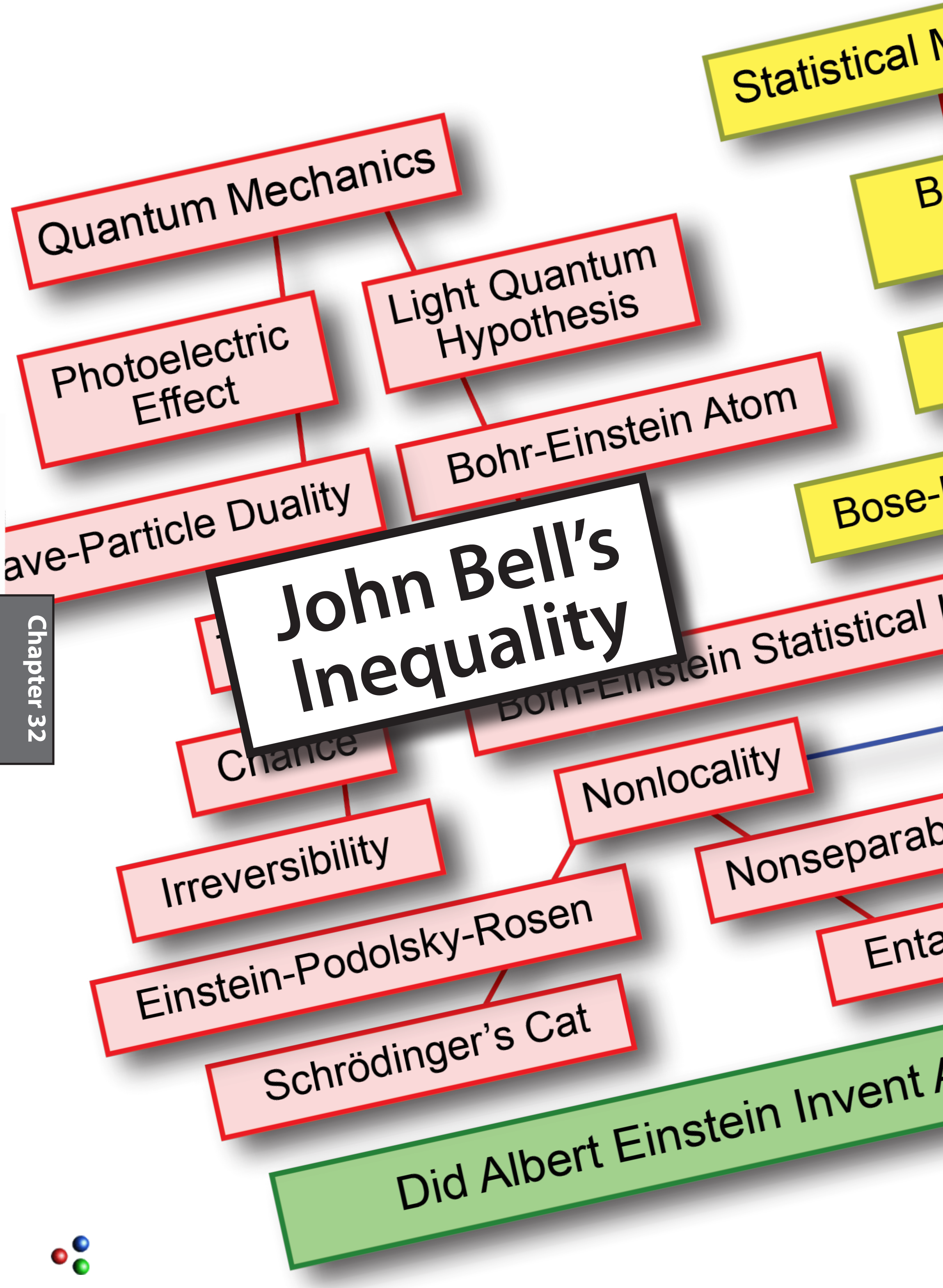
John Bell's Inequality



John Bell's Inequality

In 1964 John Bell showed how the 1935 "thought experiments" of Einstein, Podolsky, and Rosen (EPR) could be made into real experiments. He put limits on David Bohm's "hidden variables" in the form of what Bell called an "inequality," a *violation* of which would confirm standard quantum mechanics. Bell appears to have hoped that Einstein's dislike of quantum mechanics could be validated by hidden variables, returning to physical determinism.

