It is a very old doubt of mankind, how freedom and contingency can be reconciled with the series of causes and with providence. The difficulty of the matter has been increased by the dissertations of Christian authors on God's justice in procuring the salvation of men.

For my part, I used to consider that nothing happens by chance or by accident, except with respect to certain particular substances; that fortune, as distinct from fate, is an empty word; and that nothing exists unless its individual requisites are given, and that from all these taken together it follows that the thing exists. So I was not far from the view of those who think that all things are absolutely necessary; who think that security from compulsion is enough for freedom, even though it is under the rule of necessity, and who do not distinguish the infallible—that is, a truth which is certainly known—from the necessary.

But I was dragged back from this precipice by a consideration of those possibles which neither do exist, nor will exist, nor have existed. For if certain possibles never exist, then existing things are not always necessary; otherwise it would be impossible for other things to exist instead of them, and so all things that never exist would be impossible. For it cannot be denied that many stories, especially those which are called 'romances', are possible, even if they do not find any place in this series of the universe, which God has chosen—unless someone supposes that in the vast magnitude of space and time there exist the regions of the poets, where you could see wandering through the world King Arthur of Britain, Amadis of Gaul, and Dietrich von Bern, famed in the stories of the Germans. A certain distinguished philosopher of our century...
On Freedom

seems to have been close to this opinion, for he says expressly somewhere that matter takes on successively all the forms of which it is capable (*Principles of Philosophy*, Part III, art. 47)

This view is indefensible, for it would remove all the beauty of the universe and all choice, to say nothing here of other arguments by which the contrary can be shown.

Once I had recognised the contingency of things, I then began to consider what a clear notion of truth would be; for I hoped, not unreasonably, to derive from this some light on the problem of distinguishing necessary from contingent truths. However, I saw that it is common to every true affirmative proposition—universal and particular, necessary or contingent—that the predicate is in the subject, or that the notion of the predicate is in some way involved in the notion of the subject, and that this is the principle of infallibility in every kind of truth for him who knows everything *a priori*. But this seemed to increase the difficulty. For if, at a given time, the notion of the predicate is in the notion of the subject, then how, without contradiction and impossibility, can the predicate not be in the subject at that time, without destroying the notion of the subject?

A new and unexpected light finally arose in a quarter where I least hoped for it—namely, out of mathematical considerations of the nature of the infinite. There are two labyrinths of the human mind: one concerns the composition of the continuum, and the other the nature of freedom, and both spring from the same source—the infinite. That distinguished philosopher whom I mentioned above could not unravel these knots, or at any rate was unwilling to make his opinion known, but preferred to cut them with a sword. For he says (*Principles of Philosophy*, Part I, arts. 40 and 41) that we can easily involve ourselves in great difficulties if we try to reconcile God’s preordination with the freedom of the will, and that we must abstain from discussing them, since God’s nature cannot be comprehended by us. He also says (Part II, art. 35) that we ought not to doubt that matter is divided *ad infinitum*, even though we cannot understand this. But this is not enough: for it is one thing for us not to understand a thing, and another for us to understand its contradictory.
So it is at all events necessary to be able to answer those arguments which seem to imply that freedom or the division of matter imply a contradiction.

It must be known, therefore, that all creatures have impressed on them a certain mark of the divine infinity, and that this is the source of many wonders which amaze the human mind.

For example, there is no portion of matter so small that there does not exist in it a world of creatures, infinite in number. Again, every individual created substance, however imperfect, acts on all others and is acted on by all others, and contains in its complete notion (as this exists in the mind of God) the whole universe, and whatever is, was or will be. Further, every truth of fact or of individual things depends on a series of infinite reasons, and all that is in this series can be seen by God alone. This is also the reason why God alone knows contingent truths a priori, and sees their infallibility in another way than by experience.

When I had considered these more attentively, a profound difference between necessary and contingent truths came to light. Every truth is either original or derivative. Original truths are those of which a reason cannot be given; such truths are identical or immediate, and they affirm a term of itself or deny a contradictory of its contradictory. Derivative truths are again of two sorts: some are analysed into original truths, others admit of an infinite process of analysis. The former are necessary, the latter contingent. A necessary proposition is one whose contrary implies a contradiction, such as all identical propositions and all derivative propositions which are analysable into identical propositions. These are the truths which are said to be of metaphysical or geometrical necessity. For demonstration consists simply in this: by the analysis of the terms of a proposition, and by substituting for a defined term a definition or part of a definition, one shows a certain equation or coincidence of predicate with subject in a reciprocal proposition, or in other cases at least the inclusion of the predicate in the subject, in such a way that what was latent in the proposition and as it were contained in it virtually is rendered evident and express by the demonstration.
For example: e if we understand by a ternary, senary or duodenary (etc.) number one which can be divided by 3, 6 and 12 respectively, we can demonstrate the proposition ‘Every duodenary is a senary’. For every duodenary is a binary-binary ternary (for this is the analysis of a duodenary into its prime factors, $12 = 2 \times 2 \times 3$: i.e. the definition of a duodenary). Now, every binary-binary ternary is a binary ternary (which is an identical proposition), and every binary ternary is a senary (this is the definition of a senary: $6 = 2 \times 3$). Therefore every duodenary is a senary ($12$ is the same as $2 \times 2 \times 3$; $2 \times 2 \times 3$ is divisible by $2 \times 3$; $2 \times 3$ is the same as $6$; therefore $12$ is divisible by $6$).

