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Epistemic *Primacy vs.* Ontological *Elusiveness* of Spatial *Extension*: Is There an Evolutionary Role for the *Quantum*?

Massimo Pauri

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Abstract A critical re-examination of the history of the concepts of space (including spacetime of general relativity and relativistic quantum field theory) reveals a basic *ontological elusiveness* of *spatial extension*, while, at the same time, highlighting the fact that its *epistemic primacy* seems to be unavoidably imposed on us (as stated by A.Einstein "giving up the extensional continuum ... is like to breathe in airless space"). On the other hand, Planck's discovery of the *atomization* of *action* leads to the fundamental recognition of an ontology of *non-spatial, abstract entities* (Quine) for the quantum level of reality (QT), as distinguished from the necessarily *spatio-temporal*, experimental revelations (*measurements*). The *elementary quantum act* (measured by Planck's constant) has neither duration nor extension, and any *genuinely quantum* process literally does not belong in the *Raum* and time of our experience. As Heisenberg stresses: "Während also die klassische Physik ein objectives Geschehen in Raum and Zeit zum Gegenstand hat, für dessen Existenz seine Beobachtung völlig irrelevant war, behandelt die Quantentheorie Vorgänge, die sozusagen nur in den Momenten der Beobachtung *als raumzeitliche Phänomene* aufleuchten, und über die *in der zwischenzeit anschaulische physikalische Aussagen sinloss sind*". An admittedly speculative, hazardous conjecture is then advanced concerning the relation of such quantum ontology with the role of the *pre-phenomenal continuum* (Husserl) in the perception of macroscopically distinguishable objects in the *Raum* and time of our experience. Although rather venturesome, it brings together important philosophical issues. Coherently with recent general results in works on the foundations of QT, it is assumed that the linearity of quantum dynamical evolution does not apply to the central nervous system of living beings at a certain level of the evolutionary ramification and at the *pre-conscious stage* of subjectivity. Accordingly,

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M. Pauri (✉)
Physics Department, University of Parma, Parma, Italy
e-mail: massimo.pauri@unipr.it

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corresponding to the onset of a non-linear dynamic evolution, a 'primary spatial' reduction is 'continually' taking place, thereby constituting the neural precondition for the experience of distinguishable macroscopic objects in the continuous spatial extension. While preventing the theoretically possible quantum superpositions of macroscopic objects from *being perceivable by living beings*, the 'primary reduction' has no effect on the standard processes concerning quantum level entities involved in laboratory man-made experiments. In this connection, an experimental check which might falsify the conjecture is briefly discussed. The approach suggested here, if sound, leads to a *naturalization* of that part of Kant's Transcendental Aesthetics than can survive the *Euclidean catastrophe*. According to such *naturalized transcendentalism*, "space can well be transcendental without the axioms being so", in agreement with a well-known statement by Boltzman. Finally, as far as QT is concerned, the conjecture entails that a scheme for quantum measurement of the von Neumann type cannot even 'leave the ground', vindicating Bohr's viewpoint. A quantum theory of measurement, in a proper sense, turns out to be unnecessary and in fact impossible.

Keywords Spatial extension · Perception of macroscopic objects in space · Quantum ontology · Superposition principle · Macro-objectification · Violation of linearity of quantum evolution · Naturalized transcendentalism

Personne n'est exempt de dire de fadaises
Le malheur est de le dire courieusement.
Michel de Montaigne[1]

1 Introduction

The core view inspiring the present paper is the recognition that the discovery of the *quantum of action* and the *atomization of processes* by Planck is the deepest and most far-reaching acquisition in the whole history of knowledge. Nevertheless, the radical *ontological 'diminution'* implied by the atomization of action for the concept of *object*, as a perceived entity within the *extensional Raum* of our experience, has not *explicitly* affected yet our notions of space and time. On the other hand, we have historical evidence of a significant correlation between the ontological notions of space and time, on the one hand, and those of objects and processes, on the other.

In the present paper I intend to analyze from various perspectives the nature of this missing correlation on the basis of both a critical re-examination of the historical concept of space and a re-analysis of the main open problems in quantum theory (QT). Admittedly, the paper does not belong to the *main stream* of contemporary philosophy of science. In some places, I shall present viewpoints that deviate sensibly

[1] *Essays, Troisième Livre, chapitre 1*. Note that in 16th century French the word 'courieusement' had the same meaning as the contemporary word 'pretentieusement'. Yet, the first English translation of the Essays, by Charles Cotton Esq, can be found in the E.H.W. Meyerstein Collection of the English Faculty Library, Oxford, 1711. The quoted passage sounds like "No Man is free from speaking foolish things: but the worst on't is when a Man studies to play the Fool".

from what could be considered accepted wisdom in present literature, at least outside the so-called 'continental' camp. In particular, the final sections contain a main venturesome conjecture about the role of quantum theory concerning our macroscopic experience of spatial objects. On the other hand, given the lasting situation of QT, especially concerning the so-called *measurement problem* or *macro-objectification* issue, it seems that one either has to admit to total ignorance or is authorized to engage in some speculation. I do believe that any suggestion about such problems which is not absolutely crazy or evidently contradictory, and brings together some important correlated issues should be made explicit. Thus, I acknowledge from the beginning that my presentation contains a good deal of provocative speculation and that my main proposal can be defended by plausible arguments only. I would consequently like to define this paper as a (risky) adventure in *empirical metaphysics,* where the *oxymoron* is justified by the fact that, although not positively supported by any direct empirical evidence, my conjecture will however be shown to admit *falsifiability* in a definite and concrete sense by *experimental check*, a test which is becoming technologically feasible in quantum optics. Finally, I must apologize for the fact that due to the variety and complexity of the subject matter, the presentation of the paper may appear to be rather sketchy.

In conclusion, the reader is invited to take my present proposal in the spirit of Montaigne's statement quoted in the epigraph.

2 Space

2.1 A Brief Account of the Classical Debate and Its Current Relevance[2]

I assume that the reader is well aware of the historical main distinctions between the so-called *absolutist* or, better, *substantivalist*, and the *relational* view of space, the former being roughly identified with Newton's position and the latter with Leibniz's position. A simplified yet remarkably clear account of the various senses of 'absolute' in Newton's Principia (*Scholium*) can be found in [13]. Such senses of absolute mainly concern the concept of motion whilst I am here interested in the question of the ontological consistency of the concept of space. Thus I will limit myself to quoting the points relevant to the latter issue, together with its relational counterpart.

Newton *substantivalism*.[3] A_1: "Space (or Space-time) is a *substance* in that it forms a *substratum* that underlies physical events and processes, and spatiotempo-

[2] In this paper I will focus primarily on the concept of space rather than on the whole space-time. In spite of the formal operational unification of time and space within the special theory of relativity (STR), time and space maintain a deep ontological difference. Also, in General Theory of Relativity (GTR), once examined from the dynamical (Hamiltonian) point of view, time plays a special role (see [30, 31]). Even more, time has a substantially different status from space in quantum theory. *Time* (unlike the *position of something* in space) is not *observable* at any level of the theory and, furthermore, it plays a privileged role in the foundations of the very concept of probability. It is nevertheless true that many assertions concerning space scattered throughout this paper could be plainly attributed to the time continuum and the relativistic concept of space-time too.

[3] As a matter of fact, Newton's absolutist position is much more articulated. A_1 is an oversimplification based on the Leibnizian tradition. See later.

ral relations among such events and processes are *parasitic* on the spatiotemporal relations inherent in the substratum of space-time points and regions".

Leibniz *relationism.* Though Earman recognizes that there is no *direct* (Leibnitzian) relationist counterpart to Newton's *Scholium*, which is the *locus classicus* of absolutism and also that there are almost as many versions of relationism as there are relationists, for simplicity he proposes the following statements to define *relationism*; R_1: "All motion is the relative motion of bodies, and *consequently*, space-time does not have, and cannot have, *structures* that support absolute quantities of motion"; and, above all: R_2: "Spatiotemporal relations among bodies and events are *direct*; that is they are not parasitic on relations among a *substratum* of space points that underlie bodies or space-time points that underlie events."

As Earman himself warns, the philosophical discussion about the nature of space and time has the character of a Tower of Babel. I believe that the following quotations [4, 11] are instructive in this connection: (i) John Earman: 'Why Space is not a Substance (at Least Not to First Degree)', (ii) Andreas Bartels: "What is Spacetime if not a Substance? Conclusions from the New Leibnizian Argument". Moreover, there is also disagreement over what the debate is about; some authorities see the issue of whether space or space-time is a substance as central to the debate [47] while others are skeptical that the issue admits a clear and interesting formulation [32]. Finally, some authors have begun to suggest that the absolute-relational dichotomy is not the best way to parse the issues and that a *tertium quid* needs to be articulated [10, 50]. On the other hand, the tentative conclusion of Earman himself is that 'a correct account of space and time may lie outside of the ambit of the traditional absolute-relational controversy'. So I will skip all the subtleties of the various accounts to center my attention to what seems to me the main issue, namely the *ontological elusiveness* of the concept of space itself.

Prima facie, according to Earman (A_1), the Newtonian *tradition* of absolute space would therefore appear to include a *substantivalist*, or more generically, a *realistic* conception too. However, some of Newton's works themselves reveal some significant *vagueness* in relation to the notion of space. Therefore, the widespread opinion, mainly due to the Leibnizian tradition, which considers Newton's absolutism as *necessarily* implying an *ontological substantivalism of points in space*, in accordance with Earman's A_1 definition, does not appear to be justified. I shall quote some extracts taken from *De Gravitatione et Aequipondio Fluidorum*[4] which, while exemplifying the revolutionary role played by the concept of space in the process of emancipation from classical ontology, contextually and enigmatically entail some specific *ontological elusiveness* of the concept itself.

