-interval in which emission of radiation connected with the transition takes place. In fact, together with other well-known paradoxes of the quantum theory, the latter difficulty has strengthened the doubt, expressed from various sides,† whether the detailed interpretation of the interaction between matter and radiation can be given at all in terms of a causal description in space and time of the kind hitherto used for the interpretation of natural phenomena (P.Q.T., Ch. III, § 1). Without in any way removing the formal character of the theory, it nevertheless appears, as mentioned in the introduction, that a definite advance as regards the interpretation of the observable radiation phenomena may be made by connecting these phenomena with the stationary states and the transitions between them in a way somewhat different from that hitherto followed.

2. Radiation and transition processes

We will assume that a given atom in a certain stationary state will communicate continually with other atoms through a time-spatial mechanism which is virtually equivalent with the field of radiation which on the classical theory would originate from the virtual harmonic oscillators corresponding with the various possible transitions to other stationary states. Further, we will assume that the occurrence of transition processes for the given atom itself, as well as for the other atoms with which it is in mutual communication, is connected with

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* R. Ladenburg, Zs. f. Phys. **4** (1921) 451. See also R. Ladenburg and P. Reiche, Naturwiss. **11** (1923) 584.

† Such a view has perhaps for the first time been clearly expressed by O. W. Richardson, 'The Electron Theory of Matter,' 2nd ed. (Cambridge 1916) p. 507.

this mechanism by probability laws which are analogous to those which in Einstein's theory hold for the induced transitions between stationary states when illuminated by radiation. On the one hand, the transitions which in this theory are designated as spontaneous are, on our view, considered as induced by the virtual field of radiation which is connected with the virtual harmonic oscillators conjugated with the motion of the atom itself. On the other hand, the induced transitions of Einstein's theory occur in consequence of the virtual radiation in the surrounding space due to other atoms.

While these assumptions do not involve any change in the connexion between the structure of the atom and the frequency, intensity, and polarization of the spectral lines derived by means of the relation (1) and of the correspondence principle, they lead to a picture as regards the time-spatial occurrence of the various transition processes on which the observations of the optical phenomena ultimately depend which in an essential respect differs from the usual concepts. In fact, the occurrence of a certain transition in a given atom will depend on the initial stationary state of this atom itself and on the states of the atoms with which it is in communication through the virtual radiation field, but not on the occurrence of transition processes in the latter atoms.

On the one hand it will be seen that our view, in the limit where successive stationary states differ only little from each other, leads to a connexion between the virtual radiation field and the motion of the particles in the atom which gradually merges into that claimed by the classical radiation theory. In fact neither the motion nor the constitution of the radiation field will in this limit undergo essential changes through the transitions between stationary states. As regards the occurrence of transitions, which is the essential feature of the quantum theory, we abandon on the other hand any attempt at a causal connexion between the transitions in distant atoms, and especially a direct application of the principles of conservation of energy and momentum, so characteristic for the classical theories. The application of these principles to the interaction between individual atomic systems is, on our view, limited to interactions which take place when the atoms are so close that the forces which would be connected with the radiation field on the classical theory are small compared with the conservative parts of the fields of force originating from the electric charges in the atom. Interactions of this type, which may be termed

'collisions', offer, as is well known, remarkable illustrations of the stability of stationary states postulated in the quantum theory. In fact, an analysis of the experimental results based on the theory of conservation of energy and momentum is in agreement with the view that the colliding atoms before as well as after the process will always find themselves in stationary states (P.Q.T., Ch. I, § 4)*. By interaction between atoms at greater distances from each other, where according to the classical theory of radiation there would be no question of simultaneous mutual action, we shall assume an independence of the individual transition processes, which stands in striking contrast to the classical claim of conservation of energy and momentum. Thus we assume that an induced transition in an atom is not directly caused by a transition in a distant atom for which the energy difference between the initial and the final stationary state is the same. On the contrary, an atom which has contributed to the induction of a certain transition in a distant atom through the virtual radiation field conjugated with the virtual harmonic oscillator corresponding with one of the possible transitions to other stationary states, may nevertheless itself ultimately perform another of these transitions.

