

Appendix C



Quantum Physics

In the classical Newtonian picture of matter in motion, there is only one possible future, determined completely by the distribution and motion of matter at any moment. The future is certain and “causally closed.” Complete information about the future exists today, even if unknowable.

In the quantum picture, there are many possible futures. Quantum mechanics lets us exactly calculate the probability for the different futures, but it cannot tell us the actual future that will be realized. The actual future is uncertain. New information about the future is being created every day and we are co-creators of that information. The future is open.

It is important to understand that new information generated by quantum mechanics is not necessarily permanent. New information must be stably recorded, protected from erasure by the destructive forces of entropy.

As we saw in appendix B, this requires that more positive entropy must be transferred away from the new information structure than its negative entropy, to satisfy the second law.

MAX PLANCK derived the distribution of radiation at different frequencies (or wavelengths) just as Maxwell and Boltzmann had derived the distribution of velocities (or energies) of the gas particles. Both curves have a power law increase on one side up to a maximum and an exponential decrease down the other side from the maximum (the “Boltzmann factor” of $e^{-E/kT}$). This is because both curves describe particles, one matter, the other light.

Planck’s assumption that the energy of the oscillators is “quantized” was the beginning of quantum mechanics, but he did not actually believe that radiation came in the form of discrete particles, as we do today. It was ALBERT EINSTEIN in 1905 who made the hypothesis that light comes in highly



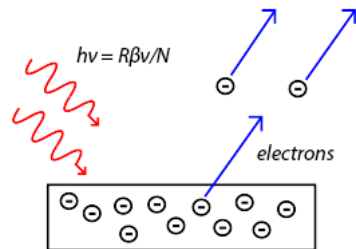
localized discrete particles, subsequently called “photons.” Later, Einstein showed that each photon, although massless, must have an associated momentum $p = h\nu/c = h/\lambda$, another fundamental connection between matter and light deriving from his most famous equation, $E = mc^2$.

But Einstein was puzzled and deeply concerned about the connection between the wave properties of light and his new insight that light consists of particles. In classical electrodynamics, electromagnetic radiation (light, radio) is well known to have wave properties, such as interference. When the crest of one wave meets the trough of another, the two waves cancel one another. How, he wondered, could discrete particles show interference effects?

Like water surface waves, light goes off in all directions as outgoing spherical waves. But if the energy of light fills a large spherical volume, Einstein wondered, how does the energy get itself collected together instantaneously to be absorbed by a single electron in a particular atom? Does the widely distributed energy move faster than the speed of light when it collapses to a single point?

In 1905, Einstein published his special theory of relativity denying that possibility. That same year he proved the existence of Boltzmann’s atoms with his explanation that the Brownian motions of visible particles in a liquid are caused by invisible atoms or molecules. His concerns about light waves versus light particles also appeared the same year, in his paper on the photoelectric effect (for which he was awarded the Nobel prize).

When ultraviolet light shines on a metal surface and ejects a single electron from one of the

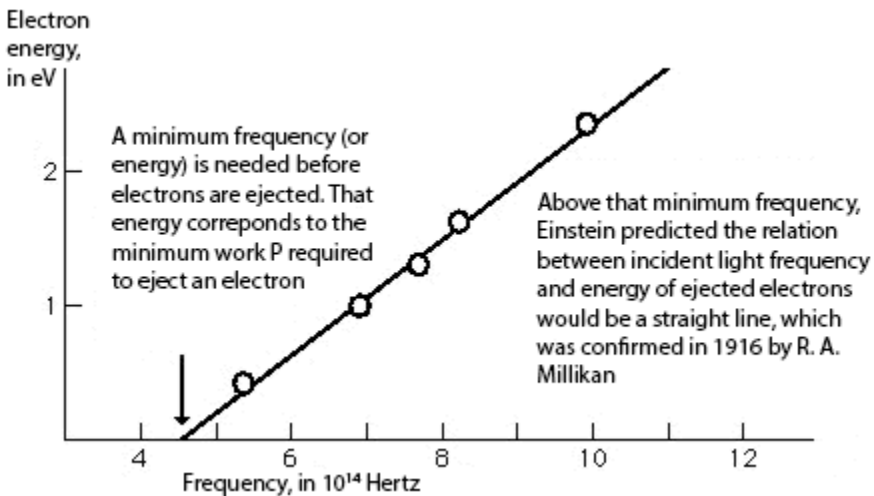


atoms in that metal, Einstein showed that some energy in the light beam acts like a single particle of light getting absorbed by a single ejected electron.

