II.

The History of Quantum Theory

THE origin of quantum theory is connected with a well-known phenomenon, which did not belong to the central parts of atomic physics. Any piece of matter when it is heated starts to glow, gets red hot and white hot at higher temperatures. The color does not depend much on the surface of the material, and for a black body it depends solely on the temperature. Therefore, the radiation emitted by such a black body at high temperatures is a suitable object for physical research; it is a simple phenomenon that should find a simple explanation in terms of the known laws for radiation and heat. The attempt made at the end of the nineteenth century by Lord Rayleigh and Jeans failed, however, and revealed serious difficulties. It would not be possible to describe these difficulties here in simple terms. It must be sufficient to state that the application of the known laws did not lead to sensible results. When Planck, in 1895, entered this line of research he tried to turn the problem from radiation to the radiating atom. This turning did not remove any of the difficulties inherent in the problem, but it simplified the interpretation of the empirical facts. It was just at this time, during the summer of 1900, that Curlbaum and Rubens in Berlin had made very accurate new measurements of the spectrum of
heat radiation. When Planck heard of these results he tried to represent them by simple mathematical formulas which looked plausible from his research on the general connection between heat and radiation. One day Planck and Rubens met for tea in Planck’s home and compared Rubens’ latest results with a new formula suggested by Planck. The comparison showed a complete agreement. This was the discovery of Planck’s law of heat radiation.

It was at the same time the beginning of intense theoretical work for Planck. What was the correct physical interpretation of the new formula? Since Planck could, from his earlier work, translate his formula easily into a statement about the radiating atom (the so-called oscillator), he must soon have found that his formula looked as if the oscillator could only contain discrete quanta of energy—a result that was so different from anything known in classical physics that he certainly must have refused to believe it in the beginning. But in a period of most intensive work during the summer of 1900 he finally convinced himself that there was no way of escaping from this conclusion. It was told by Planck’s son that his father spoke to him about his new ideas on a long walk through the Grunewald, the wood in the suburbs of Berlin. On this walk he explained that he felt he had possibly made a discovery of the first rank, comparable perhaps only to the discoveries of Newton. So Planck must have realized at this time that his formula had touched the foundations of our description of nature, and that these foundations would one day start to move from their traditional present location toward a new and as yet unknown position of stability. Planck, who was conservative in his whole outlook, did not like this consequence at all, but he published his quantum hypothesis in December of 1900.
The idea that energy could be emitted or absorbed only in discrete energy quanta was so new that it could not be fitted into the traditional framework of physics. An attempt by Planck to reconcile his new hypothesis with the older laws of radiation failed in the essential points. It took five years until the next step could be made in the new direction.

This time it was the young Albert Einstein, a revolutionary genius among the physicists, who was not afraid to go further away from the old concepts. There were two problems in which he could make use of the new ideas. One was the so-called photoelectric effect, the emission of electrons from metals under the influence of light. The experiments, especially those of Lenard, had shown that the energy of the emitted electrons did not depend on the intensity of the light, but only on its color or, more precisely, on its frequency. This could not be understood on the basis of the traditional theory of radiation. Einstein could explain the observations by interpreting Planck’s hypothesis as saying that light consists of quanta of energy traveling through space. The energy of one light quantum should, in agreement with Planck’s assumptions, be equal to the frequency of the light multiplied by Planck’s constant.

The other problem was the specific heat of solid bodies. The traditional theory led to values for the specific heat which fitted the observations at higher temperatures but disagreed with them at low ones. Again Einstein was able to show that one could understand this behavior by applying the quantum hypothesis to the elastic vibrations of the atoms in the solid body. These two results marked a very important advance, since they revealed the presence of Planck’s quantum of action—as his constant is called among the physicists—in several phenomena, which had nothing immediately to do with heat radiation. They
revealed at the same time the deeply revolutionary character of the new hypothesis, since the first of them led to a description of light completely different from the traditional wave picture. Light could either be interpreted as consisting of electromagnetic waves, according to Maxwell's theory, or as consisting of light quanta, energy packets traveling through space with high velocity. But could it be both? Einstein knew, of course, that the well-known phenomena of diffraction and interference can be explained only on the basis of the wave picture. He was not able to dispute the complete contradiction between this wave picture and the idea of the light quanta; nor did he even attempt to remove the inconsistency of this interpretation. He simply took the contradiction as something which would probably be understood only much later.

