I

THE PRINCIPLE OF SUPERPOSITION

§ 1. Waves and Particles

In the application of classical electrodynamics to atomic phenomena one meets with difficulties of a very fundamental nature, which show that the classical theory is irreconcilable with the facts. For instance, it is quite hopeless on the basis of classical ideas to try to account for the remarkable stability of atoms and molecules that is required in order that substances may have definite physical and chemical properties. These difficulties have necessitated a modification of some of the most fundamental laws of nature and have led to a new system of mechanics, called quantum mechanics, since its most striking (although not its most important) differences from the old mechanics apparently show a discontinuity in certain physical processes and a discreteness in certain dynamical variables.

Classical electrodynamics forms a self-consistent and very elegant theory, and one might be inclined to think that no modification of it would be possible which did not introduce arbitrary features and completely spoil its beauty. This is not so, however, since quantum mechanics, after passing through many stages and having its fundamental concepts changed more than once, has now reached a form in which it can be based on general laws and is, although not yet quite complete, even more elegant and pleasing than the classical theory in those problems with which it deals. This is brought about by the fact that the changes made in the classical theory are very few in number, although they are of a fundamental nature and involve the introduction of entirely new concepts, and are such that practically all the features of the classical theory to which it owes its attractiveness can be taken over unchanged into the new theory.

The necessity for a fundamental departure from the laws and concepts of classical mechanics is seen most clearly by a consideration of experimentally established facts on the nature of light. On the one hand the phenomena of interference and diffraction can be explained only on the basis of a wave theory of light; on the other, phenomena such as photo-electric emission and scattering by free electrons show that light is composed of small particles, which are called photons, each having a definite energy and momentum depending on the frequency of the light. These photons appear to have just as real an existence as electrons, or any other particles known in physics. A fraction of a photon is never observed, so that we may safely assume it cannot exist.

To obtain a consistent theory of light which shall include interference and diffraction phenomena, we must consider the photons as being controlled by waves, in some way which cannot be understood from the point of view of ordinary mechanics. This intimate connexion between waves and particles is of very great generality in the new quantum mechanics. It occurs not only in the case of light. *All* particles are connected in this way with waves, which control them and give rise to interference and diffraction phenomena under suitable conditions. The influence of the waves on the motion of the particles is less noticeable the more massive the particles and only in the case of photons, the lightest of all particles, is it easily demonstrated.

The waves and particles should be regarded as two abstractions which are useful for describing the same physical reality. One must not picture this reality as containing both the waves and particles together and try to construct a mechanism, acting according to classical laws, which shall correctly describe their connexion and account for the motion of the particles. Any such attempt would be quite opposed to the principles by which modern physics advances. What quantum mechanics does is to try to formulate the underlying laws in such a way that one can determine from them without ambiguity what will happen under any given experimental conditions. It would be useless and meaningless to attempt to go more deeply into the relations between waves and particles than is required for this purpose.

§ 2. The Polarization of Photons

Although the idea of a physical reality being describable by both particles and waves, which are connected in some curious manner, is of far-reaching importance and wide applications, yet it is only a special case of a much more general principle, the *Principle of Superposition*. This principle forms the fundamental new idea of quantum mechanics and the basis of the departure from the classical theory.

In order to lead up to an explanation of this principle, we shall first take a very simple special case of it, which is provided by a consideration of the polarization of light. It is known experimentally that when plane-polarized light is used for ejecting photo-electrons, there is a preferential direction for the electron emission. Thus the polarization properties of light are closely connected with its corpuscular properties and one must ascribe a polarization to the photons. One must consider, for instance, a beam of light plane polarized in a certain direction as consisting of photons each of which is plane polarized in that direction and a beam of circularly polarized light as consisting of photons each circularly polarized. Every photon is in a certain state of polarization, as we shall say. The difficulty is now how we are to fit in these ideas with the known facts about the resolution of light into polarized components and the recomposition of these components.

