$*13$. IDENTITY

Summary of $*13$.

The propositional function "x is identical with y" will be written "$x = y$." We shall find that this use of the sign of equality covers all the common uses of equality that occur in mathematics. The definition is as follows:

$*13 \cdot 01$. $x = y \,.=:\, (\phi) : \phi\,!\,x \,.\supset.\, \phi\,!\,y$ Df

This definition states that x and y are to be called identical when every predicative function satisfied by x is also satisfied by y. We cannot state that *every* function satisfied by x is to be satisfied by y, because x satisfies functions of various orders, and these cannot all be covered by one apparent variable. But in virtue of the axiom of reducibility it follows that, if $x = y$ and x satisfies ψx, where ψ is any function, predicative or non-predicative, then y also satisfies ψy (cf. $*13 \cdot 101$, below). Hence in effect the definition is as powerful as it would be if it could be extended to cover *all* functions of x.

Note that the second sign of equality in the above definition is combined with "Df," and thus is not really the same symbol as the sign of equality which is defined. Thus the definition is not circular, although at first sight it appears so.

The propositions of the present number are constantly referred to. Most of them are self-evident, and the proofs offer no difficulty. The most important of the propositions of this number are the following:

$*13 \cdot 101$. $\vdash : x = y \,.\supset.\, \psi x \supset \psi y$

I.e. if x and y are identical, any property of x is a property of y.

$*13 \cdot 12$. $\vdash : x = y \,.\supset.\, \psi x \equiv \psi y$

This includes $*13 \cdot 101$ together with the fact that if x and y are identical any property of y is a property of x.

$*13 \cdot 15 \cdot 16 \cdot 17$, which state that identity is reflexive, symmetrical and transitive.

$*13 \cdot 191$. $\vdash :. y = x \,.\supset_y.\, \phi y : \equiv .\, \phi x$

I.e. to state that everything that is identical with x has a certain property is equivalent to stating that x has that property.

$*13 \cdot 195$. $\vdash : (\exists y) \,.\, y = x \,.\, \phi y \,.\equiv.\, \phi x$

I.e. to state that something identical with x has a certain property is equivalent to saying that x has that property.

$*13 \cdot 22$. $\vdash : (\exists z, w) \,.\, z = x \,.\, w = y \,.\, \phi(z, w) \,.\equiv.\, \phi(x, y)$

This is the analogue of $*13 \cdot 195$ for two variables.