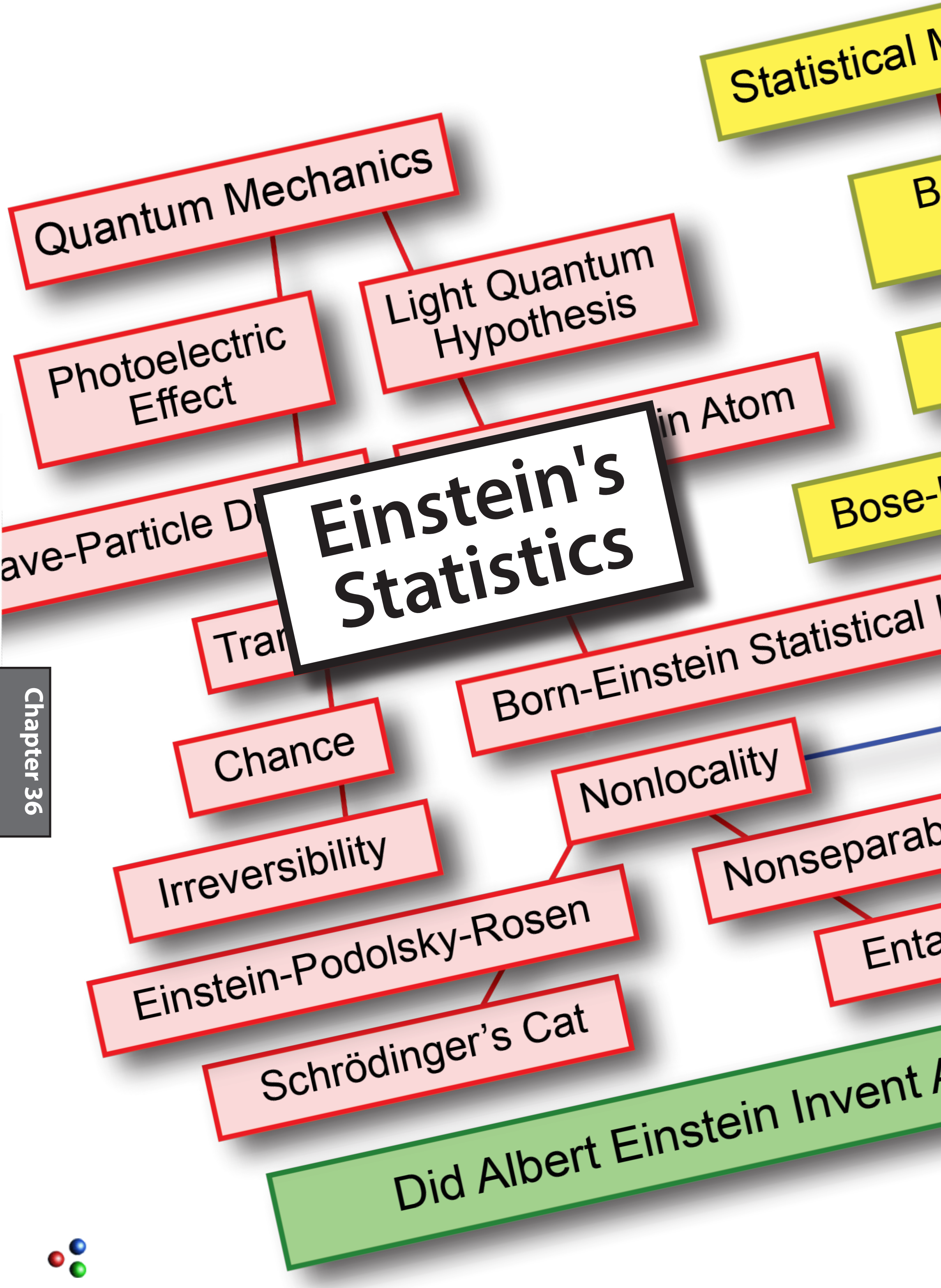
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Did Albert Einstein Invent

Einstein's Statistics

We saw in chapter 5 that Einstein rederived all of classical statistical mechanics between 1902 and 1904, going beyond the kinetic theory of gases developed by Ludwig Boltzmann in the nineteenth century. Twenty years later, Einstein discovered quantum statistics (chapter 15). All of this *before* the "founders" of quantum mechanics discovered the equations that allow us to *calculate* quantum properties to extraordinary levels of accuracy.