Suppose, for instance, that we have a beam of plane-polarized light passing through a polariscope and getting resolved into two components polarized at angles of α and $\alpha+\frac{1}{2}\pi$ with the direction of polarization of the incident beam. The intensities of the two components will be, according to classical optics, respectively $\cos^2\alpha$ and $\sin^2\alpha$ times that of the original beam. Let us say that a photon of the original beam is in the state of polarization 0 and a photon in one or other of the two components is in the state α or $\alpha+\frac{1}{2}\pi$ respectively. The question that now arises is: What must we consider happens to each individual photon when it reaches the polariscope? How do the photons in the state 0 change into photons in the states α and $\alpha+\frac{1}{2}\pi$?

This question cannot be answered without the help of an entirely new concept which is quite foreign to classical ideas. We shall therefore first consider another question of a different type, namely, what will be the result of any particular experiment which one may perform to try to determine what happens to an individual photon when it reaches the polariscope. It is only questions of this type that are really important, and quantum mechanics always gives a definite answer to them. Any answer that may be given to our first question, *i.e.* any description of the whole course of a photon during the experiment, would be simply a device to help us to remember the results of the experiments. We ought not to be surprised if no such description based on classical ideas is possible.

The most direct experiment of this kind would be to use an incident beam consisting of only a single photon and then to measure the energy in each of the two components. The result predicted by quantum mechanics is that sometimes one would find the whole of the energy in one component and the other times one would find the whole in the other component. One would never find part of the energy in one and part in the other. Experiment can never reveal a fraction of a photon. If one did the experiment a large number of times, one would find in a fraction $\cos^2\alpha$ of the total number of times that the whole of the energy is in the α-component and in a fraction $\sin^2\alpha$ that the whole of the energy is in the $(\alpha+\frac{1}{2}\pi)$-component. One may thus say that a photon has a probability $\cos^2\alpha$ of appearing in the α-component and a probability $\sin^2\alpha$ of appearing in the $(\alpha+\frac{1}{2}\pi)$-component. These values for the probabilities lead to the correct classical distribution of energy between the two components when the number of photons in the incident beam is large.

Thus the individuality of the photon is preserved in all cases, but only at the expense of determinacy. The result of an experiment is not determined, as it would be according to the classical theory, by the conditions under the control of the experimenter. The most that can be predicted is the probability of occurrence of each of the possible results. This lack of determinacy, which runs through the whole of quantum mechanics and is in sharp contradiction to the classical theory, may at first sight appear to be unsatisfactory, as implying a departure from the law of causality. It should be remarked, though, that if one makes any experimental arrangement to observe the energy of one of the components (*e.g.* by reflection by a movable mirror and measurement of the recoil momentum communicated to the mirror), it will always be impossible subsequently to recombine the two components to produce interference effects. The observation must inevitably produce, as we shall see from the general laws of quantum mechanics, a change in phase of uncertain and unpredictable amount. One may therefore, as has been pointed out by Bohr,* ascribe the lack of determinacy in the result to the uncertainty in the disturbance which the observation necessarily makes, although one cannot inquire closely into how it comes about. The apparent failure of causality is from this point of view due to a theoretically necessary clumsiness in the means of observation.

We must now consider the answer to our first question and give a description of the photon throughout the course of the experiment. A description consisting of a continuous picture in the classical sense

* See the article by N. Bohr in *Nature*, p. 580, 1928.

is not possible. The description which quantum mechanics allows us to give is merely a manner of speaking which is of value in helping us to deduce and to remember the results of experiments and which never leads to wrong conclusions. One should not try to give too much meaning to it.

It is necessary to suppose a peculiar relationship to exist between the different states of polarization, which is such that when, for instance, a photon is in the state 0, it may be considered as being partly in the state α and partly in the state $\alpha+\frac{1}{2}\pi$. Similarly it could be considered as partly in state β and partly in state $\beta+\frac{1}{2}\pi$, where β is any other angle of polarization, or as partly in the state of left-circular polarization and partly that of right-circular polarization. More generally, one could consider it partly in each of two states plane polarized in two directions that are not at right angles, though this is seldom convenient, or one could consider it partly in each of more than two states. There are thus many ways of describing the photon, which are all always permissible and equally good theoretically, although, of course, the one that says the photon is entirely in state 0 is simpler than those that say it is 'distributed' over two or more states. When we say that the photon is distributed over two or more given states the description is, of course, only qualitative, but in the mathematical theory it is made exact by the introduction of numbers to specify the distribution, which determine the *weights* with which the different states occur in it.