> Perhaps now it may be expected that I should define extension as *substance*, or *accident,* or else *nothing* at all. But by no means, for it has *its own manner of existing* which is proper to it and which fits neither substances nor accidents. It is not a substance: on the one hand, because it is not absolute in itself, but is as it were an *emanative effect of God and an affection of every kind of being*;

[4] Beth Jo Teeter Dobbs proposes that it was written in 1684 or 1685 while Newton was preparing the first edition of the *Principia* (1687). (I am indebted to the Referee for this information). See also Janiak [24]: 21,22 and 25 (*italics mine*).

> on the other hand, because it is not among the proper affections that denote substance, namely actions, such as thoughts in the mind and motions in body. … Moreover, since we *can cleanly conceive extension existing without any subject*, as when we may imagine spaces outside the world or places empty of any body whatsoever, and we believe [*extension*] to exist wherever we imagine there are no bodies and we cannot believe that it would perish with the body if God should annihilate a body, it follows that [*extension*] does not exist as an accident inhering in some subject. And hence it is not an accident. And much less may it be said to be nothing, since it is something more than an accident, and *approaches more nearly to the nature of substance.* There is no idea of nothing, nor has nothing any properties, but *we have an exceptionally clear idea of extension* by abstracting the dispositions and properties of a body so that there remains only the *uniform and unlimited stretching out of space in length, breadth and depth*. And furthermore, many of its properties are associated with this idea; these I shall now enumerate not only to show that it is something, but also to show what it is … Space is an affection of a *being just as a being.*

On the other hand, five pages later, Newton writes:

> For just as the parts of duration are *individuated* by their *order*, so that (for example) if yesterday could change places with today and become the later of the two, it would lose its *individuality* and would no longer be yesterday, but today; so the parts of space are *individuated* by their *positions*, so that if any two could change their positions, they would change their *individuality* at the same time and each would be converted *numerically* into the other as *individuals*. The parts of duration and space are understood to be the same as they really are only because of their *mutual order and position* ['propter solum ordinem et positiones inter se']; nor do they have any *principle of individuation* apart from that *order* and *position*, which consequently cannot be altered.

As can be seen, the only ontological characterization explicitly adopted by Newton is space as '*emanative effect of God* and *an affection of every kind of being*'.[5] Such *theistic* characterization has understandably provoked an interesting and lively historical-epistemological debate which may lie outside the scope of the present paper, but reveals the *extraordinary* nature of the problems raised by the conceptualization of space. The relevance of the issue to my discussion forces me to further

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[5] In this connection, Howard Stein's comments are very interesting (see [48, 49]). "I confess that, until recently, this did indeed seem to me a *quaint, even bizarre-sounding doctrine*". *Bearing in mind the notion of spacetime of GTR*, thus within the current *scientific image*, he then refers to Newton's view as an 'objective structural characteristic of the world' and adds 'I believe we may reasonably understand *emanative effect* to signify something *that is not created* or *produced by causal agency*, in accordance with causal laws, but that entailed by (or 'flows from') *the nature of something*'. However, he is afterwards forced to specify that "space is a *non-causally generated fact*, with *little or no ontological associations*" i.e. almost *nothing.* Again, *from a modern perspective*, DiSalle [8] defined Newton's position as an exemplification of *proto-structuralism*. Yet, I must object that space—*as conceived by Newton*—was *uniform*, or *homogeneous*, and strictly speaking it has no structure at all! I suggest a comparison with the rich *physical* (gravitational) *structure* of certain general-relativistic spacetimes (see [30, 31, 35] and Sect. 2.2). Of course, once formalized, Newton's space is characterized by a well-defined *mathematical* structure.

expand on the topic. We can now state that Newtonian space is an unlimited and immaterial entity in which any Euclidean construction can be implemented. However, it clearly lies outside the categories of traditional ontology. Spatial points need to be conceived as specific *entia per se* in order to be in some way 'realistically' interpreted. Certainly, they ought to be individuated *solo numero* and not—given their uniformity—according to distinctive qualities. Yet Newton explicitly excludes any '*principle of individuation* apart from that *order* and *position*'. Therefore the points in space are not *intrinsically individuated* at all apart from in a merely *verbal* way. It is like suggesting some kind of *internal relationism* in space, if that. Any true relationism indeed presupposes the independent existence of the entities to be put into relation. In this case, however, the *distinguishability* of the entities is actually *only* and *already* spatial. Therefore all we have is a mere *principle of distinguishability*. Actually, the most crucial, meaningful property of Newtonian space, which emerges in any debate on its nature, mainly consists in its *uniformity*, or *homogeneity*,[6] and this is precisely the basis of Leibniz's subsequent criticism, which is very difficult to object to.

In conclusion, the extracts quoted above from *De Gravitatione* prove the *difficulty to find an ontological grounding for space* in any case. Therefore, the thesis of space having the nature of a *substance* or *accident* appears to be untenable. On the other hand, this situation does not jeopardize the validity of the spatial-temporal foundation of mechanics at all, since it is based on the methodological components of Newton's thought rather than on his ontology.

Leibniz: Leibniz's conception is the *only fully relational position ever articulated*. As well known Leibniz identified the foundation of Being, both physical and ideal, in an infinite continuity of *substantial units*, intrinsically *single* and *isolated* ('monads'), a kind of 'metaphysical points', whose essence is a 'force' or 'activity of representation', i.e. essentially a *process*. Space and time become *phenomena*, i.e. modes of appearance of reality, generated by the process of monadic representation, and their unit only consists in the *hypostatization of order relations*. According to Leibniz, space (and time) do not possess any *substantiality*, or *objective extensionality* in the Cartesian sense: they are not *real entities*, i.e. they do not include the principle of their own reality. Therefore, in addition to its relational perspective, Leibniz's view is fundamental precisely because for the first time space and time are conceived as *idealities*, i.e.—in current terms, perhaps a little far-fetched—as essential features of *subjectity*. Obviously, the fact that Leibniz's conception can be *purely relational* is due to the *non-spatiotemporal* nature of the monads. This involves a price to pay:

[6] It is well known, however, that the points of an n-dimensional *homogeneous space* cannot have any *intrinsic individuality*. As Hermann Weyl puts it (see [52], *mine emphasis*): "There is no distinguishing objective property by which one could tell apart one point from all others in a homogeneous space. At this level, fixation of a point is possible only by a *demonstrative* act as indicated by terms like 'this' and 'there'." Quite aside from the phenomenological stance implicit in Weyl's words, there is only one way to individuate points *at the mathematical level*: namely by coordinatization, which transfers the 'individuality' of *n-tuples* of real numbers to the elements (points) of the topological set. It should be clear, however, that what is about here is a purely *mathematical individuation* rather than a *physical* one, an operation that, by its very nature, is quite arbitrary. This problem becomes a fundamental issue for the *non-homogeneous* and structurally very rich space-time of (GTR) in certain cases (see Sect. 2.2, Hole Argument).

namely that it is purely metaphysical and was never translated by Leibniz, or anyone else, in a concrete scientific programme. I report here two significant glances at Leibniz's views:

> Space, like time, is something *not substantial*, but *ideal*, and consists in possibilities, or in order of co-existents, that is, in some way, possible. And thus there are no divisions in it, but such as are made by the mind, and *the part is posterior to the whole*. In *real things*, on the contrary, *units are prior to the multitude*, and multitudes exist only through units. *The same holds for changes, which are not really continuous*' ([29], *italics mine*).

> And also

> Space being *uniform*, no reason whatsoever can exist, external or internal, for distinguishing among its parts and for making any choice whatsoever among them. Indeed, any *external reason* for distinguishing among the parts of space can only be founded upon some *internal reason*. Otherwise we should discern what is indiscernible, or choose without discerning ([2], *italics mine*).

From the perspective of the problem of objectity and realism (in the sense of Agazzi) [1]. Leibniz's position is undoubtedly *anomalous*. We are dealing with *strong metaphysical realism* according to which *space and time are not real*, and at the same time with *maximum objectivity in the sense of intersubjectivity or invariance of possible descriptions* according to the current meaning of the term. Indeed, Leibniz is an important forerunner of relativistic thought. Let us note that invariance with respect to possible descriptions, referred to Leibniz, is based on the *uniformity* of space, that is, according to his own point of view, on the characteristic itself which guarantees its *metaphysical irreality*.

There exists extensive contemporary literature which refers to Leibniz to support relational theories in connection with relativistic theories (GTR mainly). Such literature, however, actually refers to the 'Leibniz of positivists', as correctly defined by Michael Friedman, and gives rise to a falsification of the terminology of Leibniz's relationism. Apart from some structural analogies, the argumentations made by current Leibnizians are (obviously) unrelated to Leibniz's original metaphysical premises and lead to a kind of *incomplete* or *impure* reduction of spatial ontology. Their relational theses are actually forced to consider relationships between (real or possible) *empirical* objects which are *already* extended and, above all, already distinguishable as regards *extension*. Such theses cannot therefore avoid *elements of* '*spatiality*' amongst *the primitive ideas* which the reduction should lead to. Incidentally, this limitation of the relationist viewpoint shows the specific resilience of the *extensionality* of space. Similarly, current *substantivalist* theses ([4, 12] and bibliography therein) are criticizable when they expect to transfer Newtonian absolutism into the so-called '*manifold substantivalism*', i.e. the thesis according to which relativistic points in spacetime void of matter and fields be 'substances'.