In the meantime the experiments of Becquerel, Curie and Rutherford had led to some clarification concerning the structure of the atom. In 1911 Rutherford's observations on the interaction of $\alpha$-rays penetrating through matter resulted in his famous atomic model. The atom is pictured as consisting of a nucleus, which is positively charged and contains nearly the total mass of the atom, and electrons, which circle around the nucleus like the planets circle around the sun. The chemical bond between atoms of different elements is explained as an interaction between the outer electrons of the neighboring atoms; it has not directly to do with the atomic nucleus. The nucleus determines the chemical behavior of the atom through its charge which in turn fixes the number of electrons in the neutral atom. Initially this model of the atom could not explain the most characteristic feature of the atom, its enormous stability. No planetary system following the laws of Newton's mechanics would ever go back to its original configuration after a collision with another such
system. But an atom of the element carbon, for instance, will still remain a carbon atom after any collision or interaction in chemical binding.

The explanation for this unusual stability was given by Bohr in 1913, through the application of Planck’s quantum hypothesis. If the atom can change its energy only by discrete energy quanta, this must mean that the atom can exist only in discrete stationary states, the lowest of which is the normal state of the atom. Therefore, after any kind of interaction the atom will finally always fall back into its normal state.

By this application of quantum theory to the atomic model, Bohr could not only explain the stability of the atom but also, in some simple cases, give a theoretical interpretation of the line spectra emitted by the atoms after the excitation through electric discharge or heat. His theory rested upon a combination of classical mechanics for the motion of the electrons with quantum conditions, which were imposed upon the classical motions for defining the discrete stationary states of the system. A consistent mathematical formulation for those conditions was later given by Sommerfeld. Bohr was well aware of the fact that the quantum conditions spoil in some way the consistency of Newtonian mechanics. In the simple case of the hydrogen atom one could calculate from Bohr’s theory the frequencies of the light emitted by the atom, and the agreement with the observations was perfect. Yet these frequencies were different from the orbital frequencies and their harmonics of the electrons circling around the nucleus, and this fact showed at once that the theory was still full of contradictions. But it contained an essential part of the truth. It did explain qualitatively the chemical behavior of the atoms and their line spectra; the existence of the discrete station-
ary states was verified by the experiments of Franck and Hertz, Stern and Gerlach.

Bohr's theory had opened up a new line of research. The great amount of experimental material collected by spectroscopy through several decades was now available for information about the strange quantum laws governing the motions of the electrons in the atom. The many experiments of chemistry could be used for the same purpose. It was from this time on that the physicists learned to ask the right questions; and asking the right question is frequently more than halfway to the solution of the problem.

What were these questions? Practically all of them had to do with the strange apparent contradictions between the results of different experiments. How could it be that the same radiation that produces interference patterns, and therefore must consist of waves, also produces the photoelectric effect, and therefore must consist of moving particles? How could it be that the frequency of the orbital motion of the electron in the atom does not show up in the frequency of the emitted radiation? Does this mean that there is no orbital motion? But if the idea of orbital motion should in incorrect, what happens to the electrons inside the atom? One can see the electrons move through a cloud chamber, and sometimes they are knocked out of an atom; why should they not also move within the atom? It is true that they might be at rest in the normal state of the atom, the state of lowest energy. But there are many states of higher energy, where the electronic shell has an angular momentum. There the electrons cannot possibly be at rest. One could add a number of similar examples. Again and again one found that the attempt to describe atomic events in the traditional terms of physics led to contradictions.
Gradually, during the early twenties, the physicists became accustomed to these difficulties, they acquired a certain vague knowledge about where trouble would occur, and they learned to avoid contradictions. They knew which description of an atomic event would be the correct one for the special experiment under discussion. This was not sufficient to form a consistent general picture of what happens in a quantum process, but it changed the minds of the physicists in such a way that they somehow got into the spirit of quantum theory. Therefore, even some time before one had a consistent formulation of quantum theory one knew more or less what would be the result of any experiment.

One frequently discussed what one called ideal experiments. Such experiments were designed to answer a very critical question irrespective of whether or not they could actually be carried out. Of course it was important that it should be possible in principle to carry out the experiment, but the technique might be extremely complicated. These ideal experiments could be very useful in clarifying certain problems. If there was no agreement among the physicists about the result of such an ideal experiment, it was frequently possible to find a similar but simpler experiment that could be carried out, so that the experimental answer contributed essentially to the clarification of quantum theory.

The strangest experience of those years was that the paradoxes of quantum theory did not disappear during this process of clarification; on the contrary, they became even more marked and more exciting. There was, for instance, the experiment of Compton on the scattering of X-rays. From earlier experiments on the interference of scattered light there could be no doubt that scattering takes place essentially in the following way: The
incident light wave makes an electron in the beam vibrate in the frequency of the wave; the oscillating electron then emits a spherical wave with the same frequency and thereby produces the scattered light. However, Compton found in 1923 that the frequency of scattered X-rays was different from the frequency of the incident X-ray. This change of frequency could be formally understood by assuming that scattering is to be described as collision of a light quantum with an electron. The energy of the light quantum is changed during the collision; and since the frequency times Planck's constant should be the energy of the light quantum, the frequency also should be changed. But what happens in this interpretation of the light wave? The two experiments—one on the interference of scattered light and the other on the change of frequency of the scattered light—seemed to contradict each other without any possibility of compromise.