He assumed there is a “work function” or potential energy P that must be overcome to release an electron and that the energy of a photon must exceed that energy. Any excess energy E_e shows up as kinetic energy in the liberated electron.

$$E_e = h\nu - P.$$

Some part of the incoming photon energy, P , is used to release the electron. Einstein predicted the other part produces a linear relationship between the kinetic energy E_e of the electron and the frequency ν . It was over ten years before Einstein’s predictions were experimentally confirmed.



Turning up the intensity (more photons) of light with less energy (longer wavelengths) cannot eject an electron. And once the light has high enough frequency (energy), it does not matter how low the intensity of the light, electrons continue to be ejected.

It is thus the energy of a single quantum of light that becomes energy in a single electron. At this moment in 1905,



Einstein was grappling with two problems that the “founders” of quantum mechanics would themselves not see for another twenty years.

The first problem is the apparent “collapse” of the light wave. The second is called “nonlocal” behavior. Einstein’s great field theories like gravitation require what he called “local reality.”

If we can see these problems through Einstein’s young eyes, which many great quantum physicists could not, we may also see the most plausible solutions to those two problems and perhaps more. A third Einstein insight will help us understand “wave-particle duality.” A fourth will clarify “entanglement.”

In 1913, Niels Bohr developed his radical model of the atom incorporating Planck’s quantum conditions. Where classical electrodynamic theory says that electrons orbiting a central nucleus would continuously radiate energy at the orbital frequency (and the loss of energy would cause the electron to spiral in to the nucleus), Bohr postulated the atom has “stationary states” and that transitions (discontinuous “quantum jumps”) between those states result in the emission or absorption of energy with a frequency ν according to Planck’s relation $h\nu = E_m - E_n$, where E_m and E_n are the energies of the two states.

Einstein had confirmed the relation $E = h\nu$ in his photoelectric paper, but Bohr did not mention it. Bohr’s theory agreed perfectly with the frequencies of known spectral lines in the hydrogen atom and predicted many more lines that were subsequently found.

Einstein said that Bohr’s theory was an “enormous achievement” and “one of the greatest discoveries,” but Bohr did not accept Einstein’s hypothesis of a discrete light particle. The quantum jumps are discontinuous, but the emitted radiation is continuous, said Bohr.



Bohr was asked how we know to which other state a quantum jump will go. He replied we do not know. A few years later, Einstein calculated the probabilities for electronic transitions between Bohr's energy levels. He confirmed that quantum jumps are a matter of chance, just as we cannot predict the time or direction of a particle ejected from a decaying radioactive nucleus. Quantum theory is a statistical theory.

In the 1920's, LOUIS DE BROGLIE argued that if photons, with their known wavelike properties, could be described as particles, perhaps particles like electrons might show wavelike properties with a wavelength λ inversely proportional to their momentum $p = m_e v$. De Broglie's formula for a particle's wavelength, $\lambda = h/p$, is the same as Einstein's formula for the momentum of a photon, $p = h\nu/c$, because $\lambda\nu = c$.

Experiments confirmed de Broglie's assumption and led ERWIN SCHRÖDINGER to derive a "wave equation" to describe the motion of de Broglie's waves. For elementary particles, Schrödinger's quantum equation replaces the classical Newton equations of motion.

Note that Schrödinger's equation describes the motion of only the wave aspect, not the particle aspect, and so it includes interference effects in the waves. Note also that it is fully deterministic and continuous, just like Newton's equations. Schrödinger thought particles are not real, but could be explained as point-like singularities in his continuous waves.