But Bell lamented late in life...

> It just is a fact that quantum mechanical predictions and experiments, in so far as they have been done, do not agree with [my] inequality. And that's just a brutal fact of nature... that's just the fact of the situation; the Einstein program fails, that's too bad for Einstein, but should we worry about that?
>
> I cannot say that action at a distance is required in physics. But I can say that you cannot get away with no action at a distance. You cannot separate off what happens in one place and what happens in another. Somehow they have to be described and explained jointly.[1]

Bell himself came to the conclusion that *local* "hidden variables" will never be found that give the same results as quantum mechanics. This has come to be known as Bell's Theorem.

Bell concluded that all theories that reproduce the predictions of quantum mechanics will be "nonlocal." But as we saw in chapter 23, Einstein's nonlocality defined as an "action" by one particle on another in a spacelike separation ("at a distance") at speeds faster than light, simply *does not exist*. What does exist is Einstein's "impossible simultaneity" of events in a spacelike separation.

We have seen that the ideas of nonlocality and *nonseparability* were invented by Einstein, who disliked them, just as he disliked his discovery of chance. Erwin Schrödinger also disliked chance, but his wave mechanics can explain the perfect correlations of the properties of entangled particles. See chapter 29.

We explained entanglement as the consequence of "hidden constants" that are "local" in the sense that they are carried along with the moving particles, conserving all the particles' properties so they remain perfectly correlated whenever they are measured.

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[1] Transcript of CERN talk. http://www.youtube.com/watch?v=V8CCfOD1iu8

These pre-existing local constants can not explain the perfect correlation of Alice and Bob's measurements in a specific spatial direction. This we attribute to the projection of the directionless and symmetric two-particle wave function into a specific spin direction by Alice's measurement.

Experiments to test Bell's inequality have done more to prove the existence of entangled particles than any other work. As a result, many people credit Bell with the very idea of entanglement. Our efforts to restore credit to Einstein for this and most other exotic effects in quantum mechanics is therefore not an easy task.

This is particularly difficult because Einstein did not like much of what he was first person to see - single-particle nonlocality, two-particle nonseparability, and other fundamental elements of quantum mechanics, notably its statistical nature, indeterminism, and ontological chance.

We saw in chapter 30 that David Bohm developed a version of quantum theory that would restore determinism to quantum mechanics as well as explaining nonlocality. This was the beginning of a trend among young physicists to question the *foundations* of quantum mechanics. No one was more supportive of this trend than Bell, though he warned all his younger colleagues that questioning the "orthodox" Copenhagen Interpretation could compromise their academic advancement.

We have chosen Bohm, Hugh Everett, Bell, and the decoherence theorists as the leading members of the effort to challenge "standard" quantum mechanics, although there are several others. Ironically, they all base their work on trying to support Einstein's criticisms of quantum mechanics, especially his early hopes for restoring determinism, whereas Einstein in his later life had moved on to his worries about nonlocality violating relativity.

From his earliest work, Bell followed Bohm's deterministic and nonlocal alternative to standard quantum mechanics. He also followed Schrödinger's denial of quantum jumps and even the existence of particles. Decoherence theorists agree on this denial of Dirac's *projection postulate*. Like Schrödinger, they use a misinterpretation of Dirac's *principle of superposition, viz.*, that particles can be in multiple states at the same time.

Bell's Theorem

In his classic 1964 paper "On the Einstein-Podolsky-Rosen Paradox," Bell made the case for *nonlocality*.