By this time many physicists were convinced that these apparent contradictions belonged to the intrinsic structure of atomic physics. Therefore, in 1924 de Broglie in France tried to extend the dualism between wave description and particle description to the elementary particles of matter, primarily to the electrons. He showed that a certain matter wave could "correspond" to a moving electron, just as a light wave corresponds to a moving light quantum. It was not clear at the time what the word "correspond" meant in this connection. But de Broglie suggested that the quantum condition in Bohr's theory should be interpreted as a statement about the matter waves. A wave circling around a nucleus can for geometrical reasons only be a stationary wave; and the perimeter of the orbit must be an integer multiple of the wave length. In this way de Broglie’s idea connected the quantum condition, which always had been a for-
eign element in the mechanics of the electrons, with the dualism between waves and particles.

In Bohr's theory the discrepancy between the calculated orbital frequency of the electrons and the frequency of the emitted radiation had to be interpreted as a limitation to the concept of the electronic orbit. This concept had been somewhat doubtful from the beginning. For the higher orbits, however, the electrons should move at a large distance from the nucleus just as they do when one sees them moving through a cloud chamber. There one should speak about electronic orbits. It was therefore very satisfactory that for these higher orbits the frequencies of the emitted radiation approach the orbital frequency and its higher harmonics. Also Bohr had already suggested in his early papers that the intensities of the emitted spectral lines approach the intensities of the corresponding harmonics. This principle of correspondence had proved very useful for the approximative calculation of the intensities of spectral lines. In this way one had the impression that Bohr's theory gave a qualitative but not a quantitative description of what happens inside the atom; that some new feature of the behavior of matter was qualitatively expressed by the quantum conditions, which in turn were connected with the dualism between waves and particles.

The precise mathematical formulation of quantum theory finally emerged from two different developments. The one started from Bohr's principle of correspondence. One had to give up the concept of the electronic orbit but still had to maintain it in the limit of high quantum numbers, i.e., for the large orbits. In this latter case the emitted radiation, by means of its frequencies and intensities, gives a picture of the electronic orbit; it represents what the mathematicians call a Fourier expansion
of the orbit. The idea suggested itself that one should write down the mechanical laws not as equations for the positions and velocities of the electrons but as equations for the frequencies and amplitudes of their Fourier expansion. Starting from such equations and changing them very little one could hope to come to relations for those quantities which correspond to the frequencies and intensities of the emitted radiation, even for the small orbits and the ground state of the atom. This plan could actually be carried out; in the summer of 1925 it led to a mathematical formalism called matrix mechanics or, more generally, quantum mechanics. The equations of motion in Newtonian mechanics were replaced by similar equations between matrices; it was a strange experience to find that many of the old results of Newtonian mechanics, like conservation of energy, etc., could be derived also in the new scheme. Later the investigations of Born, Jordan and Dirac showed that the matrices representing position and momentum of the electron do not commute. This latter fact demonstrated clearly the essential difference between quantum mechanics and classical mechanics.

The other development followed de Broglie's idea of matter waves. Schrödinger tried to set up a wave equation for de Broglie's stationary waves around the nucleus. Early in 1926 he succeeded in deriving the energy values of the stationary states of the hydrogen atom as "Eigenvalues" of his wave equation and could give a more general prescription for transforming a given set of classical equations of motion into a corresponding wave equation in a space of many dimensions. Later he was able to prove that his formalism of wave mechanics was mathematically equivalent to the earlier formalism of quantum mechanics.

Thus one finally had a consistent mathematical formalism,
which could be defined in two equivalent ways starting either from relations between matrices or from wave equations. This formalism gave the correct energy values for the hydrogen atom; it took less than one year to show that it was also successful for the helium atom and the more complicated problems of the heavier atoms. But in what sense did the new formalism describe the atom? The paradoxes of the dualism between wave picture and particle picture were not solved; they were hidden somehow in the mathematical scheme.

A first and very interesting step toward a real understanding of quantum theory was taken by Bohr, Kramers and Slater in 1924. These authors tried to solve the apparent contradiction between the wave picture and the particle picture by the concept of the probability wave. The electromagnetic waves were interpreted not as "real" waves but as probability waves, the intensity of which determines in every point the probability for the absorption (or induced emission) of a light quantum by an atom at this point. This idea led to the conclusion that the laws of conservation of energy and momentum need not be true for the single event, that they are only statistical laws and are true only in the statistical average. This conclusion was not correct, however, and the connections between the wave aspect and the particle aspect of radiation were still more complicated.