One cannot picture in detail a photon being partly in each of two states; still less can one see how this can be equivalent to its being partly in each of two other different states or wholly in a single state. We must, however, get used to the new relationships between the states which are implied by this manner of speaking and must build up a consistent mathematical theory governing them.

In our polarizing experiment, if we choose to consider the incident photon as being partly in state α and partly in state $\alpha+\frac{1}{2}\pi$, the action of the polariscope is then quite simple. It separates the two components α and $\alpha+\frac{1}{2}\pi$ into two distinct beams, so that after the photon has passed through we must say that it is partly in one beam with the polarization α and partly in the other with the polarization $\alpha+\frac{1}{2}\pi$. There is now no way of saying the photon is wholly in one state, without a generalization of the meaning of a state, which will be made later. The simplest description is the one just given, in which the photon is distributed over two states. Other possible descriptions would require the photon to be distributed over three or more states; *e.g.* one could say it is partly in the first beam with the polarization α, partly in the second beam with the polarization β (arbitrary), and partly in the second beam with the polarization $\beta+\frac{1}{2}\pi$. Such descriptions would not, however, be of value unless the beams were subsequently passed through other polarizing instruments.

Let us consider now what happens when we determine the energy in one of the components. The result of such a determination must be either the whole photon or nothing at all. Thus the photon must change suddenly from being partly in one beam and partly in the other to being entirely in one of the beams. This sudden change may be counted as due to the disturbance of the photon which the observation necessarily makes. It is impossible to predict in which of the two beams the photon will be found. Only the probability of either result can be calculated from the previous distribution of the photon over the two beams.

This way of describing the photon during the course of the experiment leads to one important conclusion, namely, the above-mentioned circumstance that when once the energy in one of the components has been determined, it will be impossible subsequently to bring about interference between the two components. When the photon is partly in one beam and partly in the other, if the two beams are superposed interference can take place, as the mathematical theory will show. This possibility disappears when the photon is forced entirely into one of the beams by the energy observation. The other beam then no longer enters into the description of the photon, so that if any experiment is subsequently performed on the same photon it will count as being entirely in the one beam in the ordinary way.

We have obtained a description of the photon throughout the experiment, which rests on a new rather vague idea of a photon being partly in one state and partly in another. The reader may, perhaps, feel that we have not really solved the difficulty of the conflict between the waves and the corpuscles, but have merely talked about it in a certain way and, by using some of the concepts of waves and some of corpuscles, have arrived at a formal account of the phenomena, which does not really tell us anything that we did not know before. The difficulty of the conflict between the waves and corpuscles is, however, actually solved as soon as one can give an unambiguous answer to any experimental question. *The only object of theoretical physics is to calculate results that can be compared with experiment*, and it is quite unnecessary that any satisfying description of the whole course of the phenomena should be given.

With regard to the objection that the present description does not seem to take us any farther than we could, perhaps, have gone with very hazy notions of the relations between photons and electromagnetic waves, such as, for instance, those one had before the discovery of quantum mechanics, it should be remarked that the conclusion obtained above, that when once the energy of one of the beams has been measured subsequent interference between the beams would be impossible, could not have been drawn from *very* hazy notions, and also that the present discussion is really too qualitative for the advantages of the new theory to show up clearly. In § 5 the discussion on the nature of light will be renewed on a slightly more quantitative basis, which will bring out definitely the difference between the present theory and the previous hazy notions. For many elementary optical experiments, moreover, the hazy notions would suffice to give answers to questions concerning the results of observations and in such cases quantum mechanics would not give any further information. The object of quantum mechanics is to extend the domain of questions that can be answered and not to give more detailed answers than can be experimentally verified.