> The paradox of Einstein, Podolsky and Rosen was advanced as an argument that quantum mechanics could not be a complete theory but should be supplemented by additional variables. These additional variables were to restore to the theory causality and locality. In this note that idea will be formulated mathematically and shown to be incompatible with the statistical predictions of quantum mechanics. It is the requirement of locality, or more precisely that the result of a measurement on one system be unaffected by operations on a distant system with which it has interacted in the past, that creates the essential difficulty. There have been attempts [by von Neumann] to show that even without such a separability or locality requirement no 'hidden variable' interpretation of quantum mechanics is possible. These attempts have been examined [by Bell] elsewhere and found wanting. Moreover, a hidden variable interpretation of elementary quantum theory has been explicitly constructed [by Bohm]. That particular interpretation has indeed a gross non-local structure. This is characteristic, according to the result to be proved here, of any such theory which reproduces exactly the quantum mechanical predictions.
>
> With the example advocated by Bohm and Aharonov, the EPR argument is the following. Consider a pair of spin one-half particles formed somehow in the singlet spin state and moving freely in opposite directions. Measurements can be made, say by Stern-Gerlach magnets, on selected components of the spins σ_1 and σ_2. If measurement of the component $\sigma_1 \bullet a$, where a is some unit vector, yields the value + 1 then, according to quantum mechanics, measurement of $\sigma_2 \bullet a$ must yield the value — 1 and vice versa. Now we make the hypothesis, and it seems one at least worth considering, that if the two measurements are made at places remote from one another the orientation of one magnet does not influence the result obtained with the other.
>
> Since we can predict in advance the result of measuring any chosen component of σ_2, by previously measuring the same component of σ_1, it follows that the result of any such measurement must actually be predetermined. Since the initial

> quantum mechanical wave function does not determine the result of an individual measurement, this predetermination implies the possibility of a more complete specification of the state.[2]

"pre-determination" is too strong a term. The "previous" measurement just "determines" the later measurement.

As we showed in chapter 29, there are in fact many properties that are determined at the initial entanglement and are conserved from that moment to the measurement of σ1 • a. We call them "hidden constants." They are *local* quantities that travel with the particles.

Experimental Tests of Bell's Inequality

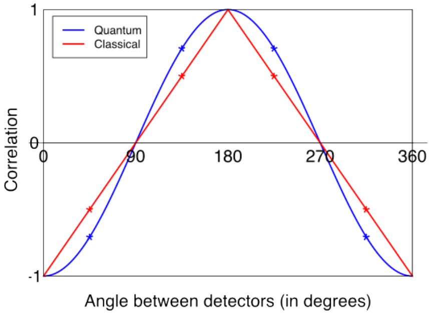
Bell experiments are usually described as the distant measurements of electron spins or photon polarizations by Alice and Bob, when their polarization or spin detectors are set at different angles.

Electrons in an entangled "singlet" spin state have spins in opposite directions. As Bell said above, when measured at the same angle (0°), spins are anti-correlated. The correlation is -1. If measured in opposite directions (180°) , the correlation is +1.

Measurements at 90° are completely uncorrelated. With photons, a vertically polarized photon will be completely absorbed by a horizontal polarizer.

Measurements will be decorrelated randomly at a small angle from 0°, say 1°. Since Bell assumes (with no physical reason) that measurements at 1° more (now 2°) are statistically independent of those in the first 1° angle, they should be no more than twice the decorrelation of the first 1° angle. Bell therefore predicts that the correlations at other angles will yield a straight-line relationship.

But it is well known that when polarizers are rotated, the correlations fall off as the cosine (amplitude) or cosine^2 (intensity). Measuring the components of spins or polarization at intermediate angles shows a "violation" of what Bell called his inequality. Instead of his



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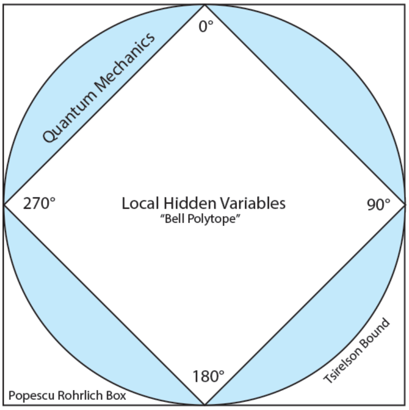
[2] Bell, 1964, p.195

physically unrealistic straight-line correlation for hidden variables, we see the quantum results tracing out a sinusoid.