There was some hope, particularly by Einstein, that Schrödinger's continuous equation would return determinism to physics, eliminating chance. It was not to be.

Schrödinger attempted to interpret his "wave function" for the electron as a probability density for electrical charge, but charge density would be positive everywhere and thus unable to interfere with itself. Moreover, fractions of the electron spread out in the wave are never found. Fractions of the energy



would have different (lower) energies and frequencies.

Long before the work of de Broglie and Schrödinger, Einstein had suggested that light waves might be thought of as a “ghost field” (*Gespensterfeld*) or a “leading field” (*Führungsfeld*) that guides the motion of the light particles. Einstein suggested that waves indicate the probable locations of light quanta.

The information about probabilities and possibilities in the wave function is *immaterial*, but that abstract information has real causal powers. The wave’s interference with itself predicts null points where no particles should be found. And experiments confirm that no particles are found there. Information philosophy views information as a kind of modern “spirit.”

MAX BORN applied Einstein’s suggestion about light to matter. He shocked the world of physics by suggesting that the absolute values of the square of the wave function ψ ($|\psi|^2$) can be interpreted as the probability of finding an electron in various position and momentum states - if a measurement is made. This allows the probability amplitude ψ to interfere with itself, producing non-intuitive phenomena such as the two-slit experiment. It is an *immaterial* wave of information about possible locations that passes through both slits.

Despite the *immaterial* probability amplitude going through two slits and interfering with itself, experimenters never find parts of electrons. They are always found whole.

Born’s statistical interpretation of the wave function says that the motion of the *immaterial* probabilities wave function is continuous and deterministic, but the motion of the material particles themselves is discontinuous and probabilistic.

Einstein and Schrödinger could never accept this.

Interpreters of quantum mechanics have found it hard to reconcile this combination of determinism and indeterminism,



of continuous wavelike and discontinuous particle-like behaviors. The information interpretation of quantum mechanics attempts that reconciliation. (See chapter 16.)

Basic Quantum Mechanics

The basic ideas of quantum mechanics are hopelessly non-intuitive. They describe quantum phenomena that are simply impossible to imagine in classical physics. This does not mean that they cannot be visualized, by which we mean illustrated, even animated with tools now available for web pages, which are much more powerful than this static printed page.

We hope that watching the animations will help you to develop new intuitions about the way the quantum world works. The classical world we experience is just the quantum world as seen at our macroscopic level, where it is averaged over a vast number of indeterministic quantum events to produce an adequately (or statistically) determined world.

We present the fundamental ideas of quantum mechanics following two great mathematical physicists, PAUL DIRAC and JOHN VON NEUMANN. Von Neumann proposed that quantum mechanics consists of just two basic processes. Dirac said the basics can be summarized in just three definitions, a *principle of superposition*, an *axiom of measurement*, and a *projection postulate*. Let's start with Dirac's three definitions, then see how they are realized in von Neumann's processes.

Finally, we present Dirac's application of the three definitions in the very simple case of a quantum system in a superposition of just two quantum states. This example of three polarizers also demonstrates von Neumann's two processes.

Almost all the conflicting interpretations of quantum mechanics today depend on either denying one or more of these basic elements of quantum mechanics or extending them to situations where they do not apply.



These three definitions and two processes are used throughout the physics chapters in support of the proposed solutions to great problems in physics.

The Principle of Superposition

The fundamental equation of motion in quantum mechanics is Schrödinger's famous wave equation that describes the evolution in time of his wave function ψ ,

$$i\hbar/2\pi \delta\psi/\delta t = H\psi.$$

For a single particle in idealized complete isolation, and for a Hamiltonian H that does not involve magnetic fields, the Schrödinger equation is a unitary transformation that is time-reversible (the principle of microscopic reversibility, see chapter 24).

MAX BORN interpreted the square of the absolute value of Schrödinger's wave function as providing the probability of finding a quantum system in a certain state ψ_n .