§ 3. Superposition and Indeterminacy

The new ideas that we have introduced in our description of the photon must be extended and applied to any atomic system, *i.e.* to any set of electrons and atomic nuclei interacting with each other and perhaps also with photons. We must first generalize the meaning of a 'state' so that it can apply to any atomic system. Corresponding to the case of the photon, which we say is in a given state of polarization when it has been passed through suitable polarizing apparatus, we say that any atomic system is in a given state when it has been prepared in a given way, which may be repeated arbitrarily at will. The method of preparation may then be taken as the specification of the state. The state of a system in the general case includes any information that may be known about its position in space from the way in which it was prepared, as well as any information about its internal condition.

We must now imagine the states of any system to be related in such a way that whenever the system is definitely in one state, we can equally well consider it as being partly in each of two or more other states. The original state must be regarded as the result of a kind of *superposition* of the two or more new states, in a way that cannot be conceived on classical ideas. Any state may be considered as the result of a superposition of two or more other states, and indeed in an infinite number of ways. Conversely any two or more states may be superposed to give a new state, even also when they refer to different positions of the system in space. Thus in our previous example of the polarization experiment, when the photon is partly in the one beam with the polarization α and partly in the other with the polarization $\alpha+\frac{1}{2}\pi$, we may still count it as being entirely in a certain single state. In fact it still satisfies the definition of having been prepared in a definite way which may be repeated at will.

When a state is formed by the superposition of two other states, it will have properties that are in a certain way intermediate between those of the two original states and that approach more or less closely to those of either of them according to the greater or less 'weight' attached to this state in the superposition process. The new state is completely defined by the two original states when their relative weights in the superposition process are known, together with a certain phase difference, the exact meaning of weights and phases being provided in the general case by the mathematical theory of the next chapter. In the case of the polarization of a photon their meaning is that provided by classical optics, *e.g.* when two perpendicularly plane polarized states are superposed with equal weights, the new state may be circularly polarized in either direction, or linearly polarized at an angle $\frac{1}{4}\pi$, or else elliptically polarized, according to the phase difference. This, of course, is true only provided the two states that are superposed refer to the same beam of light, *i.e.* all that is known about the position and momentum of a photon in either of these states must be the same for each.

It is convenient at this stage to modify slightly the meaning of the word 'state' and to make it more precise. We must regard the state of a system as referring to its condition throughout an indefinite period of time and not to its condition at a particular time, which would make the state a function of the time. Thus a state refers to a region of 4-dimensional space-time and not to a region of 3-dimensional space. A system, when once prepared in a given state, remains in that state so long as it remains undisturbed. This does not, of course, imply that it is not undergoing changes which could be revealed by experiment. In general it will be following out a definite course of changes, predictable by the quantum theory, belonging to that state. It is sometimes purely a matter of convenience whether we are to regard a system as being disturbed by a certain outside influence, so that its state gets changed, or whether we are to regard the outside influence as forming part of and coming in the definition of the system, so that with the inclusion of the effects of this influence it is still merely running through its course in one particular state. An illustration of this is our previous example of a photon being passed through a polariscope and becoming partly in each of two beams. Either we may consider the polariscope as disturbing the photon, so that after it has passed through it is in a different state; or else we may consider the polariscope as forming part of the 'field' in which the photon is moving, so that it is in the same state when it is in the incident beam as later when it is partly in each of the two component beams, and it is just following out its course in that state. The general laws of quantum mechanics apply equally well for either of these meanings of the state. There are, however, two cases when we are in general obliged to consider the disturbance as causing a change in state of the system, namely, when the disturbance is an observation and when it consists in preparing the system so as to be in a given state.

With the new space-time meaning of a state we need a corresponding space-time meaning of an *observation*. This requires that the specification of an observation shall include a definite time at which the observation is to be made, or at which the apparatus used in making the observation is to be set in motion, relatively to the time when the system was prepared. It should be noticed that it has a meaning to consider an observation being made on a system in a given state before this state is prepared. If the system is prepared at time t_0, so that after time t_0 it is in a given state, we can imagine what it would have to be like before time t_0 in order that, if left undisturbed, it may become in the given state after time t_0. Thus we can imagine the given state being produced backwards in time and can give a meaning to an observation being made before time t_0 on the system in this state.