Since the *Euclidean catastrophe* caused by the disgregation of Kantian *a priori*, a profound divergence has taken place between the levels of the *ontological grounding* of the concepts of space and time, and the *physical-mathematical description* of such concepts. Riemann's [42] fundamental works gave rise to a crucial generalization of the concept of space, from the tridimensional *Raum* of our experience to the multi-dimensional space as abstract *Mannigfaltigkeit* which, being the basis of differential geometry, constitutes an essential premise for relativistic theories. For my subsequent argument, it is important to realize that the concept of space as *Mannigfaltigkeit* introduces a complexification of the scientific image in which axiomatization acquires increasing relevance. Actually, the *new notion of space* includes a whole stratification of polyadic predicates which cannot be reduced to *extensional* ones, such as, e.g. (local and global) topology, with its companions of orientability, connectivity, enantiomorphisms, orientation-entanglement, non-commutative geometry, which would be better defined as *intensional* and require a base of *axiomatic structures.* However, as Weyl stressed [53] the transition to the new conception of space, which gives rise to the modern *scientific image* of spatiotemporality, entails overcoming a philosophical 'abyss':[7] precisely the jump from the notion of space as an *external continuum*, understood as *real* and *sensed intuition* (just the '*Raum*' of experience), to the abstract set-theoretical notion of it, by means of which theoretical physics reconstructs the continua of space and time.

On the basis of an objective *current analysis* of the classical debate, we should conclude that—setting aside the level of supernatural explanations inherited by Newton from Cambridge Neo-Platonism—*neither the absolutist viewpoint, nor the relational* one can provide a satisfactory *ontological* grounding to the notion of space. The concept of space does appear to have a peculiar fate: it has the strength to generate emancipation from Platonian-Aristotelian ontology, yet it does not have the solidity to stand metaphysically. I do believe, in fact, that a *metaphysical issue of space remains open.*

2.2 Remarks on the *Status* of Space-Time in the *Scientific Image* (GTR)

The structure of spacetime in the scientific image provided by the General Theory of Relativity (GTR) is by far more complex than one could believe on the basis of the historical view based on the manifestly covariant scenario expressed by the invariance under arbitrary transformations of coordinates (*passive diffeomorphisms*). We have shown ([30, 31, 35] and bibliography therein, also [9]). that 'the last remnant of physical objectivity of space-time' that Einstein thought (in 1916) [14] was 'taken away by the requirement of general covariance' (essentially on the basis of the famous 'Hole Argument' or *Lochbetrachtung*) can indeed be recovered, at least in a continuous family of models of GTR, by exploiting the intrinsic degrees of freedom of the gravitational field in vacuum (viz. the so-called *Dirac Observables* (DO)). The remaining functional part of the metric field (viz. the *gauge* variables) identifies a global non-inertial extended space-time laboratory (NIF) playing the function of a global Observer. The resultant physical individuation of point-events is obtained in terms of *non-local* functionals of the DO which show a *rich structure* of spacetime so that the latter—unlike Newton's space—maintains '*its own way of existence*'. Such individuation, however, turns out to be NIF-dependent so that it can only be construed as a *weak* form of *objectivity.* Correspondingly, one could interpret the resulting *ontology* of spacetime in the *scientific image* as a weak kind of *structuralism*

[7] Often defined as *Platonic Gap*.

that inherits some elements of both *substantivalism* and *relationism.* In any case any form of *substantivalism* turns out to be falsified. Our position has been examined by other authors within a more general philosophical context and redefined as *moderate structural realism* [16]. However, apart from the lack of generality of our results, I do not believe that the analysis of the ontology of spacetime of GTR can be confined to this level. In any case we have admittedly solved a problem about the characterization of the *intrinsic* spacetime structure rather than an *ontological* issue about the existence of spacetime. I shall limit myself to making the following remarks: (1) The very fact that GTR predicts singularities—in which the equivalence principle does not hold—indicates that it cannot be a universal theory of spacetime. (2) The non-locality shown by our individuation procedure is an unclear, yet profound feature of the theory: for example the total energy is a non-local thing and resides non-locally through space. The same *gravitational* energy is a rather elusive thing.[8] (3) Many authors believe now that the incompatibility between GTR and QT could only be resolved by abandoning the idea that space is a fundamental entity and that, correspondingly, the debated issue of Quantum Gravity must be dealt with by forgetting in the first place about space and movements within it, and by starting from *a purely quantum core*. From this point of view *space itself* should be viewed as an *emerging entity* and the same GTR should be considered as a purely *effective theory* arising from a quantum *substrate*. Thus gravity should turn out to be a non-fundamental, rather an induced, residual, emergent and intrinsically classical interaction, and the quantization of GTR would not lead to any fundamental theory. Of course, the explanatory power of GTR or its experimental success are not at stake here. The point is that all these features provide some indication that the ontological *elusiveness* of the concept of space infects—as it were—even the spacetime representation in the current *scientific image.*

2.3 The Kantian Legacy

Why we cannot exercise the intellect on any object absolutely apart from the continuous, or apply it even to non-temporal things unless in connexion with time, is another question.
Aristotle[9]

The philosophical configuration characterizing the unsolved classical opposition of absolutism and relationism also justifies the strength of the lasting suggestion exercised by Kant concerning the conceptions of space and time in spite of the disgregation of the *a priori* and even in spite of the fact that the Kantian notion of *pure intuition* as necessary intermediate element between sensation and intellect has

[8] It should be stressed, however, that this non-locality has nothing to do, at least at our present level of knowledge, with the quantum non-locality discussed later.

[9] In Aristotle: *On Memory and Reminiscence*, written 350 B.C. (translated by J.I. Beare): 450a 7–9 of the standard Bekker edition.

been empirically reduced in terms of the theory of vision[10] or absorbed within the constructive symbolic forms of neo-Kantians. The strength of the Kantian position concerning space and time, resides in the evidence of the fact that, within any critical formulation concerning such issues, an intuitive representation of extension and duration is always in some way already epistemically *presupposed.* The scientific irrelevance of Leibniz's fully relational view can in fact be traced to the impossibility of deriving or constructing space and time starting from elements that are—as it were—completely 'isolated'. In other words, spatial and temporal relations seem unable to take shape starting from the union of simple elements.[11] *Of course Kant's notion of intuition cannot be re-proposed today* with *all* its implications, in particular in relationship to its connection with *intellectual schemata*, as generating *a priori* synthetic judgments, which, as such, are apodictic and constitutive of the mathematical structure of space and time. Nevertheless, I do believe that the legacy of the Kantian notion of pure intuition of homogeneous extension, as an essential feature of the real (sensible) notion of space-and-time should not be jettisoned altogether. In the first place I observe that this legacy actually survives in various forms within phenomenological thinking.[12] On the other hand, the universal form of the homogeneous intuition has the nature of an *epistemic substrate* for all the particular sensible intuitions and is essentially characterized by the fact that the *sensible spatial continuum* cannot be linguistically coded.[13] In conclusion it should be clear that we must not confuse the vague continuity of perceptual wholes (let me call this the *psychological continuum*) with the *ideal continuum* of *homogeneous extension*, which is a presupposition of our notion of the continuum in general, before its bifurcation in spatial and temporal continuity. Surely, we cannot think of space and time, hence of the external continua, as 'objects'. As a matter of fact, we do not perceive space or time; we perceive and think of inhomogeneities and variations over an extended homogeneous background. The

[10] I would like to stress that, lacking a true causal theory of perception, this move only amounts to a shift of the surviving problem of the intuition of extension.

[11] The essential point being the unrestrainable and spontaneous nature of the intuition of the continuous extensionality. Recall what Einstein said in this connection: 'to give up the continuous ... is like to breath in airless space'.

[12] As a first instantiation I would like to mention Husserl's notion of *pre-phenomenal continuum* as a primary, non-inferential grasp of pure extension, which is neither based on nor reducible to perception, let alone memory or introspection. In Husserl's words: (see [23]) "There exists a *Primacy of Extension*", and "*Pure spatiality can be acquired as a fundamental form of thingness*". Also "Pure extension is conceived as a singular *substratum* for all intuitions and perceptions of particular objects". Analogous notions concerning the continuum as perfect qualitative, *non-compositional homogeneity,* can be found in Peirce, Brentano, Stumpf, Salanskis, Weyl and Poincarè too. See [40].

[13] All efforts to represent an abstract entity through a *schematization* unavoidably lead to the visualization of something prominent over a continuous background (or memory of a process acting in space). The *points* which are ideally introduced into the homogeneous extension are only preliminary elements for the *axiomatic categorization* of space. Only within the axiomatic structure do they become monadic referents of symbols and objects of quantification, and thereby '*individuals*' in that they are '*numbered*' (though almost all non computable in the algorithmic sense of the word). It is only within this construction that we can pose the problem of the potential or actual infinity of elements, while the pure intuition of the homogeneous extension appears to be necessary '*in act*' only as a singular datum. We can conclude with [40] that the continuum possesses an intrinsic *bi-modality,* that of the *homogeneous pre-phenomenal intuition* and that of the *axiomatic structure*, both fundamental.

latter—as singular intuition—realizes the idea of a homogeneous spatial extension with no body positioned *in* it. The pre-Cartesian *synthetic* continuum of geometry is just a conceptual abstraction of this intuition.

Finally, I wish to close this survey of Kantian legacy by emphasizing that we are transparently reminded of the unique way in which *the conception of space is strictly connected to the issues of subjectivity* by the following passages in the *Critique of Pure Reason*: ([26], *italics mine*). "In order that certain sensations be referred to something *outside me* (that is, to something in another region of space from that in which *I* find myself), and similarly, in order that *I* may be able to represent them as *outside* and *alongside* one another, and accordingly not only as different but as in different places, *the representation of space must be presupposed*". Thus, not only is the presupposition of the representation of space necessary to recognize things as *spatial*, *distinct* and *next to each other* but, above all, to recognize them as "*outside the subject*". Furthermore, "I am conscious of my own existence as determined in time. All determination of time presupposes something permanent in perception. This permanent cannot, however, be something in me, since it is only through this permanent that my existence in time can itself be determined. Thus perception of this permanent is possible only through a thing outside me and not through the mere representation of a thing outside me; and consequently the determination of my existence in time is possible only through the existence of actual things which I perceive outside me. ... In other words, the consciousness of my existence is at the same time an immediate consciousness of the existence of other things outside me." ([26], added in the 2nd edition p. 245.)