At present there is unfortunately no experimental evidence at hand which allows to test these ideas, but it may be emphasized that the degree of independence of the transition processes assumed here would

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* These considerations hold obviously only in so far as the radiation connected with the collisions can be neglected. Although in many cases the energy of this radiation is very small, its occurrence might be of essential importance. This has been emphasized by Franck in connexion with the explanation of Ramsauer's important results regarding collisions between atoms and slow electrons [Ann. d. Phys. Leipzig 64 (1922) 513; 66 (1922) 546], from which it seems to follow that in certain cases the electron can pass freely through the atom, without being influenced by its presence. In fact, if in these 'collisions' a change in the motion of the electron actually took place, the classical theory would involve so large a radiation, that a rational conjugation of the radiation with the possible transition processes, as claimed by the correspondence principle, could hardly be established [compare F. Hund, Zs. f. Phys. 13 (1923) 241]. On the view presented in this paper, such an explanation might on the one hand be regarded as the more natural, since the origin of radiation is not directly sought in the occurrence of transitions but in the motion of the electron. On the other hand, it must be remembered that we are here dealing with a case where, on account of the large magnitude of the classical reaction of radiation, the theory does not allow a sharp distinction between stationary motion and transition processes.

seem the only consistent way of describing the interaction between radiation and atoms by a theory involving probability considerations. This independence reduces not only conservation of energy to a statistical law, but also conservation of momentum. Just as we assume that any transition process induced by radiation is accompanied by a change of energy of the atom of the amount $h\nu$, we shall assume, following Einstein, that any such process is also accompanied by a change of momentum of the atom of an amount $h\nu/c$. If the transition is induced by virtual radiation fields from distant atoms, the direction of this momentum is the same as that of the wave propagation in this virtual field. In case of a transition by its own virtual radiation, we shall naturally assume that the change of momentum is distributed according to probability laws in such a way that changes of momentum due to the transitions in other atoms are statistically compensated for any direction in space.

The cause of the observed statistical conservation of energy and momentum we shall not seek in any departure from the electrodynamic theory of light as regards the laws of propagation of radiation in free space, but in the peculiarities of the interaction between the virtual field of radiation and the illuminated atoms. In fact, we shall assume that these atoms will act as secondary sources of virtual wave radiation which interferes with the incident radiation. If the frequency of the incident waves coincides closely with the frequency of one of the virtual harmonic oscillators corresponding to the various possible transitions, the amplitudes of the secondary waves will be especially large, and these waves will possess such phase relations with the incident waves that they will diminish or augment the intensity of the virtual radiation field, and thereby weaken or strengthen its power of inducing transitions in other atoms. Whether it is a diminishing or an augmentation of the intensity which takes place, will depend on whether the virtual harmonic oscillator, which is called into play by the incident radiation, corresponds with a transition by which the energy of the atom is increased or diminished respectively. It will be seen that this view is closely related to the ideas which led Einstein to introduce probabilities of two kinds of induced transitions between stationary states corresponding with an increase or decrease of the energy of the atom respectively. In spite of the time-spatial separation of the processes of absorption and emission of radiation characteristic for the quantum theory, we may nevertheless expect, on our view,

a far-reaching analogy with the classical theory of electrodynamics
as regards the interaction of the virtual radiation field and the virtual
harmonic oscillators conjugated with the motion of the atom. It
seems actually possible, guided by this analogy, to establish a con-
sistent and fairly complete description of the general optical pheno-
mena accompanying the propagation of light through a material
medium, which accounts at the same time for the close connexion
of the phenomena with the spectra of the atoms of the medium.