The introduction of indeterminacy into the results of observations, which we had to make in our discussion of the photon, must now be extended to the general case. When an observation is made on any atomic system that has been prepared in a given way and is thus in a given state, the result will not in general be determinate, *i.e.* if the experiment is repeated several times under identical conditions several different results may be obtained. If the experiment is repeated a large number of times it will be found that each particular result will be obtained a definite fraction of the total number of times, so that one can say there is a definite probability of its being obtained any time the experiment is performed. This probability the theory enables one to calculate. In special cases this probability may be unity and the result of the experiment is then quite determinate.

The indeterminacy in the results of observations is a necessary consequence of the superposition relationships that quantum mechanics requires to exist between the states. Suppose that we have two states A and B such that there exists an observation which, when made on the system in state A, is certain to lead to one particular result, and when made on the system in state B, is certain not to lead to this result. Two such states we call *orthogonal*. Suppose now that this observation is made on the system in a state formed by superposition of A and B. It is impossible for the result still to be determinate (except in the special case when the weight of A or B in the superposition process is zero). There must be a finite probability p that the result, that was certain for state A, will now be obtained and a finite probability $1-p$ that it will not be obtained. By continuously varying the relative weights in the superposition process we can get a continuous range of states, extending from pure A to pure B, for which the probability of the result, that was certain for state A, being obtained varies continuously from unity to zero.

It was mentioned above that an observation is not specified unless the time when it is made is given. In special cases it may so happen that the result of the observation, or the probability of any particular result being obtained, is independent of this time. If the state of the system is such that this is so for every observation that could be made on the system, then the state is said to be a *stationary* state and we should picture it as one in which the conditions are not varying.

The possibility in quantum mechanics of superposing states to get new states is connected with the fact that in the mathematical theory the equations that define a state are linear in the unknowns. It is not unnatural that one should try to establish analogies with systems in classical mechanics (such as vibrating strings or membranes), which are governed by linear equations and for which, consequently, a superposition principle holds. Such analogies have led to the name 'Wave Mechanics' being sometimes given to quantum mechanics. It must be emphasized, however, that *the superposition that occurs in quantum mechanics is of an essentially different nature from that occurring in the classical theory.* The analogies are therefore very misleading. Their inadequacy may be seen from the following special case. Suppose one compares the states of an atomic system with the states of vibration of a membrane. If one superposes any state of the vibrating membrane with itself, the result is a new state of double the amplitude. On the other hand, if one superposes an atomic state with itself according to quantum mechanics, the resulting state will be precisely the same as the original one. There is nothing in the atomic case that is analogous to the absolute value of the amplitude, as distinct from the relative amplitudes of different points, of the vibrating membrane.

§ 4. Compatibility of Observations

In general a system is disturbed when an observation is made on it, so that after the observation it is no longer in the same state as before. Only when the initial state and the observation are such that there is a probability unity, *i.e.* a certainty, for one particular result is it possible that the observation may produce no change of state. The necessity for this conclusion may be seen from the following argument.

Suppose that there is a probability p for a given result being obtained from the observation. Consider one occasion on which this result was actually found and suppose the observation was repeated immediately afterwards on the system in the state in which it was left by the first observation. There must have been a probability unity for the given result being obtained a second time, since we may assume the system could not have changed in the infinitely short time between the two observations. Thus while the first state is such that there is a probability p for a given result from a certain observation, the second state (*i.e.* the one in which the system was left by the first observation) is such that there is a probability unity for this same result from a practically equivalent observation. Hence the second state must differ from the first when p differs from unity, since the probability of a result is quite definite for each state. It must be understood that the second state here considered is the one that arose on that particular occasion referred to above when the first observation was found to give the particular result desired. There will be a different second state corresponding to each different result for this observation. They must all be different from the initial state when p differs from unity.