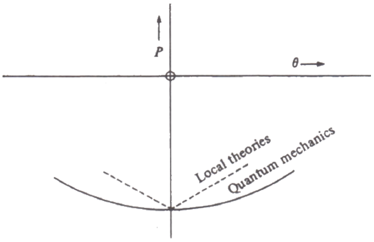
The most important intermediate angle, where the deviation from Bell's straight line is the greatest, is 22.5°.

At that angle, one-quarter of the way to 90° where the correlation will be 0, Bell's hidden variables prediction is a correlation of only 75%. The quantum physics correlation is $\cos^2(22.5°) = 85\%$.



We can display the above curves inside a unit square of possible correlations, with an inside square of Bell's local hidden variables, and then the circular region of quantum mechanics correlations, which are the same as Bell's at the corners, but move out to the circle at intermediate angles.

In 1976, Bell knew very well that the behavior of his local hidden variables at the corners has a physically unrealistic sharp "kink."[3] He said unlike the quantum correlation, which is a smooth curve stationary in θ at $\theta = 0$, the hidden variable correlation must have a kink there. He illustrated the unrealistic "kink."



What is the origin of this kink? It is buried in Bell's assumptions about his "hidden variables," that they are random, hidden in pre-existing conditions at the start of the experiment, and they can predict all the outcomes. Bell assumed that the variables can be specified completely by means of parameters λ, where λ has a "uniform probability distribution"[4] over angles, It is this uniform distribution that leads to his unrealistic straight line prediction.

Bell's inequality for hidden variables is not based on physics as much as his assumed distribution of probabilities. By contrast, there are good physical reasons to think that we can *visualize* the

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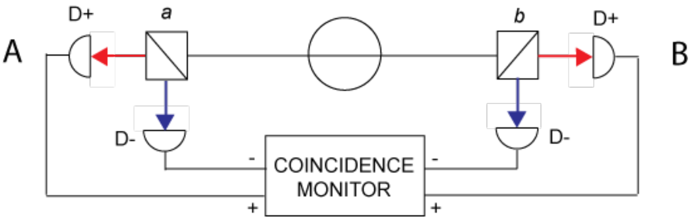
[3] Bell, 1987, p.85

[4] Bell, 1964, p.196.

angular dependence of correlations by recalling Paul Dirac's work with polarizers crossed at various angles (chapter 19). When Bob measures at the same angle as Alice, or even at angles 180° apart, the polarized light will pass straight through (a non-destructive measurement of the first kind). As we turn one polarizer away from the parallel or anti-parallel angles, some of the light is absorbed in the polarizer, but not very much at first, then falling off more quickly as we approach 90° where all the light is absorbed, There is no "kink" at 0° or 180°.

The earliest measurements were done in the hope of finding hidden variables and showing quantum physics to be "incomplete." As early as 1969 John Clauser, Michael Horne, Abner Shimony, and Richard Holt had shown Bell's hidden variable prediction had been violated and quantum physics was validated.

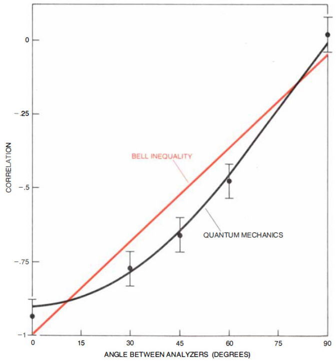
Here is the apparatus for the classic CHSH experiment.[5]



The coincidence monitor accumulates N++, N+-, N-+, and N--. As B's polarizer turns away from parallel, where perfect correlation is say, | + - > or | - + >, we start to get randomness that produces results like | + + > or | - - >. At 22.5°, Bell's straight-line hidden variables predicts 75% of measurements will be correlated + - or - +, the other 25% a random mixture of + +, - -, + -, - +.

Here are some experimental results using protons in a singlet state that confirm the 85% correlation predicted for quantum mechanics.[6]

In particular, note the confirmation of the curved sinusoidal (or cosine) shape and not Bell's physically



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[5] Clauser et al. 1969

[6] d'Espagnat, 1979, p.174

unrealistic set of straight lines with sharp kinks at the corners that Bell's inequality predicts.