The representation of space is therefore essential even to the *self-individuation* of the subject and to recognize the *plurality* of conscious subjects. I cannot disentangle such considerations from the well-known fact that conscious states themselves *do not belong to the spatial order.* They are *per sé* metaphysically irrepresentable and, in order to be referred to, they require *linguistic reference* to space (note that '*inner* sense', referring to time, is a spatial locution, like *outer* sense). What matters for my subsequent argument is that it clearly appears that *perceptibility of distinguishable objects in spatial extension* must be seen as a *necessary precondition of subjectivity.*

3 The *Quantum*

3.1 Essential Quantum Physics

I believe that the great majority of people belonging to the science community at large, mainly influenced by the complexity and operational efficacy of the *formalism* of QT, may not be fully aware of the philosophical radicality of the historical event represented by Planck's discovery. Namely the *physical fact* that the *action* is made up of *indivisible* units (*quantum*), measured by the Planck constant h. It is important to realize in the first place that the action is a theoretical entity (of the classical description) which is neither a *spatial* nor a *temporal* quantity, nor is it a property of *things*, yet it encodes both spatiotemporal and dynamic components. In other words, what turned out to be *atomized* are *processes* instead of *things*: the true *atom* ('indivisible') of contemporary physics is then the *quantum of action*. Thus we have a dramatic shift from the naïve or spatio-temporal atomism (atoms as *simple* and *indivisible spatial entities*) to action-atomism (atoms as *indivisible elementary processes*). The consequence is that *it is not the small physical size* (i.e. a spatiotemporal characterization) *that defines the quantum level,* but something more subtle concerning processes in terms of time and energy differences (see later).

Let me stress the main immediate outcomes of this revolution. Let us consider a whole as an aggregate of putative parts and an inner dynamical process of interactions among such putative parts. In traditional classical terms, this process would be conceived as a variety of exchanges of energy ΔE at some intervals of time Δt, as well as transfers of linear momentum ΔP along some intervals of space Δx. The expression $A = \Delta E \Delta t - \Delta P \Delta x = \Delta_\mu x^\mu$ is just the relativistic invariant expression[14] of the action *phase*, and the putative parts of the whole are then thought of as exchanging action among themselves. Now, the atomization of the action entails that there cannot exist real processes corresponding to exchanges of action smaller than the Planck constant. Even more, the *elementary quantum act* (corresponding to a single *quantum* of action exchanged between two putative parts of a whole) is *simple*: literally it has no spatial *extension* nor *duration*. More generally, all the *genuine quantum processes,* in a deep sense *do not belong* in the *extensional* space and time (I mean in the sense of the *Raum* and clock time of our real macroscopic experience),[15] so that they cannot be *represented* as taking place in *spatial extension* during lapses of time. Since the action is, classically, a continuous functional of the physical system's configuration, were a continuous spatiotemporal description of the interacting parts conceivable, there would be real intermediate states, and one would be able to reconstruct processes corresponding to actions of arbitrary measure. In conclusion, *stricto sensu*: (i) the parts cannot be described in any local way as entities in the *extension* (and time) of our experience so that they simultaneously lose their traditional *individuality*. In fact, the parts can no longer be conceived as *distinguishable individuals,* i.e., they are no longer *objects* in the ordinary sense of perceivable things. *If the parts were objects, the action could not be atomized*! Furthermore (ii) any genuine quantum phenomenon is an *undivided whole* and cannot be broken up into physically well-defined steps; (iii) '*Quantum ontology is one of abstract entities, though not of mental ones*' (as stressed by Quine). Only the revelations or *measurements* of the *effects* of quantum processes are *phenomena* in the *Raum* and time of our experience.[16] Revelations, however, *are not genuine quantum processes*. They are highly complicated semi-macroscopic processes magnifying quantum events to the classical level by producing irreversible traces in ordinary Raum and time so that they are indeed characterized by *quantitative measures of extension and duration* and are *perceived*

[14] Note that, unlike action, energy is not a relativistic invariant and its quantization is derivative.

[15] It is essential to be clear on this point. To assert that quantum processes do not belong to the extensional Raum and time of our experience does not mean that they remain within an unphysical realm or a blurred metaphysical Nothing at all. Simply, due to action atomization, they cannot be described in the Raum and time of our macroscopic experience. See Sect. 4.1.

[16] It is important to stress, with John Bell [5], that *at the end* in physics, and in quantum theory in particular, the only measurements we must consider are *position* observations in *space.*

as *actual occurrences*. However, *they fail to be describable by quantum theory*. At this point I must assume that the reader is sufficiently aware of the main technical elements of QT, in particular (according to a rather rough description): (A) (1) An individual quantum system S is described by a *state vector* V (or *wave function*) belonging to a complex vector space H (Hilbert Space); (2) The physical *observables* are described by suitable *operators* acting on V and furnishing the *a priori* possible values ('eigenvalues') of the *properties* of S, as described by V; (3) A typical textbook description[17] of a *measurement cycle* concerning a process of S is a *man-made laboratory procedure* defined by the following operations: (3a) 'preparation' of the *state* of the system at time t_0, as represented by a V constructed after the determination of the values of the measured properties of a so-called 'maximal set of commuting *observables*: $A, B, C, \ldots, P$'. Note that V, as interpreted in the *standard view* I am sharing here,[18] is a pure *informational entity* (a "probability catalogue", according to Schrödinger) and does not represent directly a *real physical quantity* in space and time. It is a mathematical device that collects *all* the theoretically possible information that *we* have about the state of the individual system S ('pure state' V); (3b) temporal development of the process, as described in a purely *symbolic and abstract* way by means of a *linear, deterministic* and *unitary* evolution 'operator' $\mathbf{U}$ in terms of the standard time indicated by a macroscopic clock T (time dependent Schrödinger equation in the so-called Schrödinger representation) until a given time t; (3c) 'measurement' M of a physical *property* 'p' of S at time t, which must be one among the *a priori* possible 'eigenvalues' of an operator observable P. The rules of QT then furnish the *a priori probabilities* for finding a given real 'eigenvalue' 'p' among the possible ones. Note that, by embodying the completeness of the information inscribed in V, such probabilities are *ontological* or intrinsic, and not *epistemic* probabilities that could follow, e.g., from some ignorance about the definition of V. Such intrinsic probabilities emerge from the fact that, in order to be measured, any genuine quantum entity (like S) is forced to assume manifest *object-like* appearances (i.e. a spatial and temporal *shape,* e.g. as 'particle' or 'wave') in the *extensional Raum* and time. Corresponding to the *measurement* M, the state vector V 'jumps' and coincides *by definition* with the 'eigenvector' of the observable P corresponding to a *definite* 'eigenvalue' or property 'p' of the system S. Note that M (also called *reduction of the state vector* or *wave function collapse*) is a *pragmatic ad hoc* operation (*Postulate* of the '*Wave Packet Reduction*': WPR) which *instantaneously* redefines the state as represented by the State Vector V_p, which embodies the new information about the system.

Here is where a main issue lies, however. For, if taken outside the formal set of postulates of QT, it is not clear in the first place what the difference is between a 'measurement' belonging to the list of postulates or to the set of highly idealized laboratory man-made operations, on the one hand, and the 'measurement-like' processes that are going on, more or less all the time, more or less everywhere [6], on the other hand. In the latter case, the state vector V (or the wave function) should be better interpreted as representing *something physically real*, and the variation of the state

[17] I am obviously describing a typical but simplified situation.

[18] The choice of this interpretation—-at least in certain definite circumstances—will be justified later on.

vector under such generalized 'measurement' should be intended as describing a real physical process as ruled by a *second kind of evolution law* (called **R** by Penrose). **R** would enter into play when different macroscopic effects are triggered by different microscopic situations, and—in the current status of the theory—is supposed to be described (in principle and only elliptically) in a *pure phenomenological way* by a *non-linear* and *stochastic process* (still reduction of the state vector or wave function collapse). However, for reasons that will become clear at the end, from here on I shall stick to the standard interpretation. Thus, the laboratory quantum theoretical description of any genuine quantum process begins and ends with a *spatio-temporal revelation.*The *Heisenberg inequalities* then enforce the necessary linguistic compromise between the description of the unavoidable *object-like* nature of the revelations in *extensional* Raum and time, and the fact that quantum entities or processes *are not object-like* (neither 'particles' nor 'wave', nor any other spatio-temporal model of object whatsoever). I believe that nothing better than the following passage by Werner Heisenberg can render the essence of the whole state of affairs: "Während also die klassische Physik ein objectives Geschehen in Raum und Zeit zum Gegenstand hat, für dessen Existenz seine Beobachtung völlig irrelevant war, behandelt die Quantentheorie Vorgänge, die sozusagen nur in den Momenten der Beobachtung *als raumzeitliche Phänomene* aufleuchten, und über die *in der 'Zwischenzeit' anschauliche physikalische Aussagen sinloss sind*" ([21], *italics mine*).