The quantum (discrete) nature of physical systems results from there generally being a large number of solutions ψ_n (called eigenfunctions) of the Schrödinger equation in its time-independent form, with energy eigenvalues E_n .

$$H\psi_n = E_n\psi_n,$$

The discrete energy eigenvalues E_n limit interactions (for example, with photons) to the energy differences $E_n - E_m$, as assumed by Bohr. Eigenfunctions ψ_n are orthogonal to one another,

$$\langle \psi_n | \psi_m \rangle = \delta_{nm},$$

where δ_{nm} is the Dirac delta-function, equal to 1 when $n = m$, and 0 otherwise. The sum of the diagonal terms in the matrix $\langle \psi_n | \psi_m \rangle$, when $n = m$, must be normalized to 1 to be meaningful as Born rule probabilities.

$$\sum P_n = \sum \langle \psi_n | \psi_n \rangle^2 = 1.$$



The off-diagonal terms in the matrix, $\langle \psi_n | \psi_m \rangle$, are interpretable as interference terms. When the matrix is used to calculate the expectation values of some quantum mechanical operator O , the off-diagonal terms $\langle \psi_n | O | \psi_m \rangle$ are interpretable as transition probabilities - the likelihood that the operator O will induce a transition from state ψ_n to ψ_m .

The Schrödinger equation is a linear equation. It has no quadratic or higher power terms, and this introduces a profound - and for many scientists and philosophers a disturbing - feature of quantum mechanics, one that is impossible in classical physics. This is the principle of superposition of quantum states. If ψ_a and ψ_b are both solutions of the equation, then an arbitrary linear combination of these, $\psi = c_a \psi_a + c_b \psi_b$, with complex coefficients c_a and c_b , is also a solution.

Together with Born's statistical interpretation of the wave function (remember this was Einstein's idea), the principle of superposition accounts for the major mysteries of quantum theory, some of which we hope to resolve, or at least reduce, with an objective (observer-independent) explanation of information creation during quantum processes (which can often be interpreted as measurements). See chapter 16.

The Axiom of Measurement

The axiom of measurement depends on the idea of "observables," physical quantities that can be measured in experiments. A physical observable is represented as a Hermitean operator A that is self-adjoint (equal to its complex conjugate, $A^* = A$).

The diagonal elements $\langle \psi_n | A | \psi_n \rangle$ of the operator's matrix are interpreted as giving the expectation value for A_n (when we make a measurement). The off-diagonal n, m elements describe the uniquely quantum property of interference between wave functions and provide a measure of the probabilities for



transitions between states n and m .

It is these intrinsic quantum probabilities that provide the ultimate source of indeterminism, and consequently of irreducible irreversibility (see chapter 24). The axiom of measurement is then that a large number of measurements of the observable A , known to have eigenvalues A_n , will result in the number of measurements with value A_n being proportional to the probability of finding the system in eigenstate ψ_n with eigenvalue A_n .

The Projection Postulate

The third novel idea of quantum theory is often considered the most radical. It has certainly produced some of the most radical ideas ever to appear in physics, in attempts to deny it (as the decoherence program appears to do - chapter 21, as do also Everett relative-state interpretations, many worlds theories, and Bohm-de Broglie hidden variables). The projection postulate is actually very simple, and arguably intuitive as well. It says that when a measurement is made, the system of interest will be found in one of the possible eigenstates ψ_n of the measured observable.

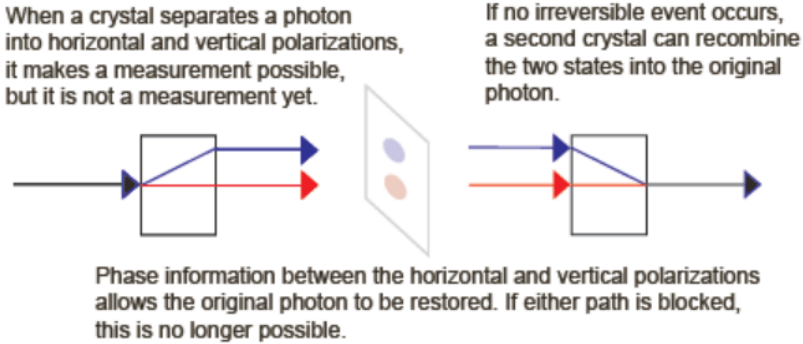
We have several possible alternatives for eigenvalues A_n . Measurement simply makes one of these eigenvalues actual, and it does so, said Max Born, in proportion to the absolute square of the probability amplitude wave function $|\psi_n|^2$. In this way, ontological chance enters physics, and it is partly this fact of quantum randomness that bothered Einstein (“God does not play dice”) and Schrödinger (whose equation of motion is deterministic).