3. Capacity of interference of spectral lines

Before we enter more closely on the general problem of the reaction
of atoms on a virtual radiation field, responsible for the phenomena
accompanying the propagation of light through material media, we
shall here briefly consider the properties of the field originating from
a single atom, as far as they are connected with the capacity of inter-
ference of light from one and the same source. The constitution of this
field must obviously not be sought in the peculiarities of the transition
processes themselves, the duration of which we shall assume at any
rate not to be large compared with the period of the corresponding
harmonic component in the motion of the atom. These processes will,
on our view, simply mark the termination of the time-interval in
which the atom will be able to communicate with other atoms through
the corresponding virtual oscillator. An upper limit of the capacity
of interference, however, will clearly be given by the mean time
interval in which the atom remains in the stationary state representing
the initial state of the transition under consideration. The estimation
of the time of duration of states based on the correspondence principle
has obtained a general confirmation from the well-known beautiful
experiments on the duration of the luminosity of high speed atoms
emerging from a luminescent discharge into a high vacuum. (Compare
P.Q.T., Ch. II, § 4.) On the present point of view these experiments
obtain a very simple interpretation. In fact it will be seen that on
this view the variation of the luminosity along the path of the atoms
will not depend on the peculiarity of the transitions, but only on the
relative number of atoms in the various stationary states in the
different parts of the path. If all the emerging atoms have the same
speed and are initially in the same state, we must thus expect that
for any spectral line conjugated with a transition from this state the
luminosity will decrease exponentially along the path at one and the

same rate. At present the experimental material at hand is hardly sufficient to test these considerations.

When we ask for the capacity of interference of spectral lines, determined by optical apparatus, the mean time of duration of the stationary states will certainly constitute an upper limit for this capacity, but it must be remembered that the sharpness of a given spectral line which is due to the statistical result of the action from a large number of atoms will depend not only on the lengths of the individual wave trains terminated by the transition processes, but clearly also on any uncertainty in the definition of the frequency of these waves. In view of the way this frequency through relation (1) is related to the energy in the stationary states, it is of interest to note that the above-mentioned upper limit of capacity of interference may be brought in close connexion with the limit of definition of the motion and of the energy in the stationary states. In fact, the postulate of the stability of stationary states imposes an *a priori* limit to the accuracy with which the motion in these states can be described by means of classical electrodynamics, a limit which on our picture is directly involved in the assumption that the virtual radiation field is not accompanied by a continuous change in the motion of the atom, but only acts by its induction of transitions involving finite changes of the energy and the momentum of the atom (P.Q.T., Ch. II, § 4). In the limiting region where the motions in the two stationary states involved in the transition process differ only comparatively little from each other, the upper limit of capacity of interference of the individual wave trains coincides with the limit of definition of the frequency of the radiation determined by (1), if the influence of the lack of definition of the energy in the two states is treated as independent errors. In the general case where the motions in these states may differ considerably from each other, the upper limit of the capacity of interference of the wave trains is closely related with the definition of the motion in the stationary state which forms the starting point of the transition process. Also here we may, however, expect that the observable sharpness of the spectral lines will be determined according to relation (1) by adding the effect of any possible lack of definition of the energy in the stationary state terminating the transition process to the effect of the lack of definition in the starting state in a similar way as independent errors. Just this influence of the lack of definition of both stationary states on the sharpness of a spectral line makes it

possible to ensure the reciprocity which will exist between the constitution of a line when appearing in an emission and in an absorption spectrum, and which is claimed by the condition for thermal equilibrium expressed by Kirchhoff's law. In this connexion it may be remembered how the apparent deviations from this law exhibited by the remarkable difference often shown by the structure of the emission and the absorption spectra of an element as regards the number of lines present are directly accounted for on the quantum theory when account is taken of the difference in the statistical distributions of the atoms over the various stationary states under different external conditions.