There are two important instantiations of quantum states, namely: (B) A 'pure' state Σ, defined by the so-called '*coherent superposition*' of two distinct 'pure' states $\Sigma = c_1 V_1 \,(+)\, c_2 V_2$, ($c_1, c_2$ complex numbers with *fixed relative phase*), is called a *superposition state* and has the following fundamental property: if an observable A possesses *with certainty* the distinct values a_1 in V_1 and a_2 in V_2, the observable A for the state Σ does not possesses an *a priori* definite value, i.e. *no intrinsic property* qualified by A. The value of A in Σ is mere *potentiality* which is *actualized* only by means of a *measurement,* that furnishes the values a_1 *or* a_2 with *probabilities* given by the *squared moduli* of c_1, or c_2, respectively. The fundamental point is that the operation $(+)$ can never be translated into a logical disjunction, so that it is *incompatible* with assertions like 'the observable A possesses (*before the measurement*) one *or* the other of the properties associated with the pure states V_1 and V_2'. The so-called *Principle of Superposition* shows the most direct consequences of the *atomicity* of the action and the intrinsic limitations of the role played by spatiotemporal concepts at the quantum level. Indeed, it provides the technical representation of the fact that *a transition to actuality is never spatiotemporally described* since the *formalism* furnishes only the probability $P(t)$ that at time t the actualization has taken place, and the probability $1 - P(t)$ that it has not taken place.[19] (C) A state E which is an *entangled state* (firstly defined in [45]) of a compound quantum system $S = S_1 \,[+]\, S_2$,

[19]Correspondingly, in the case of a *spatial superposition* of two possible paths, p_1 and p_2, followed by, say, a photon γ in an interferometer, the condition of γ being in a state of superposition is incompatible with the following statements (which *exhaust all the logical possibilities* according to a standard *spatiotemporal representation* of the process): (1) γ does follow p_1, *or* p_2; (2) γ does follow both p_1 *and* p_2; (3) γ does follow *neither* of the two. In this connection, one also realizes the fundamental fact that, unlike the mere auxiliary role played in classical physics, *complex numbers* have an essential, irreplaceable role at the quantum level of reality.

where S_1 and S_2 are non interacting and (arbitrarily) *space-like* separated, is such that the *potentialities* of the component subsystems are *interconnected,* so that *the potentiality* of one subsystem *is instantaneously actualized* as soon as *the transition to actuality is induced in the other subsystem* (for a brief and clear presentation of the issue see [36, pp. 279–299]). In other words, the subsystems cannot be considered as *separate* and *independent* entities until they are eventually *disentangled* by a *measurement*.[20] The entanglement lies at the basis of the so-called EPR type phenomena[21] and expresses an intrinsic *non-locality* of quantum theory. What matters here is that not only has such phenomenon actually been experimentally verified up to a distance of 144 km [51], but that nothing exists in quantum theory that could in principle introduce any limitation concerning such kind of extensional separation! From this lack of limitations[22] it further follows that even the *temporal order* of the disentangling measurements is observer dependent. We can conclude that, *provided* a so-called *realistic interpretation of the state vector* is excluded [38, pp. 290–296, 388–391], neither entanglement or disentanglement heed *spatial extension or temporal ordering*. In other words: no story in space as a function of time can tell us how nonlocal correlations happen, hence nonlocal quantum correlations seem to *emerge* somehow from *outside* space. Note that the essence of the entanglement is strictly ontological and cannot be explained epistemically by saying e.g. that our knowledge of S_1 is inseparable from that of S_2.

G.C. Ghirardi [20] has recently shown that *any theory* which (a) reproduces the experimental consequences of quantum theory, and (b) satisfies certain very general conditions concerning the composition of probabilities, is necessarily *non-local* (basically the probabilities must be defined before the experiments take place). *Quantum nonlocality* then holds *independently of the specific formal structure* of the theory and follows uniquely from the *atomicity* of *action.* This means, however, that *Nature itself is essentially non local,* even if this fundamental property only surfaces when few action quanta are exploited in entanglement type conditions. In other words, there is necessarily some kind of 'influence' that connects quantum microentities across spatiotemporal unlimited space-like separations. Nonlocality, however, taken at face value, is only a technical way of speech which, in my reading, is a sign of the fact that although our intuitive representation of the underlying spatial continuum has an *epistemic primacy* for the sensory experience and ordinary perception in the *Raum*, it fails to possess a sound *ontological* basis. This should be read as a confirmation of its already historically recognized *elusiveness*. It is also important to stress that non-locality and process atomization only clash with the intuition of *pre-phenomenal continuum,* and *not* with all the axiomatic structures which constitute the concept of physical space-time of the contemporary *scientific image.* Finally, although there is no *operational conflict* between quantum theory and, e.g., STR, still—as Penrose thinks—there is a conflict with the 'spatiotemporal spirit' of STR.

[20] Note that it is not correct to characterize the transition from potentiality to actuality occurring in a subsystem as an 'event', just because it fails to share one or more of the classical properties of events, notably that of being *loci* of permanent actuality.

[21] The possibility of such phenomena was indeed suggested by Einstein, Podolski and Rosen in a famous paper. See [15].

[22] In a more general, relativistic, context, from the space-like character of the relation between S_1 and S_2.

3.2 Remarks on the *Status* of Spacetime in Relativistic Quantum Field Theory

The difficulties of Relativistic Quantum Field Theory (RQFT) are well known. Above all, for our purposes, we must stress that a consistent relativistic interpretation of the quantum collapse of the vector state has not even been touched in RQFT. Quite independently of these important limitations, however, one should ask how it is that, if *extensionality* of spacetime is so *elusive*, background relativistic space-time is thoroughly been used so successfully for the phenomenological description of elementary particles. Let me call the spacetime utilized in RQFT '*micro-space time*'. My answer is that, although quantum theory imposes no limitations on the utilization of space-time continuum, its epistemological *status* has been already substantially altered in a peculiar way. From the chrono-geometric point of view, the *micro-space-time* is a universal classical deterministic macro space-time (actually Minkowski's space-time) used *without any scale limitation from below*. As a matter of fact, however, it is introduced into the theory through the group-theoretical requirement of relativistic invariance of the *statistical* results of *measurements* as regards the choice of the *macro reference frames*. Therefore, this *micro space-time* is *anchored,* so to speak, to the macroscopic *medium-sized objects* that *asymptotically* define the experimental laboratory conditions so that its concept is parasitic of the perception of distinguishable macroscopic objects in the Raum. It is in fact in this asymptotic sense that a physical meaning is attributed to the classical space-time *coordinates* on which the quantum fields' operators depend as parameters. Thus, the spatiotemporal properties of the *micro Minkowski manifold*, including its basic *causal structure*, are, as it were, *projected on it from outside*. Also, the limitations posed by quantum theory on the procedures for measuring *microscopic space-time intervals* (easily reducible to *time intervals*) are very strong indeed. Actually, it can be shown *that every* possible kind of clock is necessarily a *non-microscopic object* and, in particular, that even the so-called *atomic clocks* can be considered microscopic devices only on the basis of a rough idealization which conflicts with the fundamental concepts of measurability. There exists no way of measuring microscopic intervals (e.g. intra-atomic)[23] in a *direct operational manner*. All measurements are then based on *indirect procedures* that involve a *theoretical interpolation*. However, while this is true of nearly all physical measurements, the special aspect of the microscopic case is that the theory involved is precisely the quantum theory with its inherent spatio-temporal limitations. In this connection see [39, 43].

Thus, we are led to distinguish between *physical* space-time, on the one hand, and *physical* '*analogical*' *micro-space-time*, on the other. The former as operationally founded on the behavior of (macroscopic) *clocks*, the latter as a purely *mathematical structure* whose empirical meaning rests on a theory that in principle excludes a spatiotemporal representation of its ontology. Far from worrying about a direct and operational justification, what I wish to emphasize is that the empirical meaning of the '*analogical*' *micro-space-time* is conferred on it solely by the *macroscopic* (*classical*) *level of control* of quantum theory on the basis of an abstract extrapolation

[23] Not to mention "cosmological" intervals!

from the 'macro' space-time of STR.[24] In conclusion, the role of *this 'analogical' micro-space-time* seems essentially that of a *translator* from the symbolic and abstract structure of the theory into the language of the macroscopic irreversible traces that constitute the experimental findings within the *Raum* and time of our experience.[25] The conceptual *status* of this *external translator* then fits in very well with the role of an epistemic precondition for the formulation of relativistic quantum field theory in Bohr's sense. Finally, a further sign of the uncertain *status* of the '*analogical*' *micro spacetime* in RQFT comes from the so-called 'peaceful coexistence' between the latter and the STR, but I will not elaborate on this point further. I will simply conclude with Penrose [36, p. 290] that: "Despite the undoubted power and impressive accuracy of quantum field theory (in those few cases where the theory can be fully carried through), one is left with a feeling that deeper understandings are needed before one can be confident of any 'picture of physical reality' that it may seem to lead to".

3.3 The **Z**-Mysteries and the **X**-Mysteries of Quantum Theory

On May 15, 2010, seventy five years had elapsed since the publication of the Einstein-Podolsky-Rosen paper in Physical Review (vol. 47, May 15, 1935). From then onwards, foundational debates about quantum theory essentially centered around two distinct issues. The former deals with questions such as non-locality, non-separability, indefiniteness of property attributions, and the EPR phenomenon, with the connected idiosyncratic debate about the so-called 'spacetime realism'. It concerns the quantum behavior of entangled systems involving *only quantum-level entities,* such as electrons, photons, atoms or molecules.The latter is about the so-called *macro-objectification problem*, that is the formulation of a *quantum theory of measurement*, and a resolution of related paradoxes (*Schrödinger Cat* and the like). Roger Penrose [38, pp. 237–240] called these two classes of problems and paradoxes *Z-mysteries* and *X-mysteries*, respectively, and I will adopt his terminology throughout. The **Z**-mysteries, which are *directly experimentally supported*, depend on the *action atomization* together with the *linearity* of the theory at the underlying quantum level.

[24] Note, furthermore, that in the literature about classical field theories space-time points are usually taken to play the role of *individuals*, but it is often implicit that they can be distinguished only by the physical fields they carry. No such possibility, however, is consistently left open in a non-metaphorical way in RQFT. From this point of view, Minkowski's *micro-space-time* is in a worse position than the space-time of GTR: it lacks the existence of the Riemannian intrinsic pseudo-coordinates exploited in GTR, not to mention all the non-dynamical (better, operational and pragmatic) additional elements that have often been used for the individuation of its points, like rigid rods and clocks in rigid and un-accelerated motion, or various combinations of genidentical world-lines of free test particles, light rays, clocks, and other devices in macroscopic STR. The instrumental nature of *micro-space-time* seems to be further confirmed by the fact that the quantum field physical observables are not defined in terms of point-events but over extended regions of space-time and that the interacting quantum fields are essentially quantities that interpolate asymptotically defined states.