When Einstein derived the expressions for the probabilities of emission and absorption of photons in 1916, he lamented that the theory seemed to indicate that the direction of an emitted photon was a matter of pure chance (*Zufall*), and



that the time of emission was also statistical and random, just as ERNST RUTHERFORD had found for the time of decay of a radioactive nucleus. Einstein called it a “weakness in the theory.”

Most “interpreters” of quantum mechanics do not accept this postulate, with its idea of a “collapse.” See chapter 19.



Von Neumann’s Two Processes

In 1932, JOHN VON NEUMANN explained that two fundamentally different processes are going on in quantum mechanics.

Process 1: A *non-causal* process, in which a measured electron winds up randomly in one of the possible physical states (eigenstates) of the measuring apparatus plus electron.

The probability for each eigenstate is given by the square of the coefficients c_n of the expansion of the original system state (wave function ψ) in a set of wave functions φ_n that represent the eigenfunctions of the measuring apparatus plus electron.

$$\psi = \sum_n c_n | \varphi_n \rangle$$

$$c_n = \langle \varphi_n | \psi \rangle$$

Process 1 corresponds exactly to Dirac’s projection postulate. It also describes the “collapse” of the wave function (see chapter 19). It introduces indeterminism and ontological chance.

This is as close as we get to a description of the discontinuous motion of the particle aspect of a quantum system. According



to von Neumann, the particle simply “shows up” somewhere as a result of a measurement. The information interpretation of quantum physics says it can only “show up” if a new stable information structure is created that can be seen by an observer, after which it may constitute a measurement.

(Paul Dirac explained process 1 with a very simple quantum system that has only two states, horizontal and vertical polarization. We will describe it below. It exhibits properties of quantum mechanics that are impossible for a classical system.)

Process 2: A *causal* process, in which the electron wave function ψ evolves deterministically according to Schrödinger’s equation of motion for the wavelike aspect. This evolution describes the continuous motion of the probability amplitude wave ψ between discontinuous measurements,

$$(i\hbar/2\pi) \partial\psi/\partial t = H\psi.$$

Von Neumann claimed there is another major difference between these two processes. He said Process 1 is thermodynamically *irreversible*. (See chapter 24.) Process 2 is *reversible*. This confirms the fundamental connection between quantum mechanics and thermodynamics that is explainable by the information interpretation of quantum physics.

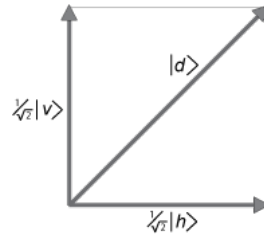
Information physics establishes that an experiment may create *irreversible* new information. If it does not, no observation and thus no measurement is possible. Most processes in the universe that create new information are never observed. Process 2 is in principle *reversible*, in practice maybe not. If so, it preserves information. The figure is an example of a reversible process.

Dirac’s Three Polarizers

In his 1930 textbook *The Principles of Quantum Mechanics*, Dirac introduced the uniquely quantum concepts of superposition, measurement, projection/collapse, and indeterminacy using polarized photons. Einstein said of Dirac,



“Dirac, to whom, in my opinion, we owe the most perfect exposition, logically, of this [quantum] theory, rightly points out that it would probably be difficult, for example, to give a theoretical description of a photon such as would give enough information to enable one to decide whether it will pass a polarizer placed (obliquely) in its way or not.”¹



Dirac’s example with an “oblique” polarizer suggests a very simple and inexpensive experiment to demonstrate the superpositions of quantum states, the projection or representation of a given state vector in another basis set of vectors, the preparation of quantum systems in states with known properties, and the measurement of various properties.