With quantum mechanics confirmed, why didn't Bell and his many supporters simply give up the search for hidden variables that he claimed could validate Einstein? How can Bell inequality tests still be considered important after so many years of success? It is probably the continued dissatisfaction with quantum mechanics

As early as 1970, Eugene Wigner, who became a lifelong supporter of attempts to provide new foundations for quantum mechanics, had clearly explained what the results would be of a Bell inequality test, well before the CHSH results were published.

> Bell does introduce, however, the postulate that the hidden variables determine the spin component of the first particle in any of the $\boldsymbol{\omega}$ directions and that this component is independent of the direction in which the spin component of the second particle is measured. Conversely, the values of the hidden variables also determine the spin component of the second particle in any of the three directions $\boldsymbol{\omega}_i$, $\boldsymbol{\omega}_2$, $\boldsymbol{\omega}3$, and this component is independent of the direction in which the component of the spin of the first particle is measured. These assumptions are very natural since the two particles may be well separated spatially so that the apparatus measuring the spin of one of them will not influence the measurement carried out on the other. Bell calls, therefore, the assumption just introduced the locality assumption...

Wigner says that the angular dependence of correlations

> can be derived also by observing that the singlet state is spherically symmetric so that the total probability of the first particle's spin being in the direction $\boldsymbol{\omega}_i$ (rather than the opposite direction) is ½|. If the measurement of the first particle's $\boldsymbol{\omega}_i$ component gives a positive result, the measurement of this component of the second particle necessarily gives a negative result. Hence, the measurement of the spin of this particle in the $\boldsymbol{\omega}_2$ direction gives a positive result with the probability $\cos^{2}½\theta$, where θ is the angle between the — $\boldsymbol{\omega}_i$ and the $\boldsymbol{\omega}_2$ direction.[7]

John Bell surely knew enough physics to recognize that his straight line "inequality" would never be found and that the sinusoidal correlations of quantum mechanics would be confirmed. Yet he encouraged young experimenters to try, in the vain hopes that they would overturn quantum mechanics and become world famous.

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[7] Wigner, 1970, p.1007

As it turned out. they (and so Bell) did become world famous, not for disproving quantum mechanics, but for discovering the kind of nonlocality and nonseparability that Einstein had seen and feared.

Experimenters noted the low quality of the results and significant sources of errors in older laboratory technology, which might contain “loopholes” that would allow “Einstein’s” hidden variables and return to determinism. Their search continued for decades, attracting vast amounts of publicity for the "age of entanglement."

Most all the loopholes have now been closed, but there is one loophole that can never be closed because of its metaphysical/philosophical nature. That is the “(pre-)determinism loophole.” Bell called it “superdeterminism.

If every event occurs for reasons that were established at the beginning of the universe, then the experimenters lack any free will or “free choice” and all their experimental results are meaningless.

Bell’s Superdeterminism

During a mid-1980’s interview by BBC Radio 3 organized by P. C. W. Davies and J. R. Brown, Bell proposed the fanciful idea of “superdeterminism” that could explain the correlation of results in two-particle experiments without the need for faster-than-light signaling. The two measurements by Alice and Bob need only have been pre-determined by causes reaching both experiments from an earlier time.

> Davies: I was going to ask whether it is still possible to maintain, in the light of experimental experience, the idea of a deterministic universe?
>
> Bell: You know, one of the ways of understanding this business is to say that the world is super-deterministic. That not only is inanimate nature deterministic, but we, the experimenters who imagine we can choose to do one experiment rather than another, are also determined. If so, the difficulty which this experimental result creates disappears.
>
> Davies: Free will is an illusion - that gets us out of the crisis, does it?
>
> Bell: That’s correct. In the analysis it is assumed that free will is genuine, and as a result of that one finds that the intervention

> of the experimenter at one point has to have consequences at a remote point, in a way that influences restricted by the finite velocity of light would not permit. If the experimenter is not free to make this intervention, if that also is determined in advance, the difficulty disappears.[8]

Bell’s superdeterminism would deny the important “free choice” of the experimenter (originally suggested by Niels Bohr and Werner Heisenberg) and later explored by John Conway and Simon Kochen. Conway and Kochen claim that the experimenters’ free choice requires that electrons themselves must have free will, something they call their “Free Will Theorem.”