[25] Furthermore, it is true that this micro-space-time appears to be 'really' classically Minkowskian with respect to all non-gravitational interactions up to the length scale of $\approx 10^{-16}$ cm; yet, it should be noted that this fact is checked by collision experiments that involve only a limited number of *real* micro-entities.

They refer to counterintuitive and unavoidable features of microprocesses[26] which—I believe—should be incorporated in any future possible theory. The X-*mysteries*, instead, call into play the fundamental problem of the *objectivity of the macroworld* as *would be described by quantum theory.* For the rules of QT imply that not only must there be indefiniteness of property attributions to the microstructure in case of superpositions, but also that—if no additional and pragmatic assumptions are made (viz. WPR)—this lack of definiteness should be propagated to the macro-realm. As a matter of fact, the main difficulty lies in the attempt to find—within the theory as it stands—a definite *scale* at which the quantum level of activity, characterized by the persistence of quantum superpositions of different alternatives, actually gives way to the classical level. Thus, besides the *atomization of the action*, the **X**-mysteries follow from the requirement that the *present structure of the quantum formalism,* which was invented for the level of molecules, atoms and the subatomic level, be valid without modifications *even at the level of macroscopic extensional world*, and, in particular, for the description of the very *macroscopic measuring detectors*.[27] If, on the other hand, the measuring detectors are excluded from the quantum theoretical description *without* further qualification, *and* the informational interpretation of the state vector is refuted toward a 'realistic' view, it seems difficult to explain how the apparatuses can behave as required by WPR itself, an issue which appears to admit a pragmatic solution only. Finally, there exists Bohr's original solution which assumes the purely classical behavior of the apparatuses and a strict informational interpretation of the state vector. This view, however, does not resolve in a transparent way the question of why, and at what stage, 'classical' behavior might arise for large and complicated structures such as detecting apparatuses.[28]

A conclusive and very general result about the restrictions following from the linearity of quantum evolution for the measurement process, was obtained by Bassi and Ghirardi [3], who showed that such linearity necessarily entails that, at the end of any measurement process on an individual quantum system, *there must be* 'superpositions of macroscopically distinct states of the detector, and, in general, of a macro-system', *with the consequence that the result cannot match our definite spatial perceptions*. This in turn implies that '*the reality of definite perceptions in spatial extensionality requires that linearity of QT must be broken somehow and somewhere*'. It is essential to stress the generality of this result, which is immunized against all criticisms based, e.g., on the so-called quantum decoherence argument, as well as against subtleties concerning technical details of measurements.

It is impressive that decades of debate about the **X**-mysteries have not yielded any agreed upon, satisfactory solution. Therefore I shall end this section by stressing

[26] More precisely, not only processes at the underlying quantum level, but also processes such as the EPR phenomena involving revelations of microentities in *extension* at macrodistances, so long as energy differences ΔE between alternative possibilities remain very small (in terms of $\hbar/\Delta T$, where ΔT is the time scale of the phenomenon).

[27] Whose accurate theoretical description is an almost impossible task due to the number of degrees of freedom involved and the unavoidable entanglement with the environment which is out of control.

[28] Yet Bohr asserted very explicitly that: 'For the discussion of the quantum description problem *it is not relevant* that the existence of the quantum of action is ultimately responsible for the properties of the materials of which the measuring instruments are built and on which the functioning of the recording devices depends', see [7].

(with Shimony) [45] that "if sustained efforts to provide a realistic account of the actualization of quantum mechanical potentialities should prove to be unsuccessful, then there would be *a posteriori* reasons to conclude that the program itself is misconceived and to return *via some new route* to Bohr, by justifying a dualistic picture of the world: quantum and classical".

4 The *Quantum* and Space

4.1 The Main Conjecture: Is There a *Primary* '*Spatial*' *Reduction* on an Evolutionary Basis?

Est aliquid praeter extensionem, imo extensione prius.
G.W. Leibniz [28]

Let me first summarize the relevant conclusions reached in the previous sections:

(1) The ontological *elusiveness* or, rather, *inconsistency* of spatial extensionality resulting from the classical debate; also, the very ontological vagueness of the extensionality of spacetime in GTR.
(2) The Kantian legacy within the phenomenological tradition, concerning the epistemic *primacy* and *unavoidability* of the *pre-phenomenal extensional continuum.*
(3) The relevance of *perception* of distinct material objects in the extensional *Raum* of our experience, and its role as a *precondition for subjectivity* and self-consciousness in particular.
(4) The suggestions according to which the spacetime of the classical *scientific image* could be an *emergent reality* from a *quantum substrate.*
(5) The fact that micro entities lack *object-like properties* and that *genuine quantum processes do not belong to the spatial extension of the Raum*, except for the results of specific 'measurement procedures' which take place according to large scale irreversible amplifications.
(6) The intrinsic *non-locality* of Nature at the quantum level independently of the specific formalism of the theory; a property that, extending at macroscopic distances, gives a further impressive blow against the ontological robustness of the *extensionality* of space: non-local quantum correlations somehow seem to *emerge from outside space* and time.
(7) The nature of the **X**-mysteries and the conflicting views about their clarification.
(8) The fact that linearity of quantum evolution is a very well *confirmed phenomenon* only at the quantum level.
(9) The generality of the implications of Bassi and Ghirardi's results concerning the **X**-mystery of macro-objectification.

(10) Some heterodox suggestions made by physicists like Wigner,[29] Shimony [45, 46], and in a certain sense even Roger Penrose[30] about the special role that conscious or, more generally, living beings could play in the quantum riddle.[31]
(11) Finally, the fundamental recognition that '*quantum ontology is one of abstract entities*' (Quine), which I take mainly to mean that quantum ontology is a *non-spatial ontology* (again, in the sense of the *Raum* of our experience) and, therefore, in some sense *inscrutable*. This entails, however, in my opinion, that the macro-objectification problem, far from being a purely technical case, has all the marks of a deep technical *and* philosophical issue concerning the human (and animal) forms of acquaintance of 'material' reality.

To summarize, at this point, I will accordingly make the following assumptions:

(a) The linearity of the quantum dynamical evolution **U** does not apply in the central nervous system of living beings at a certain level of the evolutionary ramification, even very low.[32] For the sake of clarity, I assume that this happens in the human brain at a level that—from the point of view of subjectivity—corresponds to a *pre-conscious or unconscious* stage. From my point of view—*which is neither trivially subjectivistic nor instrumentalist*—it is essential to exclude any *active* role of consciousness or, more generally, subjectivity, in this mechanism. The basic justification of this conjecture is that, since we perceive distinct macroscopic objects in the extensional *Raum*, the linear nature of **U** somewhere and somehow must be broken. Thus, apart from realistic collapse theories like GRW, the choice of locating the violation within the neural system of living organisms seems to be the most natural one. Obviously, I will not even try to sketch the concrete quantum action on neural or subneural structures of living organisms in order to 'understand' perception.
(b) Corresponding to the onset of a non-linear dynamic evolution, a *primary* '*spatial*' *reduction is* '*continually*' *taking place*[33] to the effect that, starting from the *non-extensional and abstract structure* of the quantum world, the result be that of

[29] See [54]: 'The way out of the difficulties of the measurement problem is to postulate that the equations of motion of QT cease to be linear, in fact they are grossly non-linear if conscious beings enter the picture ... it follows that the being with a consciousness must have a different role in quantum mechanics than the inanimate measuring devices.'

[30] Penrose has repeatedly suggested that a fundamental link should exist between the possible violation of the linear Schroedinger evolution and perception processes (see [37, 38]).

[31] More specifically, Shimony [46] writes: 'The scientific community is contemplating seriously the idea that one should give up the unrestricted validity of the linear nature of the dynamical evolution. If this is the case, the physical processes which present themselves as the most natural candidates ... are mesoscopic processes, processes involving the neural structure of the brain and processes in which definite perceptions seem to require for fundamental reasons that reductions actually take place'.

[32] It is not relevant here to identify a limit or scale of complexity concerning the gradation of subjectivity that is distributed almost with continuity within the living world. I am prepared to believe that subjectivity and even consciousness are a matter of degree, and not simply something that exists or does not. As Penrose notes, 'what evidence do we have that lizards and codfish do not possess some low level form of subjectivity? ... After all, all I ask is that they sometimes simply feel!' (See [37, p. 383].)

[33] Technically, this unorthodox reduction model could mimic what has been called *continual observation* within the formalism of the so-called *measurements with* '*effect*' *values of Ludwig*. See [41].

constituting the *neural precondition* for the *intuition of extensional space* and for the *experience of distinguishable macroscopic objects* (as well as of any other *spatially distinguishable source of perception*), at the subjective level.

(c) At this point, my further and consequent suggestion is that—*as perceived* by living organisms—*all quantum superpositions of alternative* '*macroscopically*' *distinguishable states are accordingly suppressed.* Therefore, *one* or *the other alternative* would be resolved out, explaining the fact that *events do actually occur*, whether or not they had done so before the existence of living beings (the gain of using the inverted commas here and before will be explained shortly). In this way, consequently, *the* ***X****-mysteries of QT should be crossed out,* at least for what concern effects which are in principle perceivable by living organisms.