Any measuring apparatus is also a state preparation system. We know that after a measurement of a photon which has shown it to be in a state of vertical polarization, for example, a second measurement with the same (vertical polarization detecting) capability will show the photon to be in the same state with probability unity. Quantum mechanics is not always uncertain. There is also no uncertainty if we measure a vertically polarized photon with a horizontal polarization detector. There is zero probability of finding the vertically polarized photon in a horizontally polarized state.

Since any measurement increases the amount of information, there must be a compensating increase in entropy absorbed by or radiated away from the measuring apparatus.

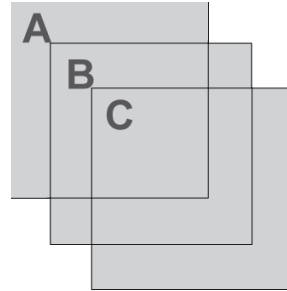
The natural basis set of vectors is usually one whose eigenvalues are the observables of our measurement system. In Dirac’s bra and ket notation, the orthogonal basis vectors in our example are $|v\rangle$, the photon in a vertically polarized state,

1 Ideas and Opinions, p.270



and $|h\rangle$, the photon in a horizontally polarized state. These two states are eigenstates of our measuring apparatus.

The interesting case to consider is a third measuring apparatus that prepares a photon in a diagonally polarized state 45° between $|v\rangle$ and $|h\rangle$, the “oblique” polarizer.



Dirac tells us this diagonally polarized photon can be represented as a *superposition* of vertical and horizontal states, with complex number coefficients that represent “probability amplitudes,” as shown in equation 1.

$$|d\rangle = (1/\sqrt{2})|v\rangle + (1/\sqrt{2})|h\rangle \quad (1)$$

Note that vector lengths are normalized to unity, and the sum of the squares of the probability amplitudes is also unity. This is the orthonormality condition needed to interpret the (squares of the) wave functions as probabilities, as proposed by Max Born, following Einstein’s idea that waves show the probable locations for light quanta.



When these complex number coefficients are squared (actually when they are multiplied by their complex conjugates to produce positive real numbers), the numbers represent the probabilities of finding the photon in one or the other state, should a measurement be made. Dirac’s bra vector $\langle|$ is the complex conjugate of the corresponding ket vector $| \rangle$.

It is the probability amplitudes that interfere in the two-slit experiment. To get the probabilities of finding a photon, we must square the probability amplitudes. Actually we must calculate the expectation value of some operator that represents



an observable. The probability P of finding the photon in state $|\psi\rangle$ at a position (in configuration space) r is

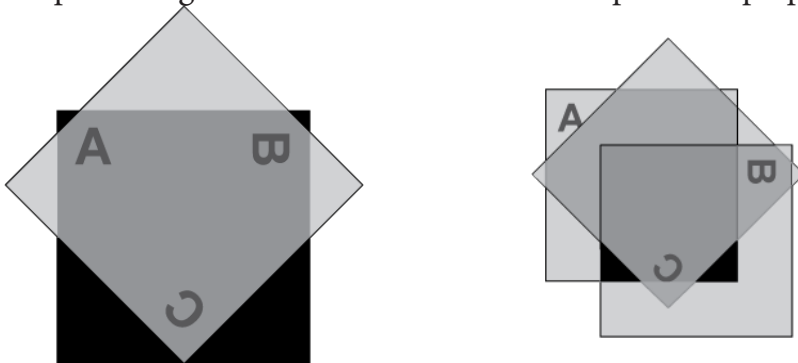
$$P(r) = \langle \psi | r | \psi \rangle.$$

No single experiment can convey all the wonder and non-intuitive character of quantum mechanics. But we believe Dirac's simple examples of polarized photons can teach us a lot. He thought that his simple examples provide a good introduction to quantum physics and we agree.

We use three squares of polarizing sheet material with white labels A, B, and C to illustrate Dirac's explanation of quantum superposition of states and the collapse of a mixture of states to a pure state upon measurement or state preparation.