A problem closely related to the sharpness of spectral lines originating from atoms under constant external conditions, is the problem of the spectrum to be expected from atoms under the influence of external forces which vary considerably within a time-interval of the same order of magnitude as the mean duration of the stationary states. Such a problem is met with in certain of the experiments by Stark on the influence of electric fields on spectral lines. In these experiments the emitting atoms move with large velocities, and the time-intervals in which they pass between two points where the intensity of the electric field differs very much, are only a small fraction of the mean time of duration of the stationary states connected with the investigated spectral lines. Nevertheless Stark found that, except for a Doppler effect of the usual kind, the radiation from the moving atoms was influenced by the electric field at any point of the path in the same way as the radiation from resting atoms subject to the constant action of the field at this point. While, as emphasized by various authors,* the interpretation of this result obviously presents difficulty on the usual quantum theory description of the connexion between radiation and transition processes, it is clear that Stark's results are in conformity with the picture adopted in this paper. In fact, during the passing of the atoms through the field, the motion in the stationary states changes in a continuous way, and in consequence also the virtual harmonic oscillators corresponding with the possible transitions. The effect of the virtual radiation field originating from the moving atoms will therefore not be different from that which would occur if the atoms along their whole path had moved in

* Compare K. Försterling, Zs. f. Phys. **10** (1922) 387; A. J. Dempster, Astrophys. Journ. **57** (1923) 193.

a field of constant intensity, at any rate if – as in Stark's experiments – the radiation originating from the other parts of their paths is prevented from reaching those parts of the apparatus on which the observation of the phenomenon depends. In a problem of this kind it will also be seen how a far reaching reciprocity in the observable phenomena of emission and absorption is ensured on account of the symmetry exhibited by our picture as regards the coupling of the radiation field with the transition processes in the one or in the other direction.

4. Quantum theory of spectra and optical phenomena

Although on the quantum theory the observation of the optical phenomena ultimately depends on discontinuous transition processes, an adequate interpretation of these phenomena must, as already emphasized in the introduction, nevertheless involve an element of continuity similar to that exhibited by the classical electrodynamical theory of the propagation of light through material media. On this theory the phenomena of reflexion, refraction, and dispersion are attributed to a scattering of light by the atom due to the forced vibration in the individual electric particles, set up by the electromagnetic forces of the radiation field. The postulate of the stability of stationary states might at first sight seem to involve a fundamental difficulty on this point. The contrast, however, was to a certain extent bridged over by the correspondence principle, which, as mentioned in § 1, led to comparing the reaction of an atom on a radiation field with the scattering which, according to the classical theory, would arise from a set of virtual harmonic oscillators conjugated with the various possible transitions. It must still be remembered that the analogy between the classical theory and the quantum theory as formulated through the correspondence principle is of an essentially formal character, which is especially illustrated by the fact that on the quantum theory the absorption and emission of radiation are coupled to different processes of transition, and thereby to different virtual oscillators. Just this point, however, which is so essential for the interpretation of the experimental results on emission and absorption spectra, seems to afford a guidance as regards the way in which the scattering phenomena are related with the activity of the virtual oscillators concerning emission and absorption of radiation. In a later paper it is hoped to show how on the present view a quanti-

tative theory of dispersion resembling Ladenburg's theory can be established.* Here we shall confine ourselves to emphasizing once more the continuous character of the optical phenomena, which seemingly does not permit an interpretation based on a simple causal connexion with transition processes in the propagating medium.