Einstein did not care much for the details of calculation, except to prove a fundamental theory. He oversaw the transition from classical statistics to quantum statistics. Just two years later, after Werner Heisenberg had developed matrix mechanics and Erwin Schrödinger created wave mechanics, Einstein generously allowed his friend Max Born to take full credit for the "statistical interpretation" of quantum mechanics, which Einstein had seen qualitatively at least a decade earlier (chapter 20).

To be sure, Born identified Einstein's *qualitative* probability with the calculated squared modulus of Schrödinger's wave function $|\psi|^2$. This made the statistical interpretation *quantitative*.

As we have seen so well, Einstein was very unhappy about the ontological implications of the statistics he discovered. He said many times to Born over the next few decades, "God does not play dice," But over those decades Born never noticed that Einstein had embraced *indeterminism* in quantum mechanics . Einstein's criticisms were mostly directed to *nonlocality* (chapter 23).

In his early work on statistical mechanics, Einstein showed that small *fluctuations* in the motions of gas particles are constantly leading to departures from equilibrium. It is like the departures from the smooth analytic bell curve for any finite number of events. The entropy does not rise smoothly to a maximum and then stay there indefinitely. It fluctuates randomly. The second law is not absolute.

The second law of thermodynamics is unique among the laws of physics because of its *irreversible* behavior. Heat flows from hot into cold places until they come to the same equilibrium temperature. The one-directional nature of *macroscopic* thermodynamics (with its gross “phenomenological” variables temperature, energy, entropy) is in fundamental conflict with the assumption that *microscopic* collisions between molecules, whether fast-moving or slow, are governed by dynamical, deterministic laws that are time-reversible. But is this not correct.

At the atomic and molecular level, if collisions were time reversible, there would be no arrow of time, but we see that Einstein’s 1916 work on transition probabilities for emission and absorption of radiation shows that particle collisions are not reversible when the interaction with radiation is taken into account (chapter 12).

Statistical “laws” grow out of examples in which there are very large numbers of entities. Large numbers make it impractical to know much about the individuals, but we can say a lot about *averages* and the probable distribution of values around the averages. This is the "quantum-to-classical transition."

Boltzmann's Principle

Einstein's work in statistical mechanics was grounded in Boltzmann's relationship between entropy and probability.

$$S = k \ln W$$

The entropy S is the logarithm of the number of ways W the particles can be arranged in the available phase-space cells, multiplied by a constant k that Max Planck called Boltzmann's constant. Boltzmann knew this relationship, but wrote the constant as the universal gas constant R divided by the number N of particles in one molecular weight of a gas.

If there is only one way that a given macroscopic system can be arranged in phase space, then $W = 1$, $\ln W = 0$, and entropy is the absolute minimum, $S = 0$.

Quantum Mechanics a Statistical Theory

Einstein and the "founders" of quantum mechanics engaged in fruitless debates for many years about the "completeness" of quantum mechanics. Quantum mechanics is incomplete because it cannot determine the exact properties of individual systems. For Einstein, that limits quantum mechanics to statistics.

> The statistical character of the present theory would then have to be a necessary consequence of the incompleteness of the description of the systems in quantum mechanics.[1]

Einstein's "objective reality" depends on the applicability of *absolute* conservation laws to individual quantum systems, even though properties of individual systems can only be studied *statistically*.

The great second law of thermodynamics is only a *statistical* law. The approach of entropy to a maximum at thermal equilibrium is always subject to statistical fluctuations

Quantum Statistics

Perhaps Einstein's most profound work in statistics was his 1924 discovery of *quantum statistics*. Prompted by a new derivation of Planck's radiation distribution law by Satyendra Nath Bose, Einstein showed that the distribution of photons differs from Boltzmann's molecular distribution by the addition of a -1 in the denominator.

Shortly after Einstein's paper, Paul Dirac showed that fermions (spin 1/2 particles) also depart from the Boltzmann distribution, by the addition of a +1 in the denominator.

No (bosons) = $(1 / (e^{h\nu / kT} - 1)$.

No (atoms/molecules) = $(1 / (e^{h\nu / kT})$.

No (fermions) = $(1 / (e^{h\nu / kT} + 1)$.

Einstein in 1905 proved material particles (atoms) exist and hypothesized that light particles exist. In 1924 he discovered bosons. His quantum statistics gave us the first examples of the two fundamental kinds of particle in the standard model of particle physics - fermions and bosons. See chapter 15.

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[1] Schilpp, 1949, p.87