(d) This *primary reduction* would concern the *state of affairs* of physical processes (including the so-called 'measurement-like' processes) which 'are going on more or less all the time, more or less everywhere' in Bell's sense, though, of course, always *as perceivable in principle by living organisms.* The important point is, however, that such *primary reduction* would have no effect on the standard quantum processes concerning *micro-entities* which are the object of *laboratory man-made experiments*. So far no isolated microscopic system has been observed as *not* behaving in accordance with the linear **U** evolution, and correspondingly as undergoing wave function collapses. In other words, all the standard reduction processes of QT—which we could rename as *secondary or z-reductions*—will continue to take place, in connection with the standard *informational interpretation* of the wave function. In this way, *all the* ***Z****-mysteries of QT would survive unaltered.* Accordingly, "the procedures of measurements will maintain an essential influence on the conditions on which the very definition of the physical quantities in question rests" (*à la* Bohr). Note, moreover, that the distinction between genuine *quantum measurements* and the rest of the *state of affairs* is accordingly established in principle.

It is now time to try to redress and clarify some evident 'lop-sidedness' of this conjecture with some important qualifications. Beforehand, however, while once again acknowledging that my conjecture is rather hazardous, I wish to point out that, when confronted with the 'oddness' of QT, Heisenberg himself was prepared to admit something analogous when he wrote: "The *a priori* character of the classical concepts may be similar to that of the forms of behavior that in animal psychology are called inherited or innate schemes. *It is quite plausible that for some primitive animals space and time are different from those of the species man and, above all, may not belong to the world independently of man*" [22, p. 91].

In a series of works on the issue of *Emergence*, Herbert Feigl, Wilfrid Sellars and Paul Meehl [17, 33] introduced a remarkable distinction concerning physical language. Precisely: (1) *Physical-1*: an event, entity or process is *physical-1* if it belongs in the coherent and adequate descriptive and explanatory account of our *spatio-temporal causal world*. (2) *Physical*-2: an event, entity or process is *physical-2* if it is definable in terms of theoretical primitives adequate to completely describe the actual states, *though not necessarily the potentialities*, of the universe *before the appearance of life* (say, perhaps, our world in the pre-Cambrian period). It is evident that although useful for examining the logical structure of the concept of *Emergence*, the *physical*-2 language is a purely *metaphoric* instrument in the present case since its semantical content is empirically beyond reach *in principle*.[34] It can nevertheless be instrumental for some comments on my conjecture. First of all, although we are doomed to using the physical-1 language, *even in the formulation of a modified QT*, an implicit use of language physical-2 would be obligatory for the *very formulation* of my conjecture, and the use of 'inverted commas' (in 'b' and 'c') had the very purpose of hiding this impossibility. Let us denote with S_Q and T_Q the notions of space and time, respectively, which in language physical-2 are the necessarily abstract and symbolic *precursors* of the corresponding concept in our physical-1 *scientific image.* We are assuming that the latter emerge in some way from the former as the concept of *macroscopic* body emerges from its quantum structure. In this sense, 'spatial', 'continually' and 'macroscopic' in 'b' and 'c' above are the physical-2 counterparts of what becomes physically-1 describable *after* the *primary reduction* has taken place.[35] It should be recognized and stressed that we here have a huge *explanatory gap.* An *explanatory gap* of some sort, however, is present whenever one tries to lay a bridge from the *scientific image* to *subjectivity* in any of its forms. The specific philosophical content of such *explanatory gap*, however, is just a matter of dispute. Yet, I will not elaborate on this topic further in the present paper.

4.2 The Main Conjecture is *Falsifiable*: A Possible Negative Experimental Test

Of course, we cannot practically obtain quantum superpositions of *macroscopic states* to put the conjecture to the empirical test. However, an experimental test could be imagined if the following situation could be realized. Assume that at the neuronal or subneuronal level of the central nervous system a perceptual process could be triggered by a purely *quantum*, and sufficiently persistent, *coherent superposition* of different *microscopic states*[36] which, emerging from *macroscopic different spatial locations,* when considered by themselves would be mutually exclusive and would give rise to distinguishable spatial perceptions. Such a situation could indeed put the conjecture to the test, as we shall presently see. It is very interesting to find that a situation of this kind has been proposed in literature [19] and declared[37] actually feasible by quantum optics techniques. I shall limit myself to the essential, inviting the reader to see Ghirardi's paper [19]. First of all, it is known that the threshold for human visual perception is approx 7 photons. According to the accepted physiological knowledge, this is the *only firmly established case* in which *a truly microscopic system can*

[34] Imagine that a physical process that occurred on a planet at a distance corresponding in time to our pre-Cambrian period had been generated by *amplification* of a true *quantum* event and (possibly?) involved a *physical-2* "macroscopic" *superposition*. That physical process would be *humanly unobservable in principle,* yet no differently from to an analogous process that had occurred on a planet at a distance in time of four light years.

[35] It is worth noting that it is not by chance that, e.g., in the GRW model, the (real) reduction can be made only on *spatial observables*; a circumstance that enlights the essential role played by *space* within the problematic transition from the *shifty* quantum level to the classical one. See [18].

[36] *Microscopic states* here means states describing *a few* quantum entities (e.g. photons, electrons); see later.

[37] I hereby thank prof. Francesco De Martini for an illuminating discussion on the experimental aspects of the problem.

trigger a definite perception directly. We then consider a bunch of say 10 photons coming from a definite spatial region A and another bunch of 10 photons coming from a macroscopically distinct region B. Each of the two bunches propagate independently at different times towards the eye of a human observer O who will have two different perceptions, seeing a luminous spot in A and one in B, even though the states refer to microscopic systems. At this point, the experimentalist prepares a *quantum state* which is the *coherent superposition* of the two states considered and *which is assumed to persist* up to the moment in which O 'looks' at it. Using a photomultiplier, we can firstly test that the superposition is still there; this is a test that, 'making a measurement', will reduce the superposition into a 50% result of photons coming from A *or* from B. We then replicate the experiment with O replacing the photomultiplier. We will have *two different possibilities*: either (i) O replicates the behavior of the photomultiplier seeing a luminous spot in A *or* a luminous spot in B, each with $1/2$ probability and reducing the superposition; or, (ii) O will end up with a *confused spatial perception* and *unable to distinguish the macroscopically separate regions A and B*, a result that would prove that O's neural system *does not reduce the superposition*. The conclusion is that even if case (i) does not *obviously* constitute *per sé* a *proof* of the main conjecture, case (ii) will blatantly *disprove* it. The existential and philosophical import of this experience, however, will be incalculably disrupting!

5 Concluding Remarks: A Naturalized *Transcendentalism*

What results from my conjecture, if sound, is a *naturalization* of that part of Kant's *Transcendental Aesthetics* that can survive the Euclidean catastrophe.[38] The result is a new kind of *transcendentalism* to which a famous statement by Boltzmann aptly applies, namely [25]: "Space can well be transcendental without the axioms being so". It is a benign form of transcendentalism which, unlike Kant's, is open to include living organisms as entities having an *internality* and therefore a 'unity per sé', whereas Kant can only admit a 'unity' depending on the human transcendental Self, i.e. organisms as 'entia rationis' like machines.[39] Living organisms, however, act *immediately* and *ontologically* in nature without waiting to be 'constructed' by the intellect.

Another gain is the understanding of the ontological *elusiveness* of the concept of space. In conclusion, we could rephrase Kant's fundamental question upon the

[38] For the sake of comparison, it is worth recalling here, freely quoting from [34], some well-known Kantian theses in terms that appear particularly adjusted to my previous quantum argument: (1) space is a *pure form* of perception which we bring to the matter of appearances and impose on it. (2) Only by *making appearances spatial can we experience them as objective, outer and interrelated*, but we make them spatial without the aid of empirical instruction from the appearances. (3) *Appearances* come from '*things in themselves*' and what they instruct in us is always *a-posteriori*; therefore, if space is *a priori,* it does not come from things. (4) *Things themselves are not spatial*, *they are not extended* and they have no spatial relationship to each other. In conclusion, space is purely a gift of the mind, not of things. It is ideal, not real.

[39] See Kant's frustrated efforts in this direction ('the phenomenon of a phenomenon') in *Opus Postumum* (1936–1938) [27].

synthetic *a priori* judgments by asking 'How is QT possible (as a theory of *atomized processes*) if our sensible experience presupposes the intuition of the *phenomenal continuum* as epistemic precondition?' Answer: 'QT is possible because—as far as we know—it concerns an *ontology of abstract entities* (Quine) which can be described only in *symbolic* terms, together (of course) with well-experienced *physical correspondence rules*'. The specific nature of this transcendentalism depends on its being generated by a natural process like the suggested *primary reduction*. Of course, in my conjecture, a fundamental role is played by Natural Evolution.

From the point of view of QT, the conjecture, if sound, entails that *a scheme for quantum measurement of the von Neumann type* cannot even 'leave the ground'. As a matter of fact, macroscopic apparatuses are continuously being reduced[40] so that their effectiveness, as measuring quantum devices interacting with the quantum system to be measured, cannot be described by a WPR of entangled states in a Hilbert space. In other words, the conjecture supports—on a different basis[41]—Bohr's attitude concerning the measurement process.[42] *A quantum theory of measurement,* in this sense, *turns out to be unnecessary and in fact impossible.*

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References

1. Agazzi, E.: Temi e Problemi di Filosofia della Fisica. Abete, Roma (1974)
2. Alexander, H. (ed.): The Leibniz-Clarke correspondence. Fourth paper. Manchester University Press, Manchester (1956)
3. Bassi, A., Ghirardi, G.C.: A general argument against the universal validity of the superposition principle. Phys. Lett. A **275**, 373–381 (2000)
4. Bartels, A.: What is spacetime if not a substance? Conclusions from the new Leibnizian argument. In: Mayer, U., Schmidt, H.J. (eds.) Semantical Aspects of Spacetime Theories, pp. 41–51. B.I. Wissenshaftverlag, Mannheim (1994)
5. Bell, J.S.: Speakable and Unspeakable in Quantum Mechanics: Collected Papers in Quantum Philosophy, p. 166. Cambridge University Press, Cambridge (1987)
6. Bell, J.S.: Against 'Measurement. In: Milled, A. (ed.) Sixty-Two Years of Uncertainty, pp. 17–31. Plenum, New York (1990)
7. Bohr, N.: Discussion with Einstein on epistemological problems in atomic physics. In: Schillp, P.A. (ed.) Albert Einstein, Philosopher-Scientist, p. 199. The Library of Living Philosophers, Evanston (1949)
8. DiSalle, R.: On dynamics, indiscernibility, and spacetime ontology. Br. J. Philos. Sci. **45**, 265–287 (1994)

[40]One should be careful in not misunderstanding this sentence as a nonsensical assertion preventing the description of macroscopic bodies as having a quantum structure in terms of molecules, atoms, elementary particles etc. Fundamental scientific advancements as the formulations of Bose-Einstein condensation and Superfluidity are a impressive vindication of the quantum nature of macroscopic bodies. Their description, however, is not a purely quantum one and is based on skilled theoretical treatments which in any case involve a *semi-classical approximation*. So they are not *literally* 'macroscopic quantum objects'.