An instructive example of these considerations is offered by the experiments on absorption spectra. In fact, the pronounced absorption by monatomic vapours for light of frequencies coinciding with certain lines in the emission spectra of the atoms strictly cannot be said, as often done for brevity, to be caused by the transition processes which take place in the atoms of the vapour induced by wave trains in the incident radiation possessing the frequencies of the absorption lines. The appearance of these lines in the spectroscope is due to the decrease of the intensity of the incident waves in consequence of the peculiarities of the secondary spherical wavelets set up by each of the illuminated atoms, while the induced transitions appear only as an accompanying effect by which a statistical conservation of energy is ensured. The presence of the secondary coherent wave-trains is at the same time responsible for the anomalous dispersion connected with the absorption lines, and is especially clearly shown by the phenomenon, discovered by Wood†, of selective reflexion from the wall of a vessel containing metallic vapour under sufficiently high pressure. The occurrence of included transitions between stationary states is on the other hand directly observed in the fluorescent radiation, which for an essential part originates from the presence of a small number of atoms which through the illumination have been transferred to a stationary state of higher energy. As is well known, the fluorescent radiation can be suppressed through the admixture of foreign gases. As regards the part played by atoms in the higher stationary states this phenomenon is explained by collisions which cause a considerable increase of the probability of the atoms to return into their normal state. At the same time any part of the fluorescent radiation due to the coherent wavelets will, through the admixture of foreign gases, just as the phenomena of absorption, dispersion, and reflexion, undergo such changes as can be brought in connexion with

* *Note added during the proof.* The outline of such a theory is briefly described by Kramers in a letter to 'Nature' published in April, 1925.

† R. W. Wood, Phil. Mag. **23** (1915) 689.

a broadening of the spectral lines *. It will be seen that a view on absorption phenomena differing essentially from that just described can hardly be maintained, if it can be shown that the selective absorption of spectral lines is a phenomenon qualitatively independent of the intensity of the source of radiation, in a similar way to what has already been found to be the case for the usual phenomena of reflexion and refraction, whose transitions in the medium do not occur to a similar extent (compare P.Q.T., Ch. III, § 3).

Another interesting example is offered by the theory of the scattering of light by free electrons. As has been shown by Compton by means of reflexion of X-rays from crystals, this scattering is accompanied by a change of frequency, different in different directions, and corresponding with the constitution of the radiation which on the classical theory would be emitted by an imaginary moving source. As mentioned, Compton has reached a formal interpretation of this effect on the theory of light-quanta by assuming that the electron may take up a quantum of the incident light and simultaneously re-emit a light-quantum in some other direction. By this process the electron acquires a velocity in a certain direction, which is determined, just as the frequency of the re-emitted light, by the laws of conservation of energy and momentum, an energy of $h\nu$ and a momentum $h\nu/c$ being ascribed to each light-quantum. In contrast to this picture, the scattering of the radiation by the electrons is, on our view, considered as a continuous phenomenon to which each of the illuminated electrons contributes through the emission of coherent secondary wavelets. Thereby the incident virtual radiation gives rise to a reaction from each electron, similar to that to be expected on the classical theory from an electron moving with a velocity coinciding with that of the above-mentioned imaginary source and performing forced oscillations under the influence of the radiation field. That in this case the virtual oscillator moves with a velocity different from that of the illuminated electrons themselves is certainly a feature strikingly unfamiliar to the classical conceptions. In view of the fundamental departures from the classical space-time description, involved in the very idea of virtual oscillators, it seems at the present state of science hardly justifiable to reject a formal interpretation as that under consideration as inadequate. On the contrary, such an interpretation seems unavoidable in order to account for the effects observed, the description of which

* See for instance, Chr. Füchtbauer and G. Joos, Phys. Zs. **23** (1922) 73.

involves the wave-concept of radiation in an essential way. At the same time, however, we shall assume, just as in Compton's theory, that the illuminated electron possesses a certain probability of taking up in unit time a finite amount of momentum in any given direction. By this effect, which in the quantum theory takes the place of the continuous transfer of momentum to the electrons which on the classical theory would accompany a scattering of radiation of the type described, a statistical conservation of momentum is secured in a way quite analogous to the statistical conservation of energy in the phenomena of absorption of light discussed above. In fact, the laws of probability for the exchange of momentum by interaction of free electrons and radiation derived by Pauli are essentially analogous to the laws governing transition processes between well-defined states of an atomic system. Especially the considerations of Einstein and Ehrenfest, referred to in § 1, are suited to bring out this analogy.