Following Bell’s ideas, Nicholas Gisin and Antoine Suarez argue that something might be coming from “outside space and time” to correlate results in their own experimental tests of Bell’s Theorem. Roger Penrose and Stuart Hameroff have proposed causes coming “backward in time” to achieve the perfect EPR correlations, as has philosopher Huw Price.

In his 1996 book, *Time’s Arrow and Archimedes’ Point*, Price proposes an Archimedean point “outside space and time” as a solution to the problem of nonlocality in the Bell experiments in the form of an “advanced action.”[9]

Rather than a “superdeterministic” common cause coming from “outside space and time” (as proposed by Bell, Gisin, Suarez, and others), Price argues that there might be a cause coming backwards in time from some interaction in the future. Penrose and Hameroff have also promoted this idea of “backward causation,” sending information backward in time in Benjamin Libet’s experiments and in the EPR experiments.

John Cramer’s Transactional Interpretation of quantum mechanics and other Time-Symmetric Interpretations like that of Yakir Aharonov and K. B Wharton also search for Archimedean points “outside space and time.”

All these wild ideas designed to return physical determinism are in many ways as extravagant as Hugh Everetts "many worlds."

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[8] *The Ghost in the Atom*, P.C.W. Davies and J. Brown, ch.3, p.47

[9] Price, 1997

Bell's Preferred Frame

A little later in the same BBC interview, Bell suggested that a *preferred* frame of reference might explain nonseparability and entanglement. And there is something valuabe in this picture.

> [Davies] Bell's inequality is, as I understand it, rooted in two assumptions: the first is what we might call objective reality - the reality of the external world, independent of our observations; the second is locality, or non-separability, or no faster-than-light signalling. Now, Aspect's experiment appears to indicate that one of these two has to go. Which of the two would you like to hang on to?
>
> [Bell] Well, you see, I don't really know. For me it's not something where I have a solution to sell! For me it's a dilemma. I think it's a deep dilemma, and the resolution of it will not be trivial; it will require a substantial change in the way we look at things. But I would say that the cheapest resolution is something like going back to relativity as it was before Einstein, when people like Lorentz and Poincare thought that there was an aether - a preferred frame of reference - but that our measuring instruments were distorted by motion in such a way that we could not detect motion through the aether. Now, in that way you can imagine that there is a preferred frame of reference, and in this preferred frame of reference things do go faster than light. But then in other frames of reference when they seem to go not only faster than light but backwards in time, that is an optical illusion.[10]

The standard explanation of entangled particles usually begins with an observer A, often called Alice, and a distant observer B, known as Bob. Between them is a source of two entangled particles. The two-particle wave function describing the indistinguishable particles cannot be separated into a product of two single-particle wave functions, at least until the wave function is measured..

The problem of faster-than-light signaling arises when Alice is said to measure particle A and then puzzle over how Bob's (later) measurements of particle B can be perfectly correlated, when there is not enough time for any "influence" to travel from A to B.

Now as John Bell knew very well, there are frames of reference moving with respect to the laboratory frame of the two observers in

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[10] Ghost in the Atom, ch.3, p.48-9

which the time order of the events can be reversed. In some moving frames Alice measures first, but in others Bob measures first.

Back in the 1960's, C. W. Rietdijk and Hilary Putnam considered observers A and B in a "spacelike" separation and moving at high speed with respect to one another. Roger Penrose developed a similar argument in his book *The Emperor's New Mind*. He called it the Andromeda Paradox.[11]

If there is a preferred or "special" frame of reference, surely it is the one in which the origin of the two entangled particles is at rest. Assuming that Alice and Bob are also at rest in this special frame and equidistant from the origin, we arrived in chapter 29 at the simple picture in which any measurement that causes the two-particle wave function Ψ_{12} to collapse makes both particles appear simultaneously at determinate places (just what is needed to conserve energy, momentum, angular momentum, and spin).

Bell became world-famous as the major proponent of quantum entanglement, understood as the instantaneous transmission of a signal between quantum systems, however far apart.