Hence when once an observation of a system in a given state has been made, one cannot in general make a second observation and suppose it to apply to the same state. The first observation spoils the state of the system, which must then be prepared again before one can make the second. The two observations may, however, be such that, although the first one alters the state of the system, yet it does so in such a way as not to make any difference to the probability of any given result being obtained with the second. By the probability of a given result being obtained with the second is here meant its probability at the beginning of the experiment, before one knows what the result of the first observation is, and not its probability after a particular result has been obtained with the first observation. Two observations for which this is so when they are made (or at least when the first is made) with the minimum of disturbance allowed by theory, which can be attained in practice only under the most favourable conditions, are called *compatible.* Three or more observations are called compatible when any two are compatible. Two or more observations may be compatible only with respect to one particular state as initial state before any of the observations, or they may be compatible with respect to all initial states. In future when it is said that two or more observations are compatible, the second alternative is to be understood unless the contrary is stated.

The condition for the compatibility of two observations is, according to the laws of quantum mechanics, a symmetrical condition between them. If one of two compatible observations, α_1 say, is made at the time t_1 and the other, α_2 say, at the time t_2 which is later than t_1, then, according to the definition given above, the probability of a given result being obtained for α_2 must be the same whether this observation is made on the system in the initial state or in the state ensuing after observation α_1. The symmetry condition now requires that the probability of a given result being obtained for α_1 must be the same whether this observation α_1 is made on the system in the initial state or in the state ensuing after observation α_2, it being necessary to suppose this latter state, which is prepared at time t_2, to be produced backwards in time, in the way mentioned in the preceding section, in order that the observation α_1 at time t_1 may be made on it. By the probability of a result for the state ensuing after a certain observation, is meant in each case the average probability for each state that can ensue after this observation, each of these states being weighted in the averaging process with the probability that it does ensue after this observation.

It has been pointed out that the state of a system after any observation has been made on it is such that this observation, if made on the system in this final state, would for a certainty give one particular result. Suppose now that a number of compatible observations $\alpha_1, \alpha_2, \ldots$ are made on the system. Then the final state must be such that, if any of the observations α_r is made on the system in this final state, there will be a certainty for one particular result, since there was a certainty for one particular result as soon as the observation α_r was made in the preparation of the final state, and this will not be affected by the subsequent observations $\alpha_{r+1}, \alpha_{r+2}, \ldots$, owing to the compatibility condition. The existence of states for which the result of any of the observations is a certainty forms one of the main properties of compatible observations. The order of the observations need not, of course, be their order in time, since we are allowed to consider an observation being made on a state before it is prepared.

The case of greatest interest of the compatibility of two observations is when they both refer to the same instant of time. The compatibility condition is now that if either is made a very short time before the other, the probability of any given result being obtained with the second shall be the same as if the first had not been made.

It is often convenient to count two or more compatible observations, particularly when they are simultaneous, as a single observation, the result of such an observation being expressible by two or more numbers. We shall frequently have to consider the greatest possible number of independent compatible simultaneous observations being made on a system and shall, for brevity, call such a set of observations a maximum observation. *When a maximum observation is made on a system, its subsequent state is completely determined by the result of the observation and is independent of its previous state.* This may be considered as an axiom, or as a more precise definition of a state.

The state of a system after a maximum observation has been made on it is such that there exists a maximum observation (namely, an immediate repetition of the maximum observation already made) which, when made on the system in this state, will for a certainty lead to one particular result (namely, the previous result over again). Any state can be specified only as the state ensuing after a given maximum observation has been made for which a given result was obtained, or in some equivalent way. We can therefore draw the conclusion that for any state there must exist one maximum observation which will for a certainty lead to one particular result, and conversely, if we consider any possible result of a maximum observation, there must exist a state of the system for which this result for the observation will be obtained with certainty.

§ 5. Further Discussion on Photons

When quantum mechanics is applied to a system composed of simply a freely moving corpuscle, the equations that define a state of the system are, as we shall find from the mathematical theory, the ordinary equations for wave motion. It is this circumstance that gives to the corpuscle many of the properties of waves and allows us to consider a corpuscle in a given state as associated with, or controlled by, a given wave. In order to show more definitely the nature of the relations between the waves and the corpuscle, a typical example will be given of the conflict between the wave and the corpuscular theories of light and of the solution which quantum mechanics provides.