Here are the three polarizing sheets. They are a neutral gray color because they lose half of the light coming through them. The lost light is absorbed by the polarizer, converted to heat, and this accounts for the (Boltzmann) entropy gain required by our new information (Shannon entropy) about the exact polarization state of the transmitted photons.

Here polarizers A and B are superimposed to show that the same amount of light comes through two polarizers, as long as the polarizing direction is the same. The first polarizer prepares



the photon in a given state of polarization. The second is then certain to find it in the same state. Let's say the direction of light polarization is vertical when the letters are upright.



If one polarizer, say B, is turned 90° , its polarization direction will be horizontal and if it is on top of vertical polarizer A, no light will pass through it, as we see in figure 3. We can still see unpolarized light from letter A.

The Wonder and Mystery of the Oblique Polarizer

As you would expect, any quantum mechanics experiment must contain an element of “Wow, that’s impossible!” or we are not getting to the non-intuitive and unique difference between quantum mechanics and the everyday classical mechanics. So let’s look at the amazing aspect of what Dirac is getting to, and then we will see how quantum mechanics explains it.

We turn the third polarizer C so its polarization is along the diagonal. Dirac tells us that the wave function of light passing through this polarizer can be regarded as in a mixed state, a superposition of vertical and horizontal states. As Einstein agreed, the information as to the exact state in which the photon will be found following a measurement does not exist.

We can make a measurement that detects vertically polarized photons by holding up the vertical polarizer A in front of the oblique polarizer C. Either a photon comes through A or it does not. Similarly, we can hold up the horizontal polarizer B in front of C. If we see a photon, it is horizontally polarized.

From equation (1) we see that the probability of detecting a photon diagonally polarized by C, if our measuring apparatus (polarizer B) is measuring for horizontally polarized photons, is $1/2$. Similarly, if we were to measure for vertically polarized photons, we have the same 50% chance of detecting a photon.

Going back to polarizers A and B crossed at a 90° angle, we know that no light comes through when we cross the s.

If we hold up polarizer C along the 45 degree diagonal and place it in front of (or behind) the cross polarizers, nothing changes. No light is getting through.

But here is the amazing, impossible part. If you insert polar-



izer C *between* A and B , some light now gets through. Note that C is slipped between A (in the rear) and B (in front).

If B , crossed with A , blocks all light, how can *adding* another polarization filter add light? It is much less light than through C alone. We shall see why.

The Quantum Physics Explanation

Let's start with the A polarizer in the back. It *prepares* the photons in the vertical polarization state $|v\rangle$. If we now had just polarizer B , it would measure for horizontal photons. None through A are horizontal, so no photons get through B .

Measurements are von Neumann process 1.

When we interpose C at the oblique angle, it measures for diagonal photons. The vertically polarized photons coming through A can be considered in a superposition of states at a 45 degree angle and a -45 degree angle. Photons at -45 degrees are absorbed by C . Those at +45 degrees pass through C .

C makes a *measurement* of 45 degree photons. It can also be viewed as a *preparation* of 45 degree photons. Only half the photons come through polarizer C , but they have been prepared in a state of diagonal polarization $|d\rangle$.

The original vertical photons coming through A had no chance of getting through B , but the diagonal photons passing through C (half the original photons) can now be regarded as in a linear superposition of vertical and horizontal photons, and the horizontal photons can now pass through B . Those vertically polarized will get absorbed by B , as usual.

Recall from equation (1) that $|d\rangle$ is a superposition of the basis vectors $|v\rangle$ and $|h\rangle$, with coefficients $1/\sqrt{2}$, which when squared give us probabilities $1/2$. Fifty percent of these photons emerging from C will pass through B . One quarter or 25% of the original A photons make it through.

This happens if we send just one photon through at a time,



just as with the two-slit experiment. Just as we can not say that the photon passes through slit A or B (only probabilities are moving in von Neumann's process 2), we cannot say that our photons are in one state or another. They are in the mysterious linear combination that can collapse instantaneously into one state when a measurement is made.