> In a theory in which parameters are added to quantum mechanics to determine the results of individual measurements, without changing the statistical predictions, there must be a mechanism whereby the setting of one measuring device can influence the reading of another instrument, however remote. Moreover, the signal involved must propagate instantaneously, so that such a theory could not be Lorentz invariant.[12]

Einstein would surely have rejected this argument, as he had rudely dismissed that of David Bohm, because it violates relativity with an "impossible simultaneity." Bell's continued defense of hidden variables was motivated in part by his objections to John von Neumann's "proof" that hidden variables are "impossible." He was also a critic of von Neumann's theory of measurement, especially the "collapse" in von Neumann's "process 1" and the need for a "conscious observer."

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[11] Penrose, 1989, p.303

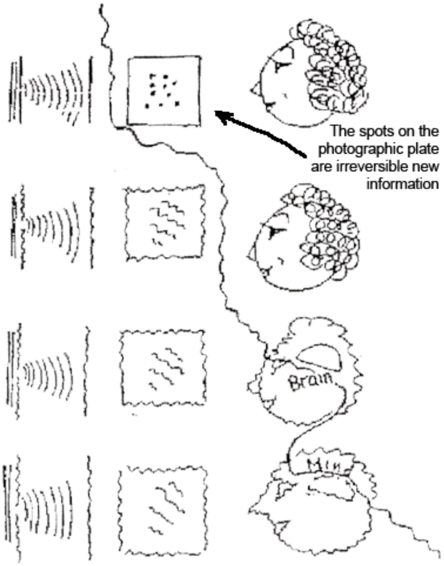
[12] Bell, 1964, p.199

As we saw in chapter 25, von Neumann developed WERNER HEISENBERG's idea that the collapse of the wave function requires a "cut" (*Schnitt* in German) between the microscopic quantum system and the observer. Von Neumann said it did not matter where this cut was placed along the "psycho-physical" path between the experiment, the observer's eye, and the observer's mind, because the mathematics would produce the same experimental results. Bell called this a "shifty split."

Bell's "Shifty Split"

We can identify Bell's "shifty split" with the "moment" at which the boundary between the quantum and classical worlds occurs. It is the moment that *irreversible* observable *information* enters the universe.

In Bell's drawing of possible locations for his "shifty split" we can identify the correct moment - when irreversible new information appears, independent of an observer's mind.



The spots on the photographic plate are irreversible new information

In our information solution to the problem of measurement, the timing and location of Bell's "shifty split" (the "cut" or "Schnitt" of Heisenberg and von Neumann) are identified with the interaction between quantum system and classical apparatus that leaves the apparatus in an *irreversible* stable state providing information to the observer.

As Bell should have seen, it is therefore not a "measurement" by a conscious observer that is needed to "collapse" wave functions. It is the irreversible interaction of the quantum system with another system, whether quantum or approximately classical. The interaction must be one that changes the information about the system. And that means a local entropy decrease and overall entropy increase to make the information stable enough to be observed by an experimenter and therefore be a measurement.

We can identify the “cut” as the moment information is recorded in the universe, and so available to an observer. In Bell's diagram, it is the appearance of spots on the photogra[phic plate or CCD.

Are There Quantum Jumps?

In 1987, Bell contributed an article to a centenary volume for Erwin Schrödinger entitled “Are There Quantum Jumps?” Schrödinger had always denied such jumps or any collapses of the wave function. Bell’s title was inspired by two articles with the same title by Schrödinger in 1952 (Part I, Part II).[13]

Just a year before Bell’s death in 1990, physicists assembled for a conference on “62 Years of Uncertainty” (referring to WERNER HEISENBERG’s 1927 principle of indeterminacy).

John Bell’s contribution to the conference was an article called “Against Measurement.” In it he attacked the statistical interpretation of quantum mechanics.