A problem similar to that of the scattering of light by free electrons is presented by the scattering of light by an atom, even in the case where the frequency of the radiation is not large enough to induce transitions by which an electron is wholly removed from the atom. In fact, in order to secure statistical conservation of momentum, we must, as emphasized by various authors,* assume the occurrence of transition processes by which the momentum of the scattering atom changes by finite amounts without, however, the relative motion of the particles of the atom being changed, as in transition processes of the usual type considered in the spectral theory. It will also be seen, on our picture, that transition processes of the type mentioned will be closely connected with the scattering phenomena, in a way analogous with the connexion of the spectral phenomena with the transition processes by which the internal motion of the atom undergoes a change. Due to the large mass of the atomic nucleus the velocity change which the atom undergoes by these transitions is so small, that it will not have a perceptible effect on the energy of the atom and the frequency of the scattered radiation. Nevertheless, it is of principal importance that the transference of momentum is a discontinuous process, while the scattering itself is an essentially continuous phenomenon, in which all the illuminated atoms take part, independent of the intensity of the incident light. The discontinuous changes in momentum of the atoms, however, are the cause of the observable reactions on the atoms

* W. Pauli, Zs. f. Phys. **18** (1923) 272; A. Smekal, Naturwissensch. **11** (1923) 875.

described as radiation pressure. This view fulfils clearly the conditions for thermal equilibrium between a (virtual) radiation field and a reflecting surface, derived by Einstein * and considered as an argument for the light-quantum theory. At the same time it needs hardly be emphasized that it is also consistent with the apparent continuity exhibited by actual observations on radiation pressure. In fact, if we consider a solid, a change of $h\nu/c$ in its total momentum will be totally imperceptible, and for visible light even vanishingly small compared with the irregular changes of this momentum of a body in thermal equilibrium with the surroundings. In the discussion of the actual experiments it may, however, be noted at the same time, that the frequency of the occurrence of such processes may often be so large that the problem arises whether the time involved in the transitions themselves can be neglected, or, in other words, whether the limit has been reached inside which the formulation of the principles of the quantum theory can be maintained (compare P.Q.T., Ch. II, § 5).

The last considerations may illustrate how our picture of optical phenomena offers a natural connexion with the ordinary continuous description of macroscopic phenomena for the interpretation of which Maxwell's theory has shown itself so wonderfully adapted. The advantage in this respect of the present formulation of the principles of the quantum theory over the usual representation of this theory will perhaps be still more clearly illustrated if we consider the phenomenon of emission of electromagnetic waves, say from an antenna as used in wireless telegraphy. In this case no adequate description of the phenomenon is offered on the picture of emission of radiation during separate successive transition processes between imaginary stationary states of the antenna. In fact, when the smallness of the energy changes by the transitions, and the magnitude of the energy radiation from the antenna per unit time, are taken into account, it will be seen that the duration of the individual transition processes can only be an exceedingly small fraction of the period of oscillation of the electricity in the antenna, so that there would be no justification in describing the result of one of these processes as the emission of a train of waves of this period. On the present view, however, we will describe the action of the oscillation of the electricity in the antenna as producing a (virtual) radiation field which through probability laws again induces changes in the motion of the electrons which may

* A. Einstein, Phys. Zs. **10** (1909) 817.

be regarded as continuous. In fact, even if a distinction between different energy steps $h\nu$ could be kept upright, the size of these steps would be quite negligible compared with the energy associated with the antenna. It will in this connexion be observed that the emphasizing of the 'virtual' character of the radiation field, which at the present state of science seems so essential for an adequate description of atomic phenomena, automatically loses its importance in a limiting case like that just considered, where the field, as regards its observable interaction with matter, is endowed with all the attributes of an electromagnetic field in classical electrodynamics.

Related papers

5a J. C. Slater, *Radiation and Atoms*. Nature 113 (1924) 307.
5b H. Geiger und W. Bothe, *Über das Wesen des Comptoneffektes; ein experimenteller Beitrag zur Theorie der Strahlung*. Z. Phys. 32 (1925) 639, received April 25, 1925.