But the paper of Bohr, Kramers and Slater revealed one essential feature of the correct interpretation of quantum theory. This concept of the probability wave was something entirely new in theoretical physics since Newton. Probability in mathematics or in statistical mechanics means a statement about our degree of knowledge of the actual situation. In throwing dice we do not know the fine details of the motion of our hands which determine the fall of the dice and therefore we say that the proba-
bility for throwing a special number is just one in six. The probability wave of Bohr, Kramers, Slater, however, meant more than that; it meant a tendency for something. It was a quantitative version of the old concept of "potentia" in Aristotelian philosophy. It introduced something standing in the middle between the idea of an event and the actual event, a strange kind of physical reality just in the middle between possibility and reality.

Later when the mathematical framework of quantum theory was fixed, Born took up this idea of the probability wave and gave a clear definition of the mathematical quantity in the formalism, which was to be interpreted as the probability wave. It was not a three-dimensional wave like elastic or radio waves, but a wave in the many-dimensional configuration space, and therefore a rather abstract mathematical quantity.

Even at this time, in the summer of 1926, it was not clear in every case how the mathematical formalism should be used to describe a given experimental situation. One knew how to describe the stationary states of an atom, but one did not know how to describe a much simpler event—as for instance an electron moving through a cloud chamber.

When Schrödinger in that summer had shown that his formalism of wave mechanics was mathematically equivalent to quantum mechanics he tried for some time to abandon the idea of quanta and "quantum jumps" altogether and to replace the electrons in the atoms simply by his three-dimensional matter waves. He was inspired to this attempt by his result, that the energy levels of the hydrogen atom in his theory seemed to be simply the eigenfrequencies of the stationary matter waves. Therefore, he thought it was a mistake to call them energies; they were just frequencies. But in the discussions which took
place in the autumn of 1926 in Copenhagen between Bohr and Schrödinger and the Copenhagen group of physicists it soon became apparent that such an interpretation would not even be sufficient to explain Planck's formula of heat radiation.

During the months following these discussions an intensive study of all questions concerning the interpretation of quantum theory in Copenhagen finally led to a complete and, as many physicists believe, satisfactory clarification of the situation. But it was not a solution which one could easily accept. I remember discussions with Bohr which went through many hours till very late at night and ended almost in despair; and when at the end of the discussion I went alone for a walk in the neighboring park I repeated to myself again and again the question: Can nature possibly be as absurd as it seemed to us in these atomic experiments?

The final solution was approached in two different ways. The one was a turning around of the question. Instead of asking: How can one in the known mathematical scheme express a given experimental situation? the other question was put: Is it true, perhaps, that only such experimental situations can arise in nature as can be expressed in the mathematical formalism? The assumption that this was actually true led to limitations in the use of those concepts that had been the basis of classical physics since Newton. One could speak of the position and of the velocity of an electron as in Newtonian mechanics and one could observe and measure these quantities. But one could not fix both quantities simultaneously with an arbitrarily high accuracy. Actually the product of these two inaccuracies turned out to be not less than Planck's constant divided by the mass of the particle. Similar relations could be formulated for other experimental situations. They are usually called relations of un-
certainty or principle of indeterminacy. One had learned that the old concepts fit nature only inaccurately.

The other way of approach was Bohr's concept of complementarity. Schrödinger had described the atom as a system not of a nucleus and electrons but of a nucleus and matter waves. This picture of the matter waves certainly also contained an element of truth. Bohr considered the two pictures—particle picture and wave picture—as two complementary descriptions of the same reality. Any of these descriptions can be only partially true, there must be limitations to the use of the particle concept as well as of the wave concept, else one could not avoid contradictions. If one takes into account those limitations which can be expressed by the uncertainty relations, the contradictions disappear.

In this way since the spring of 1927 one has had a consistent interpretation of quantum theory, which is frequently called the "Copenhagen interpretation." This interpretation received its crucial test in the autumn of 1927 at the Solvay conference in Brussels. Those experiments which had always led to the worst paradoxes were again and again discussed in all details, especially by Einstein. New ideal experiments were invented to trace any possible inconsistency of the theory, but the theory was shown to be consistent and seemed to fit the experiments as far as one could see.

The details of this Copenhagen interpretation will be the subject of the next chapter. It should be emphasized at this point that it has taken more than a quarter of a century to get from the first idea of the existence of energy quanta to a real understanding of the quantum theoretical laws. This indicates the great change that had to take place in the fundamental concepts concerning reality before one could understand the new situation.