> In the beginning, Schrödinger tried to interpret his wavefunction as giving somehow the density of the stuff of which the world is made. He tried to think of an electron as represented by a wavepacket — a wave-function appreciably different from zero only over a small region in space. The extension of that region he thought of as the actual size of the electron — his electron was a bit fuzzy. At first he thought that small wavepackets, evolving according to the Schrödinger equation, would remain small. But that was wrong. Wavepackets diffuse, and with the passage of time become indefinitely extended, according to the Schrödinger equation. But however far the wavefunction has extended, the reaction of a detector to an electron remains spotty. So Schrödinger’s ‘realistic’ interpretation of his wavefunction did not survive.[14]
>
> Then came the Born interpretation. The wavefunction gives not the density of stuff, but gives rather (on squaring its modulus) the density of probability. Probability of what exactly? Not of the electron being there, but of the electron being found there, if its position is ‘measured.’
>
> Why this aversion to ‘being’ and insistence on ‘finding’? The founding fathers were unable to form a clear picture of things

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[13] Schrödinger, 1952

[14] Miller, 2012, p.29. We saw this in chapter18.

> on the remote atomic scale. They became very aware of the intervening apparatus, and of the need for a 'classical' base from which to intervene on the quantum system.

As we saw in chapter 20, It was Einstein who first interpreted the light wave as the probability of finding particles and as "guiding" the motion of particles. Once the Schrödinger wave function was invented, Max Born said that $|\psi|^2$ gives us precisely the probability of finding particles. Why did Bell dislike this powerful idea?

> In the picture of de Broglie and Bohm, every particle is attributed a position $x(t)$. Then instrument pointers — assemblies of particles have positions, and experiments have results. The dynamics is given by the world Schrödinger equation plus precise 'guiding' equations prescribing how the $x(t)$s move under the influence of Ψ.

In the Bohmian mechanics picture, particles are traveling along distinct paths. Einstein's "objective reality" is a similar view. If the particles are conserving "constants of the motion," they correlate properties in Bell experiments without nonlocal "hidden variables."

We have seen how the "guiding" wave function produces perfectly correlated spin directions for Alice and Bob measurements,.in chapter 29. How it can guide individual particles to produce the statistical interference patterns in the two-slit experiment we will explain in the next chapter.

On the 22nd of January 1990, Bell gave a talk at CERN in Geneva summarizing the situation with his inequalities. He gives three reasons for not worrying.

- Nonlocality is unavoidable, even if it looks like "action at a distance." [It also looks like an "impossible simultaneity"]
- Because the events are in a spacelike separation, either one can occur before the other in some relativistic frame, so no "causal" connection can exist between them.
- No faster-than-light signals can be sent using entanglement and nonlocality.

Bell concluded:

> So as a solution of this situation, I think we cannot just say 'Oh oh, nature is not like that.' I think you must find a picture

> in which perfect correlations are natural, without implying determinism, because that leads you back to nonlocality. And also in this independence as far as our individual experiences goes, our independence of the rest of the world is also natural. So the connections have to be very subtle, and I have told you all that I know about them. Thank you.

John Bell Today

Bell is revered as a founder of the "second revolution" in quantum mechanics. He is also a major figure in the call for new "foundations of quantum mechanics." Bell's Theorem has been described as the founding result of quantum information theory.

His fame rests on the idea that there is something wrong with quantum mechanics and that Einstein's call for additional variables to "complete" quantum mechanics is part of the solution.

Einstein was bothered by the claim of the Copenhagen Interpretation that nothing can be known about an "objective reality" independent of human observers. Even more extreme was the anthropo-centered idea that human observers are creating reality, that nothing exists until we measure it.

We have seen that the "free choice" of the experimenter does indeed create aspects of physical reality, in Bell's case it is the preferred angles of Alice and Bob that are the core idea of entangled particles in a spacelike separation that acqure values instantaneously, simultaneously, appearing to violate Einstein's principle of relativity..

Einstein worried about this nonlocality from his *annus mirabilis* in 1905 to the end of his life. But Bell's "inequality,"a physically unrealistic straight-line and linear dependence of correlations between Alice and Bob as they rotate their polarizers, is nothing Einstein would ever have accepted. For Bell to call it "Einstein's program," and pronounce it a failure, is a great disservice to Einstein.

Nevertheless, it is poetic justice that Bell returns Einstein to the center of attention in "quantum physics 2.0," the second revolution.

Two entangled particles are now known as "EPR pairs," in four possible "Bell states." These pairs are also called "qubits," the fundamental unit of quantum computing and communication.