[41]Note that my conjecture is immunized against the critique raised by Shimony [46, Vol. 1] against Bohr's implicit renunciation to the traditional program of mutual support between epistemology and metaphysics.

[42]Note that even the intrinsic distinction between true measurements and 'measurement-like' processes reinforces Bohr's attitude.

9. Dorato, M., Pauri, M.: Holism and structuralism in classical and quantum general relativity. In: Rickles, D., French, S., Saatsi, J.T. (eds.) The Structural Foundations of Quantum Gravity, pp. 121–151. Clarendon, Oxford (2005)
10. Earman, J.: Was Leibniz a relationist. In: French, P., Wettstein, H. (eds.) Studies in Metaphysics. Midwest Studies in Philosophy, vol. 4. University of Minnesota Press, Minneapolis (1979)
11. Earman, J.: Why space is not a substance (at least not to first degree). Pac. Philos. Q., **67**, 225–244 (1986)
12. Earman, J., Norton, J.: What price spacetime substantivalism? The hole story. Br. J. Philos. Sci. **38**, 515–525 (1987)
13. Earman, J.: World Enough and Space-Time. MIT Press, Cambridge (1989)
14. Einstein, A.: Die Grundlage der allgemeinen Relativitätstheorie. Ann. Phys. **49**, 769–822 (1916)
15. Einstein, A., Podolsky, B., Rosen, N.: Can quantum-mechanical description of physical reality be considered complete?. Phys. Rev. **47**, 777 (1935)
16. Esfeld, M., Lam, V.: Moderate structural realism about spacetime. Synthèse, **160**, 27–46 (2006)
17. Feigl, H.: The 'Mental' and the 'Physical. In: Feigl, H., Scriven, M., Maxwell, G. (eds.) Minnesota Studies in the Philosophy of Science, vol. 2, pp. 370–497. University of Minnesota Press, Minneapolis (1958)
18. Ghirardi, G.C., Rimini, A., Weber, T.: Unified dynamics for microscopic and macroscopic systems. Phys. Rev. D **34**, 470 (1986)
19. Ghirardi, G.C.: Quantum superpositions and definite perceptions: envisaging new feasible experimental tests. Phys. Lett. A **262**(1), 1–4 (1999). *Erratum*: ibid. **263**, 465 (1999)
20. Ghirardi, G.C.: Does quantum nonlocality irremediably conflict with special relativity? Found. Phys. **40**, 1379 (2010)
21. Heisenberg, W.: Atomtheorie und Naturerkenntnis. Mitt. Univ. Bundes, Gött. **16**(1), 9–20 (1934)
22. Heisenberg, W.: Physics and Philosophy. Dover, New York (1958)
23. Husserl, E.: Ding und Raum: Vorlesungen 1907. Husserliana, vol. 16. Nijhoff, The Hague (1973). Edited by U. Claesges
24. Janiak, A.: Newton: Philosophical Writings: De Gravitatione. Cambridge University Press, Cambridge (2004)
25. Kahl, R.: Selected Writings of Hermann von Helmholtz. Wesleayan Press, Middletown (1971)
26. Kant, I.: Kritik der reinen Vernunft. Johann Friedrich Hartknoch, Riga (1781); 2nd edn. 1787. English translation by N.K. Smith: The Critique of Pure Reason. Macmillan, London (1963)
27. Kant, I.: Opus Postumum, Handschriftlicher Nachlass, vols. VIII, IX. de Gruyter, Berlin (1936–1938). Sez III
28. Leibniz, G.W.: Specimen dynamicum. In: Gerhardt, C.J. (ed.) Leibnizens Mathematische Schriften, vol. VI, p. 235. Halle (1850–1863)
29. Leibniz, G.W.: Philosophical Papers and Letters. Reidel, Dordrecht (1956). Edited by L.E. Loemker
30. Lusanna, L., Pauri, M.: Explaining Leibniz equivalence as difference of non-inertial appearances: dissolution of the hole argument and physical individuation of point-events. Stud. Hist. Philos. Mod. Phys., **37**(4), 692–725 (2006)
31. Lusanna, L., Pauri, M.: Dynamical emergence of instantaneous 3-spaces in a class of models of general relativity. In: Petkov, V. (ed.) Relativity and Dimensionality of the World. Springer, Berlin (2007)
32. Malament, D.: Review of Sklar 1976. J. Philos. **73**, 306–323 (1976)
33. Meehl, P.E., Sellars, W.: The concept of emergence. In: Feigl, H., Scriven, M. (eds.) Minnesota Studies in the Philosophy of Science, vol. I, pp. 239–252, University of Minnesota Press, Minneapolis (1956)
34. Nerlich, G.: The Shape of Space. Cambridge University Press, London (1976)
35. Pauri, M., Vallisneri, M.: Ephemeral point-events: is there a last remnant of spatiotemporal objectivity? Festschrift in honor of Roberto Torretti on his 70th Birthday. Dialogos, **79**, 263–303 (2002). Special Issue
36. Penrose, R.: The Emperor's New Mind—Concerning Computers, Minds, and the Laws of Physics. Oxford University Press, New York (1989)
37. Penrose, R.: The Emperor's new mind—concerning computers, minds, and the laws of physics. In: Behavioural and Brain Sciences, vol. 13, pp. 643–705. Cambridge University Press, Cambridge (1990). With Open Peer Commentary by other authors. 'The nonalgorithmic mind', author's response to criticisms of the Précis, in Behavioural and Brain Sciences, *ibid.*, 692–705 (1990)
38. Penrose, R.: Shadows of the Mind—A Search for the Missing Science of Consciousness. Oxford University Press, Oxford (1994)
39. Peres, A.: Measurement of time by quantum clocks. Am. J. Phys., **48**(7), 552 (1980)

40. Petitot, J.: Continu et objectivité. La bimodalité objective du continu et le platonisme transcendental. In: Salanskis, J.-M., Sinaceur, H. (eds.) Le Labyrinthe du Continu (Colloque de Cerisy). Springer, Berlin (1993)
41. Prosperi, G.M.: Theory of measurement in quantum mechanics and macroscopic description. Int. J. Theor. Phys. **33**, 115 (1994)
42. Riemann, B.: Über die Hypothesen, welche die geometrie zu grunde liegen. In: Weber, H. (ed.) Gesammelte Mathematische Werken. Dover, New York (1953)
43. Salecker, H., Wigner, E.P.: Quantum limitations of the measurement of spacetime distances. Phys. Rev. **109**, 571 (1958)
44. Schrödinger, E.: Die gegenwärtige Situation in der Quantemmechanik. Naturwissenschaften **23**, 807–812, 823–828, 844–849 (1935). English translation by Trimmer, J.D.: The present situation in Quantum Mechanics. Proc. Am. Phillos. Soc. **124**, 3–338 (1980)
45. Shimony, A.: Search for a Naturalistic World View, vol. 1 and vol. 2, pp. 291–309. Cambridge University Press, Cambridge (1993)
46. Shimony, A.: On mentality, quantum mechanics and the actualization of potentialities. In: The Large, the Small and the Human Mind, Reply to Penrose, pp. 144–160. Cambridge University Press, Cambridge (1997)
47. Sklar, L.: Incongruous counterparts, intrinsic features, and the substantiality of space. J. Philos. **71**, 187–215 (1976)
48. Stein, H.: Newtonian space-time. Tex. Q. **10**, 174–200 (1967)
49. Stein, H.: Newton's metaphysics. In: Cohen, I.B., Smith, G.E. (eds.) The Cambridge Companion to Newton, pp. 256–307. Cambridge University Press, Cambridge (2002)
50. Teller, P.: Space-time as a physical quantity. In: Aichinstein, P., Kagon, M.P. (eds.) Kelvin's Baltimore Lectures and Modern Theoretical Physics. MIT Press, Cambridge (1987)
51. Ursin, R., et al.: Entanglement-based quantum communication over 144 K. Nat. Phys. **3**, 481–486 (2007) www.nature.com/naturephysics
52. Weyl, H.: Groups, Klein's Erlangen program. Quantities. In: The Classical Groups, Their Invariants and Representations, 2nd edn. pp. 13–23. Princeton University Press, Princeton (1946). Chap. I, §4
53. Weyl, H.: Das Kontinuum. Kritische Untersuchungen über die Grundlagen der Analysis. de Gruyter, Berlin (1932)
54. Wigner, E.P.: Remarks on the mind-body question. In: Good, I.J. (ed.) The Scientist Speculates. Heinemann, London (1961). Reprinted in J.A. Wheeler, W.H. Zurek (eds) Quantum Theory and Measurement. Princeton University Press, Princeton (1983)