Consider a beam of light to be split into two components of equal intensity, which are made to interfere. According to the old corpuscular theory we would say that each of the two components contains an equal number of photons and we should then require that a photon in one component could interfere with one in the other. Under certain conditions they would have to annihilate one another, and under others to produce four photons. This contradicts the idea of photons being discrete particles and is, besides, in disagreement with the conservation of energy, which should hold for each process in detail and not be merely statistically true.

The answer that quantum mechanics gives to the difficulty is that one should consider each photon to go partly into each of the two components, in the way allowed by the idea of the superposition of states. Each photon then interferes only with itself. Interference between two different photons can never occur. The solution of Maxwell's equations that forms the wave picture of the phenomenon represents *one* of the photons and not the whole assembly of photons. The relative intensities that this solution gives for the light at different points determine the relative probabilities of that photon being found at these points when an experiment is made to find its position. Only the relative intensities at different points are of importance; *the absolute intensity has no interpretation.* One must not try to establish any connexion between the absolute intensity of the waves and the total number of particles, which is in sharp distinction to the older ideas of the relations between waves and particles.

The quantum-mechanical views do not, of course, get over the difficulty of enabling us to picture something having properties between those of waves and corpuscles, but they serve to remind us, by their way of saying a photon is partly in one component and partly in the other, of the close connexion between the components and so prevent us from intuitively drawing wrong conclusions, as we do on the older views when we picture each component as having its own photons. For instance, we are reminded, by the requirement that the total probability of a photon being anywhere must be and must remain unity, that in whatever way the two component beams interfere, if they neutralize each other in one place they must reinforce each other in another so that conservation of energy is preserved. We thus get into no difficulty with the detailed conservation of energy.

§ 6. Definition of Superposition

A definition of the superposition of states will now be given. *We say that a state A may be formed by a superposition of states B and C when, if any observation is made on the system in state A leading to any result, there is a finite probability for the same result being obtained when the same observation is made on the system in one (at least) of the two states*

B and C. The Principle of Superposition says that any two states B and C may be superposed in accordance with this definition to form a state A and indeed an infinite number of different states A may be formed by superposing B and C in different ways. This principle forms the foundation of quantum mechanics. It is completely opposed to classical ideas, according to which the result of any observation is certain and for any two states there exists an observation that will certainly lead to two different results.

From our definition of superposition some elementary theorems follow immediately. For example, the states B and C themselves are particular cases of states formed by superposition of B and C. Again, if we superpose two states A and B obtaining a state P, which is then superposed on another state C, the resulting state Q will have the property that, if any observation is made on the system in this state leading to any result, there will be a finite probability of this same result being obtained when the observation is made on the system in one of the two states P and C, and hence there must be a finite probability of this result being obtained when the observation is made on the system in one of the three states A, B, and C. Thus the property possessed by the state Q is symmetrical in the three states A, B, and C, so that when superpositions are made successively their order is unimportant. This, of course, is necessary for the word 'superposition' to be suitable for describing the relations between the states.

Another example of a deduction from the definition of superposition is the following: If an observation of the system in a state A is certain to lead to one particular result and if this observation for another state B is certain to lead to the same result, then the observation is also certain to lead to this result for any state obtained by superposition of A and B. This is because it cannot lead to any other result, as the probability of this other result for both the states A and B is zero.

One could proceed to build up the theory of quantum mechanics on the basis of these ideas of superposition with the introduction of the minimum number of new assumptions necessary. Although this would be the logical line of development, it does not appear to be the most convenient one, as the laws of quantum mechanics are so closely interconnected that it would not be easy, and would in any case be somewhat artificial, to separate out the barest minimum of assumptions from which the rest could be deduced. The method that will be here followed will therefore be first to give all the simple general laws in the form in which they are most easily expressed and remembered, and then to work out their consequences. This will mean that we shall continually be deducing results that are obviously necessary for the physical meaning of the theory to be tenable, or that follow from the foregoing ideas of superposition. Such deductions will then merely show the reasonableness and self-consistency of our fundamental assumptions.