But in the case of contingent truths, even though the predicate is in the subject, this can never be demonstrated of it, nor can the proposition ever be reduced to an equation or identity. Instead, the analysis proceeds to infinity, God alone seeing—not, indeed, the end of the analysis, since it has no end—but the connexion of terms or the inclusion of the predicate in the subject, for he sees whatever is in the series; indeed, this very same truth has arisen in part from his own intellect and in part from his will, and expresses in its own way his infinite perfection and the harmony of the whole series of things.

However, there have been left to us two ways of knowing contingent truths; one is the way of experience and the other the way of reason. The way of experience is when we perceive a thing clearly enough by our senses; the way of reason is derived from the general principle that nothing happens without a reason, or, that the predicate is always in some way in the subject. So we can regard it as certain that everything is done by God in the most perfect way, that he does nothing which is contrary to reason, and that nothing ever happens without the man who understands it understanding its reason—why, that is, the state of things is as it is rather than otherwise. So reasons can be given for the actions of minds no less than for the actions of bodies, although in the case of the choices that minds make there is no necessity. Sins arise from the original limitation of things; but God does not so much decree sins as admit to existence certain possible substances,
already involving in their complete notion, under the aspect of possibility, a free sin, and so involving the whole series of things which they will be in. For there can be no doubt that there are hidden reasons, transcending the understanding of every creature, why one series of things (although it includes sin) is preferred by God to another. However, God decrees only perfection, or what is positive; but limitation, and the sin which arises from it, is permitted by him, since given certain positive decrees it cannot absolutely be rejected, and reasons known to wisdom require that it should be redeemed by a greater good which cannot otherwise be obtained. But this cannot be considered here.

But to fix our attention better, so that the mind does not wander through vague difficulties, there occurs to me an analogy that holds between truth and proportions that seems to clarify the whole issue splendidly and put it in a clear light. Just as, in every proposition, the smaller number is contained in the larger, or an equal in an equal, so in every truth the predicate is in the subject. Further, in every proportion between homogeneous quantities one can carry out a kind of analysis of equal or congruent terms and subtract the smaller from the larger, by removing from the larger a part equal to the smaller; and similarly a residue can be subtracted from what has been subtracted, and so on either to a given point, or to infinity. So also, in the analysis of truths, one always substitutes for a term its equivalent, so that the predicate may be analysed into the terms which are contained in the subject. Now, in the case of proportions the analysis is sometimes completed and one arrives at a common measure, which is contained in each term of the proportion an integral number of times. Sometimes, however, the analysis can be continued to infinity, which occurs in the comparison of a rational number and a surd, or of a side and the diagonal in a square. Similarly, truths are sometimes demonstrable or necessary, and sometimes they are free or contingent; the latter cannot be reduced by any analysis to an identity, as to a common measure. This is the essential distinction both between proportionals and between truths.

However, incommensurable proportions have been mas-
tered by the science of geometry, and we even have demonstrations about infinite series. Much more are contingent or infinite truths subject to the knowledge of God; they are known by him, not by a demonstration indeed (for that would imply a contradiction) but by an infallible vision. But this vision that God has must not be conceived as a kind of experiential knowledge, as if he saw something in things which are distinct from himself, but rather as a priori knowledge (through the reasons for truths). For he sees things which are possible in themselves by a consideration of his own nature, but he sees existent things by the consideration of his own free will and his own decrees, of which the first is to do everything in the best way and with supreme reason. What is termed ‘mediate knowledge’ is simply the knowledge of contingent possibles.

When these matters have been properly considered, I do not believe that any difficulty can arise in this topic whose solution cannot be derived from what has been said above. If one admits this concept of necessity—which all do admit—namely, that those propositions are necessary whose contrary implies a contradiction, it is readily apparent that the nature of demonstration and analysis can be explained, and also that there must be truths which are not reduced by any analysis to identical truths or to the principle of contradiction, but which furnish an infinite series of reasons, which God alone can see through. This is the nature of everything which is called free and contingent, and in particular that which involves space and time. It has been shown adequately above that this follows from the infinity of the parts of the universe and the mutual penetration and connexion of things.